2 CLASSICAL FIELD THEORY

2.1 Classical Dynamics (Recap)

a) Classical Dynamics of Particles – Hamiltonian H

H^*

$$H = H[\{\underset{\sim}{x}_i(t), p_i(t)\}] = K.E + P.E$$

For each particle $n = 1, 2, \ldots, N$ have one position $\underset{\sim}{x}_n(t)$ & one momentum $p_n(t)$ (in d-space dimensions $\in \mathbb{R}^d$)

e.o.m (Equation of Motion)

$$\frac{\partial H}{\partial \underset{\sim}{x}_n} = -\dot{p}_n, \quad \frac{\partial H}{\partial p_n} = \dot{\underset{\sim}{x}}_n$$

$2(Nd)$ equations in time

Example: SHO (Simple Harmonic Oscillator) 1D

LL, p132 (40.4)

$$H = \sum_n \left(\frac{p_n^2}{2m_n} + \frac{1}{2} m_n \omega_n^2 x_n^2 \right)$$

Density $H = \int d^d\underset{\sim}{x}\, \mathcal{H}$

EFS find $\ddot{x}_n + \omega_n^2 x_n = 0$ as e.o.m

with solution $x_n(t) = Ae^{-i\omega_n t} + Be^{+i\omega_n t}$
$= A' \cos(\omega_n t) + B' \sin(\omega_n t)$

LL = Landau & Lifschitz "Mechanics"

b) Classical Dynamics of Particles - Lagrangian L

$$L = L[\{\underset{\sim}{x}_n(t), \dot{\underset{\sim}{x}}_n(t)\}] = K.E. - P.E.$$

Now use $\dot{\underset{\sim}{x}}_n(t)$ & $\underset{\sim}{x}_n(t)$ as independent quantities needed to specify solutions

LL p3 (2.6) e.o.m. - Euler-Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\underset{\sim}{x}}_n}\right) - \frac{\partial L}{\partial \underset{\sim}{x}_n} = 0$$

($\mathcal{L}$ density only with fields)

EFS check SHO e.o.m obtained from

$$L = \sum_n \left(\frac{m}{2}\dot{\underset{\sim}{x}}_n^2 - \frac{1}{2} m \omega_n^2 \underset{\sim}{x}_n^2\right)$$

LL p131 40.2 N.B. $H(p,q,t) = \sum_n p_n \dot{q}_n - L(q_n \dot{q}_n t)$ $\quad p_n = \frac{\partial L}{\partial \dot{q}}$

LINEARITY

PS2 Q

Quadratic H or L $\Leftrightarrow$ Linear e.o.m
$O(x^2), O(p^2)$ $O(x)$ p.d.e

Linear equation means two solutions (of p.d.e.)
e.g. $\underline{x}_1(t)$ & $\underline{x}_2(t)$
can be added to give a new solution

$$\underset{\sim}{x}_{new}(t) = a\,\underset{\sim}{x}_1(t) + b\,\underset{\sim}{x}_2(t)$$

Interpretation

There is NO interaction between solutions
$\Rightarrow$ Two waves pass through each other and continue on exactly the same as before, no losses, no changes.

Find Movie?

Non-linear e.o.m. can change this
e.g. period doubling, generation of new waves of different frequencies.

Later with wave $\leftrightarrow$ particle duality will see non-linearity = interacting particles
linear e.o.m. = free or non-interacting particles.