Fp 1

5.6 Feynman Rules in Momentum Space for G

eo23 5/12/16

Physical particles are e/states of momentum so best to work in terms of p^μ. JUST INSERT $\int d^4p\, \Delta(p)$ into F. rules

For $G(\{p\}) = \int d^4 y_i\, e^{-ip_i y_i}\, G(\{y_i\})$

Rules

0) Draw all possible distinct diagrams.

1) Each internal line ℓ is associated with

a) Momenta k_e^μ - choose to flow in one direction (but your choice)

b) $\int d^4 k_e$

c) $\Delta_F(k_e)$ for relevant field F

Follows since $\Delta_F(x_i - x_j) = \int d^4 k_e\, e^{-ik_e(x_i - x_j)}\, \Delta_F(k_e)$

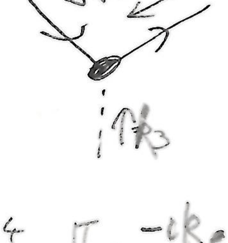
ep. L23 4/12/17

2) Each internal vertex has

a) $-ig$ (coupling constant)

b) Conserve 4-momenta at vertex

$\delta^4(\sum_e k_e)$ where k_e^μ is INTO vertex

e.g. SYTh  $\equiv -ig\, \delta^4(k_1 + k_2 + k_3)$

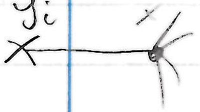
Follows since $\int d^4 x_i \prod_e e^{-ik_e(x_i - x_{other})} = \int d^4 x\, e^{-i(\sum_e k_e) x_i}$
$= \delta^4(\sum_e k_e)$

e: edges at vertex

③ External Legs/Vertices

Since $G(\{p\}) = \left(\prod_i \int d^4y_i\, e^{-ip_iy_i}\right) G(\{y_i\})$

we get a factor of



$$\Rightarrow \underbrace{\int d^4y_i\, e^{-ip_iy_i} \cdot \underbrace{\Delta_F(y_i - x)}_{\int d^4k\, e^{-ik(y_i-x)}\,\Delta_F(k)}}_{\Delta_F(p)\, e^{+ipx}}$$

NO INTEGRATION FOR EXTERNAL MOMENTUM

Goes into x vertex $\delta^4(p + k_1 + k_2 \ldots)$

$\Rightarrow$

$= \Delta_F(p)$ for $G(\{p\})$

CUT

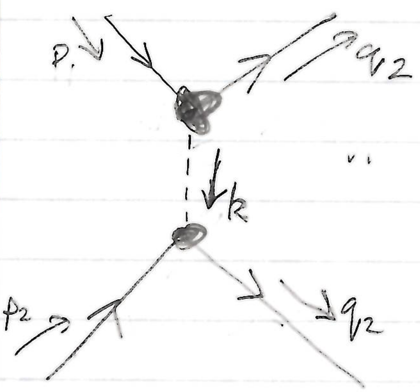
EPS **N.B.** For $\mathcal{M}$ we have $\int d^3y\, e^{-ipy}\, 2\omega(p)\, G(y, \ldots)$ factor which sets $p^0 = \omega_p$ & REMOVES $\Delta_F(p)$ on external legs

$= 1 \quad p^0 = \omega(p)$

④ Symmetry Factor

Multiply by $\frac{1}{S}$ as before.

Example $\psi\psi \Rightarrow \psi\psi$ Scattering

$A =$ 

$$= \Delta(p_1)\Delta(q_1) \times -ig\,\delta^4(p_1-k-q_1) \times \int d^4k\, \Delta(k) \times -ig\,\delta^4(p_2+k-q_2) \times \Delta(p_2)\Delta(q_1)$$

$$G_A(p_1 p_2 q_1 q_2) = \delta^4(p_1+p_2-q_1-q_2)$$ ← Overall E/p cons

$$\times \Delta(p_1)\Delta(p_2)\Delta(q_1)\Delta(q_2)$$ ← ext legs

$$\times (-ig)^2 \Delta(k)$$ ← 1PI / Truncated part.

For $\mathcal{M}$

$$\mathcal{M}_A(\psi\psi \Rightarrow \psi\psi) = \delta^4(p_1+p_2-q_1-q_2)\,(-ig)^2\Delta(k)\Big|_{p_1^{\mu=0} = \Omega(p_1)\ \text{etc}}$$

EOL24 11/12/15

N.B. Equally good for

a) $\psi(p_1)\bar\psi(q_1) \to \psi(p_2)\bar\psi(q_2)$

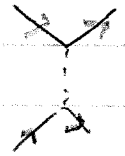
b) $\bar\psi(q_1)\bar\psi(q_2) \to \bar\psi(p_1)\bar\psi(p_2)$

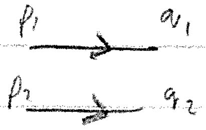
NO scattering sense of time / initial / final until project $G \to \mathcal{M}$

Loop Momenta

Connected Components/Diagrams

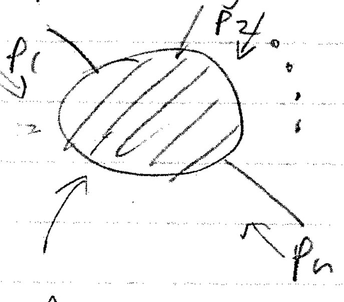
A connected diagram is one where you can find a path along edges & vertices to every other part.

e.g.  is connected $(C=1)$

e.g. 

has two connected subdiagrams (components) here (—) (—) $(C=2)$

Each connected component conserves momentum separately



$\propto \delta^4\left(\sum_{i=1}^{n} p_i\right)$

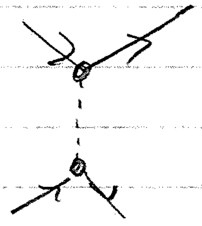
Shading = One connected component

Loop Momenta

Weinberg I (6.3.4) Number of momentum integrations $\int d^4 p_\ell$ left after δ functions at vertices accounted for is L

$$L = \underbrace{I}_{\#\text{internal lines}} - (\underbrace{V}_{\#\text{Vertices}} - \underbrace{C}_{\#\text{components}}) = I - V + C$$

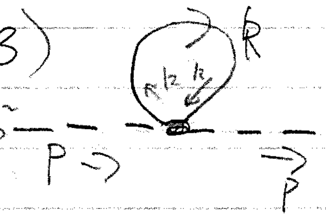
e.g. (a)



$I = 1 \quad V = 2 \quad C = 1$

$\Rightarrow L = 0$

(b) $\langle 0 | T \phi(x) \phi(y) | 0 \rangle$



$I = 1 \quad V = 1 \quad C = 1$

$\Rightarrow L = 1$

- Loops = momenta NOT constrained by δ function
- Loops generate U.V. infinities of QFT

e.g. (b) $\sim \int d^4p \, \frac{i}{p^2 - m^2 + i\varepsilon} \sim \int dp \, p \sim \Lambda^2$

for $p \sim \Lambda \gg m$