# Cross Sections

Tim Evans
29/11/16

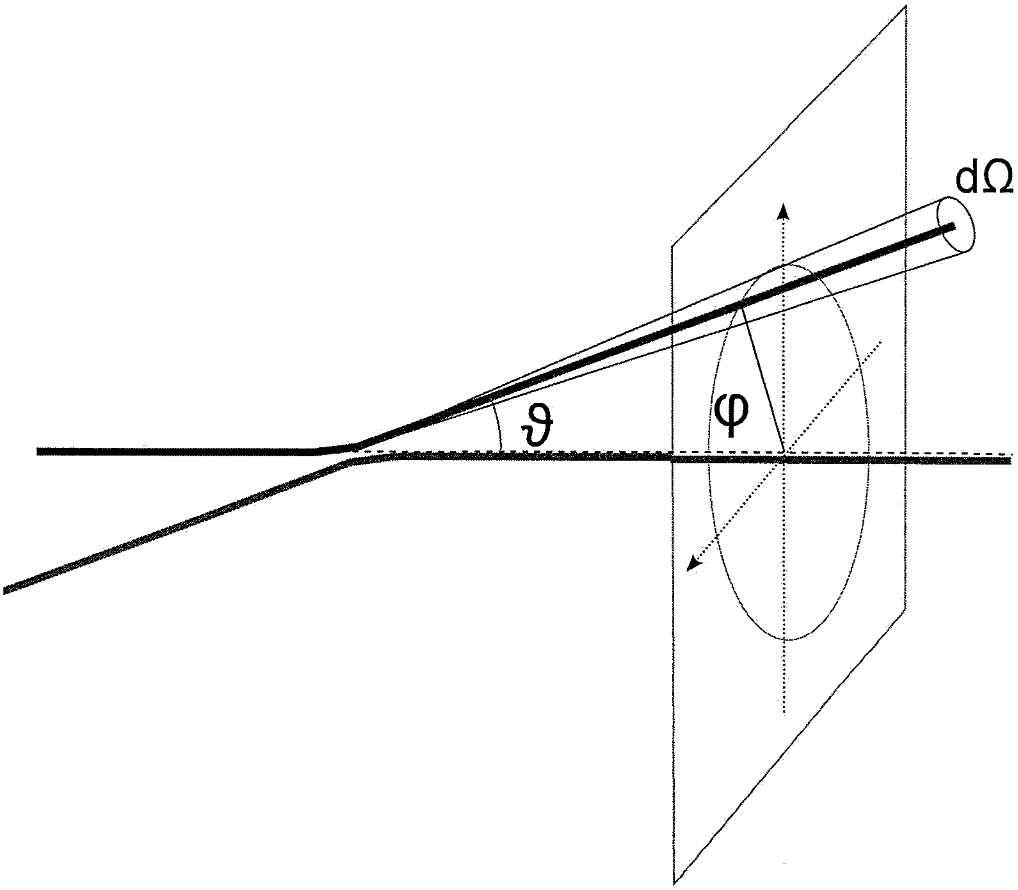
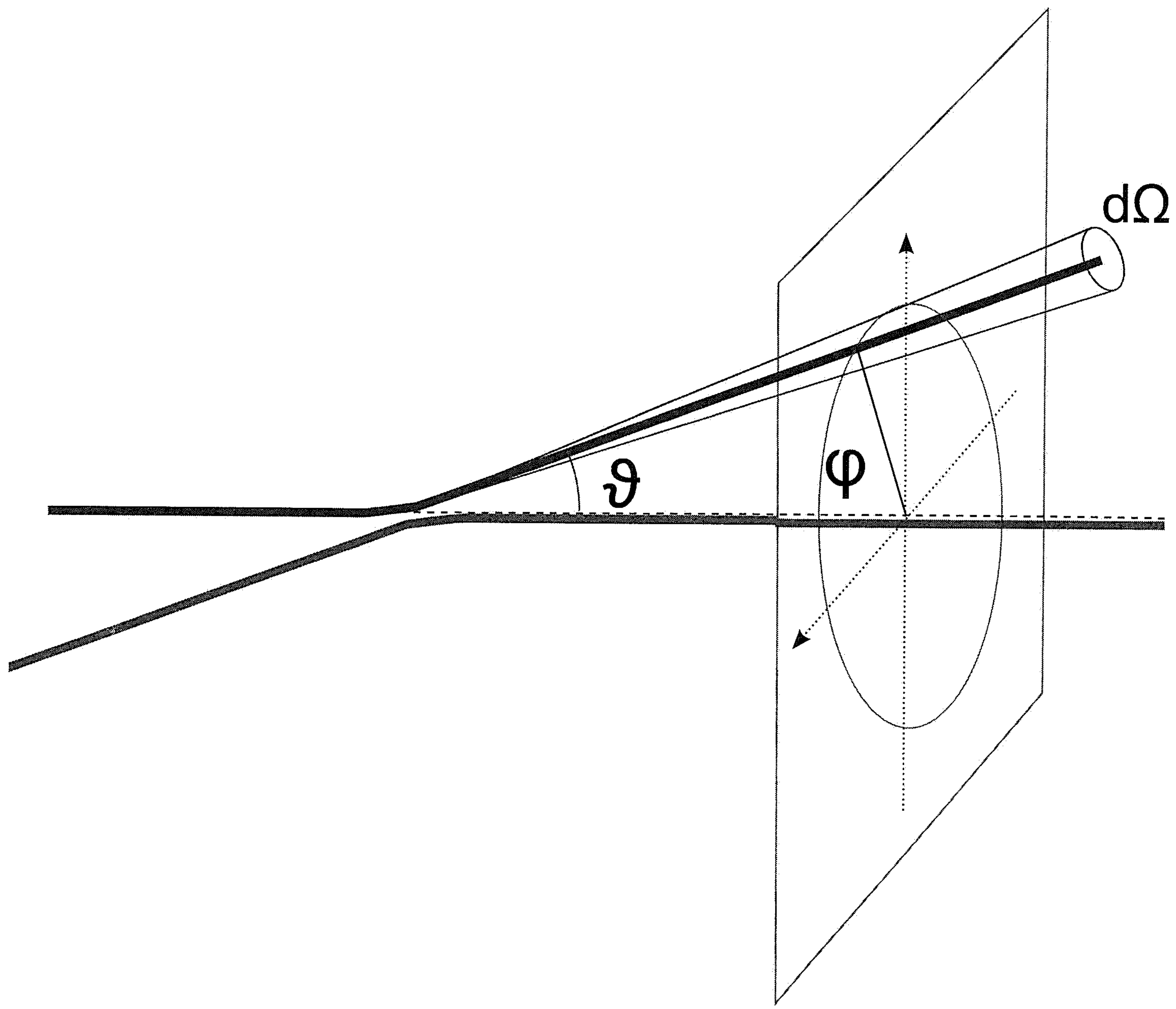


Figure 1: Figure to illustrate the definitions of the angles $\theta$ and $\phi$ used when defining the solid angle $d\Omega$, i.e. a small area on a unit sphere. The angles are measured from the point at which the interaction takes place, and are relative to the axis along which the two particles collide. The two lines represent the classical paths of two equal mass incoming particles scattering off each other. Note this is a particular frame for some observer and the coordinates are not invariant under Lorentz transformations.

Differential scattering cross section, in centre of mass frame with total energy $E_{\rm cm}$, for scattering of two equal mass particles into small area (solid angle) $d\Omega$ is $d\sigma$ where

$$d\sigma = \frac{|\mathcal{M}|^2}{64\pi^2 E_{\rm cm}} d\Omega\,. \tag{1}$$


dΩ
ϑ
φ

## CROSS SECTION σ

Collide two beams of particles (here 4 + 4)

Sometimes they miss, sometimes they collide

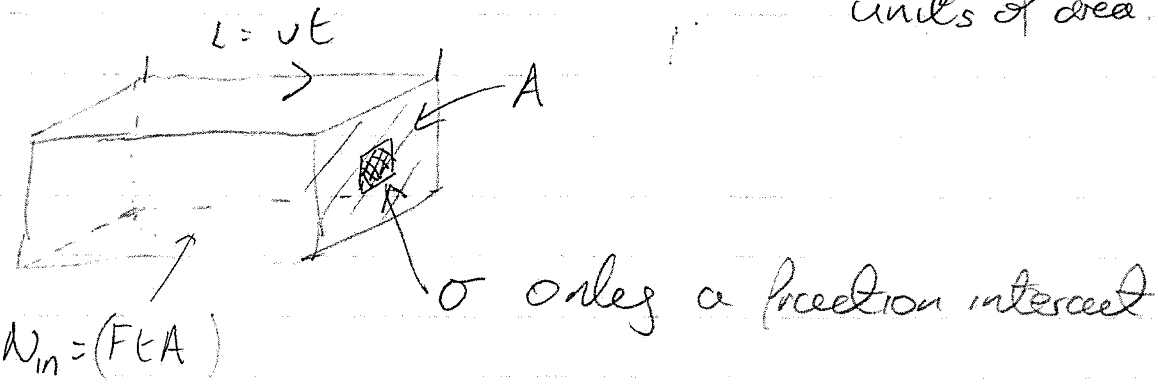
Suppose we measure $N$ scattering events

⇒ Rate of scattering = $\frac{dN}{dt}$

This should be proportional to FLUX $F$ of incoming particles, # particles per unit area per unit time

$$\frac{dN}{dt} = F \underset{\uparrow}{\sigma}$$

Definition of cross section
units of area



c.o.L24
15/12/14

In general scattered particles occur at different rates in different directions so define DIFFERENTIAL CROSSECTION $\frac{d\sigma}{d\Omega}$

as crossection $d\sigma$ in solid angle $d\Omega$ at $(\theta, \phi)$

$$\text{i.e.}\quad \sigma = \int_{\Omega} d\Omega \, \frac{d\sigma}{d\Omega} = \int_{\Omega} d\sigma$$

$$= \int_0^{\pi} d\theta \sin\theta \int_0^{2\pi} d\phi \cdot \frac{d\sigma}{d\Omega} \quad \text{if all directions detected.}$$

Long calculation involving kinematics etc. see Tong §3.6 or more details in Peskin + Schroder §4.5.

Key points

① Define $\langle f|S-\underset{\uparrow}{1}|i\rangle = i\underset{\uparrow}{\mathcal{A}_{fi}}\,\delta^4\left(\sum_i p_i - \sum_f q_f\right)$

Drop free propagation (NOT sc.)

factor out overall Energy/Momentum

$$\text{but } \frac{d\sigma}{d\Omega} \propto |\mathcal{M}|^2 = |\mathcal{A}^2|\,\delta^4(\Sigma_p - \Sigma_q)\,\underbrace{\delta^4(p^\mu=0)}_{VT=\infty!})$$

Interpret such $(\delta(p^\mu=0))$ through

$$\delta^4(p^\mu=0) = \int d^4x \, e^{-ipx}\Big|_{p^\mu=0}$$

$$= \int d^3\underline{x} \int dt$$

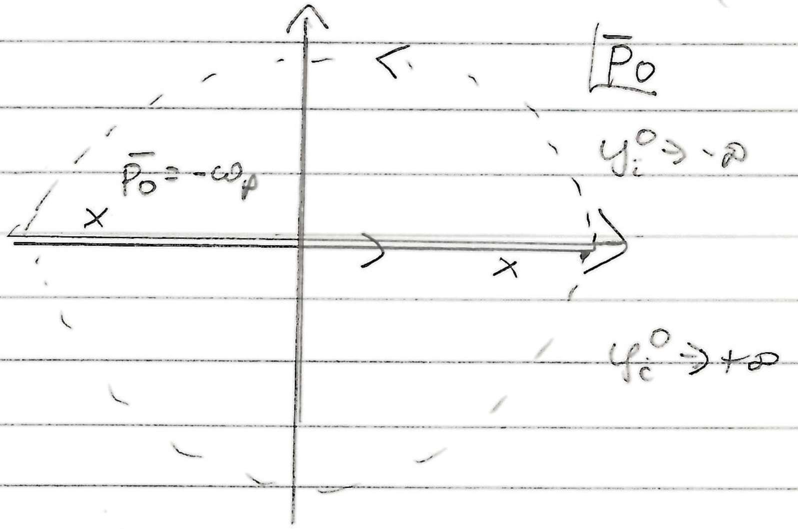
$$= \underset{\text{VOLUME,}}{V} \quad \underset{\text{TOTAL TIME}}{T}$$

* IN FACT AFTER DO KINEMATICS FIND FORMULA IS PER UNIT TIME & VOLUME

Key Point ② $\mathcal{M} \Leftrightarrow G$

$$\mathcal{M}(p,\ldots) = \int d^3 y_i\, e^{-ip_i y_i}\, 2\omega_i\, (\mathrm{Ext}_{\mathrm{rest}})\, \underbrace{G_c(y_i,\ldots)}_{\int d^4\bar{p}\, e^{-i\bar{p} y_i}\, \underbrace{G_c(\bar{p},\ldots)}_{\Delta(\bar{p})\, G_A(\bar{p}}}$$

$$= \int d\bar{p}_0\, e^{-i(\omega_p+\bar{p}^0) y_i^0}\, \delta^3(\vec{p}+\vec{\bar{p}})\, 2\omega_p \frac{i}{\bar{p}^2-\omega_p^2+i\epsilon}\, G_A(\bar{p},\ldots)\, (\mathrm{Ext}_{\mathrm{rest}})$$



$$= \underbrace{2\pi i \times \frac{1}{2\pi}\, e^{-i(0) y_i^0}\, 2\omega_p \frac{i}{-2\omega_p}}_{1}\, (\mathrm{Ext}_{\mathrm{rest}})\, G_A(\bar{p},\ldots$$

↑ Amputated Green Function

F. Rules For $\mathcal{M}$ in $p^\mu$ space :

- Exactly as before EXCEPT no $\Delta(p)$ for external legs

**Example** $\psi\psi \to \psi\psi$ crossection.

(1) Scattering Cross Section in CM (Centre of Mass) Frame for two particle scattering of equal mass particles e.g. $\psi\psi \to \psi\psi$ or $\psi\psi \to \psi\bar{\psi}$ we have (see Tong §3.6, Peskin & Schroder §4.5)

$$\frac{d\sigma}{d\Omega} = \frac{|\mathcal{A}_{fi}|^2}{64\pi^2 E_{CM}} \qquad [PS\,(4.84),\ Tong\,(3.92)]$$

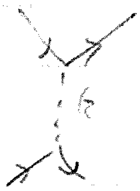
↑ Centre of Mass Energy

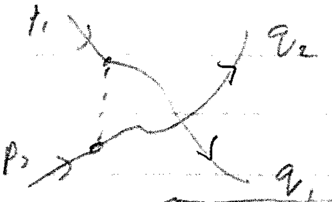
Tong (3.56½) p63

$$|\mathcal{A}_{fi}| = g^2 \left| \frac{1}{(p_1-q_1)^2 - m^2 + i\varepsilon} + \frac{1}{(p_1-q_2)^2 - m^2 + i\varepsilon} \right|$$

* $G \to \mathcal{M}$ no ext legs

* Vacuum $\Rightarrow |\Omega\rangle$





e.o. L25 9/12/16

M ↔ ?

$$\frac{E_{CM}}{2} = \Omega_p = p_i^{\mu=0} = q_f^{\mu=0}$$

$\Rightarrow$(a) $k^{\mu=0} = 0$ NO Energy for $\phi$

$\Rightarrow$(b) $|\underset{\sim}{p}_i| = |\underset{\sim}{q}_f|$

$$\Rightarrow \quad \underset{\sim}{p}_1 = -\underset{\sim}{p}_2 \ , \quad \underset{\sim}{q}_1 = -\underset{\sim}{q}_2$$

Let $\underset{\sim}{p}_1 \cdot \underset{\sim}{q}_1 = |p|^2 \cos\theta$

$$\Rightarrow |\mathcal{A}| = g^2 \left| \frac{1}{2\underset{\sim}{p}^2(1-\cos\theta) + m^2} + \frac{1}{2\underset{\sim}{p}^2(1+\cos\theta) + m^2} \right|$$

---

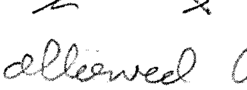
- Angular dependence
- $|\underset{\sim}{p}|^2$ dependence

} could reveal value of m BUT

CUT

20 L25 16/12/16

$\psi\bar{\psi} \to \psi\bar{\psi}$ much better as



is large when $k^2 \simeq m^2$ allowed by kinematics.

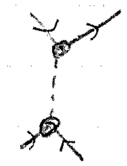
$k = p_1 + p_2$ $\quad k^2 > m^2$

⇒ (i) Angular dependence
(ii) Variation with energy $E_p \Leftrightarrow |\underline{p}|^2$
(iii) $\phi$ field mass dependence

A careful fit of data (if accurate enough!) will show if this model works
⇒ can predict mass $m$ for $\phi$ field
& predict $\psi\psi^+$ interaction mediated by $\phi$ particle

<u>Notes</u>

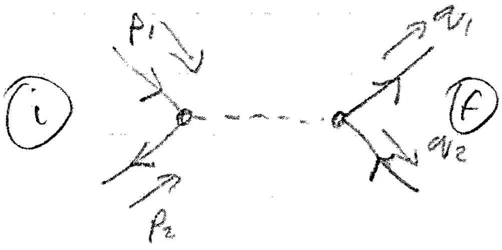
① Always need energy $\gtrsim m$

If $|\underline{p}|^2 \ll m^2$ ⇒  $\simeq \frac{(-ig)^2 \, -i}{m^2} = -i\frac{\lambda}{2}$

Find low energy behaviour equivalent to theory of $\mathcal{L}_{int} = \frac{\lambda}{4}(\psi^+\psi)(\psi^+\psi)$

CUT?

② Better to look at $\psi\bar{\psi} \to \psi\bar{\psi}$ as



$\sim \frac{i}{(p_1+p_2)^2 - m^2 + i\varepsilon} \sim \frac{i}{E_{cm}^2 - m^2}$

So $|A_{fi}| \to \infty$ as $E_{cm} \sim m^2$

In practice finite life time regulates