2.5 Conserved Quantities

Noether's Theorem

e.g. rotation by $\delta\theta$
infinitesimal symmetry transformation

Every generator of a continuous symmetry is associated with a conserved charge.

described by real valued param like $\theta \in \mathbb{R}$

These charges are generalisations of electric charge

e.g. Isospin in particle physics is linked to an $SU(2)$ INTERNAL symmetry of u/d quarks
Pions have 1, 0, -1 values.
Proton/Neutron have $+\frac{1}{2}$/$-\frac{1}{2}$ values.

(e.g. Translations in time $\Leftrightarrow$ energy conservation
space $\Leftrightarrow$ momentum)

Here we will focus on simplest group of $U(1)$ phase symmetries

e.o. L6?
PS2 ↑

U(1) Complex Field Symmetry

Consider $S = \int d^4x \{ (\partial_\mu \Phi^*)(\partial^\mu \Phi) - V(\Phi^* \Phi) \}$

where $\Phi(x) \in \mathbb{C}$ & V is any function
e.g. $V(z) = m^2 z$

Φ & Φ^* NOT mixed.

(GLOBAL transformation

$\Rightarrow$ Under $\Phi \to \Phi' = e^{i\theta}\Phi$ where $\partial_\mu \theta = 0$
$x \to x' = x$ so constant $\theta \in \mathbb{R}$

$\Rightarrow \partial_\mu \Phi' = \partial_\mu (e^{i\theta}\Phi) = e^{i\theta}(\partial_\mu \Phi)$

$$\Rightarrow (\partial_\mu \Phi'^*)(\partial^\mu \Phi') = e^{-i\theta}(\partial_\mu \Phi^*) \cdot e^{i\theta}(\partial^\mu \Phi)$$

$$= (\partial_\mu \Phi^*)(\partial^\mu \Phi)$$

So derivative term invariant.

Next $\Phi^* \Phi \to \Phi^{*\prime} \cdot \Phi' = e^{-i\theta}\Phi^* \cdot e^{i\theta}\Phi$

$$= \Phi^* \Phi$$

$\Rightarrow V(\Phi^{*\prime}\Phi') = V(\Phi^* \Phi)$ invariant

$\int d^4x$ unchanged

$\Rightarrow S' = S$ action invariant

i.e. if $\Phi(x)$ is a sol^n of e.o.m.
then so is $\Phi'(x) = e^{i\theta}\Phi(x)$ $\forall \theta \in \mathbb{R}$
$\downarrow$
easy to check directly!

Conserved Current -

Consider general infinitesimal changes in fields $\delta\Phi$ Provided $\partial_\mu(\delta\Phi) = \delta(\partial_\mu\Phi)$

$$\Rightarrow \delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\Phi}\cdot\delta\Phi + \frac{\partial\mathcal{L}}{\partial\Phi^*}\delta\Phi^* + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\Phi)}\delta(\partial_\mu\Phi) + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\Phi^*)}\delta(\partial_\mu\Phi^*)$$

$$= \underbrace{\left[\frac{\partial\mathcal{L}}{\partial\Phi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\Phi)}\right)\right]}_{\text{e.o.m.}}\delta\Phi + \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial\partial_\mu\Phi}\cdot\delta\Phi\right) + \text{c.c.}$$

$\Rightarrow \delta\mathcal{L} = \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial\partial_\mu\Phi}\cdot\delta\Phi\right) + \text{c.c.}$ IF $\Phi(x)$ satisfies e.o.m. = "on-shell".

Suppose change is the symmetry transformation

$$\Phi \to \Phi' = e^{i\theta}\Phi$$

$$\Rightarrow \delta\Phi = \Phi' - \Phi \simeq i\theta\Phi \quad \& \quad \partial_\mu(\delta\Phi) = \partial_\mu(\delta\Phi)$$

Also $\delta\mathcal{L} = 0$ by symmetry

$$\Rightarrow 0 = \partial_\mu\left((\partial^\mu\Phi^*)\cdot i\theta\Phi\right) + \text{c.c}$$

But θ arbitrary

$$\Rightarrow \partial_\mu J^\mu = 0 \quad \text{where}$$

$$J^\mu = i\Phi(\partial^\mu\Phi^*) - i\Phi^*(\partial^\mu\Phi)$$

EFS find J^μ with ϕ_1/ϕ_2

This is the CONSERVED 4-current (Φ "on-shell") for Φ particles + $\bar{\Phi}$ antiparticles, $\Leftrightarrow$ part.

Later we will see
Single Φ field $e^{i\theta}$ symmetry
- Φ particles $\neq$ Φ anti-particles
- # particles − # anti-particles conserved

But single $\phi \in \mathbb{R}$ (no cont. symmetry)
- ϕ particle = own anti-particle
- # ϕ particles not conserved

e.o.L6 17/10/16