QFT PT4.4 LT2

- I am Tim EVANS, Theory Group
- WEB PAGE: Theoretical Physics - PG Study - MSc - Info for Current-QFT
- Office Hours: 2016 Tue, Fri 10am H609

- Rapid Feedback: 2016
  Jonathan BAIRD
  EVEN WEEKS
  - Hand in level 3 office WEDS
  - Presentation Friday

  Except Week 2
  - Hand in MON
  - Presentation THURS.

  NOT for assessment, written feedback only.

  I nominate 2 or 3 Q's each week

- Problem Sheets
  - MUST DO = *
  - SHOULD DO (No mark)
  - OPTIONAL (long) #(!)

- Syllabus
  - issued at end
  - No changes expected except ~~Fermions~~

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INTRODUCTION

Natural Units

For relativistic examples we will use natural units for length, time, mass

$$c = \hbar = 1 \quad \text{where } [c] = LT^{-1}$$
$$[\hbar] = L^2MT^{-1}$$

- This means we can measure length, time, mass and hence energy ($E=mc^2$) in terms of ONE unit.
- In particle physics they use eV

Metric & Space-Time

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \text{ie. } x^\mu y_\mu = \sum_{\mu,\nu} g_{\mu\nu} x^\mu y^\nu = x_0 y_0 - x_1 y_1 - x_2 y_2 - x_3 y_3$$

We will usually assume 1+3 dimension space-time

Convention Repeated Indices = Sum

$$a_i b_i = \sum_i a_i b_i = \underline{a} \cdot \underline{b}$$

Vector

Also $\underline{\underline{A}}$

Matrix

1.1 WHY QUANTUM THEORY?

- World is NOT classical (Newton, deterministic, ...)
- On smallest scales we see QUANTUM effects
  e.g. photoelectric effect, superfluidity/conductivity
- Classical world only appears as large distance & long time averages, typically bigger than atomic
  - Compton Wave length $\lambda = \frac{\hbar}{mc}$ sets scale for Q⇔Cl transition

Why not Quantum Mechanics (QM)?

Chang ch1

MANY PARTICLES
- QM describes one particle $\int |\psi|^2 d\underset{\sim}{x} = 1$
  or a few particles,
  $[\hat{x}_{i\alpha}, \hat{p}_{j\beta}] = i\hbar\,\delta_{ij}\,\delta_{\alpha\beta}$ $\alpha = 1, 2, \ldots N$
  where x_i, p_i = position, momentum of i-th particle

Kaku §1.8

Language of FIRST QUANTISATION $\hat{x}, \hat{p}$,
WAVES $- \psi$

- QFT is QM for ∞ number of particles but is a better language & notation
  SECOND QUANTISATION $\hat{\phi}, \hat{\pi}$

Needed because:-
- Relativity allows particle production if energy $E \geq$ mass m
  ⇒ Quantum fluctuations can allow states of many (infinite) numbers of particles AND superpositions with different numbers
- Condensed matter have "holes" & massless modes leading to similar issues

QFT is more than $(QM)^\infty$

There are subtle mathematical differences between QM for FINITE # particles & QFT = QM for INFINITE # particles

These lead to STRIKING new phenomena in QFT vs QM

In particular SYMMETRY BREAKING - Higgs
= CONDENSATES - BEC cold atoms
= SUPERFLUID / SUPERCONDUCTIVITY

1.2 What is a field?

classical (function)
A field is a quantity defined at every point of space & time

$$\phi_a^{\mu}(\underline{x}, t) \in \mathbb{R} \text{ (or } \mathbb{C}\text{) if classical}$$

Labels coming from space-time symmetry - typically greek alphabet

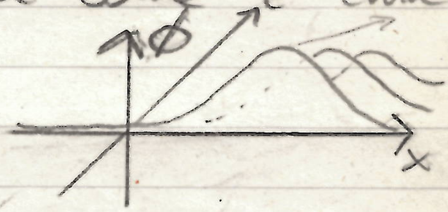
Labels coming from "Internal Symmetries" - typically Latin alphabet

Labels of space-time

We will focus on SCALAR FIELDS = SPIN 0 which don't have such labels

Barely used here but see Unification course

- A crude/poor picture is that the more stuff (energy/particles/charge) at some point the larger ϕ is
  BUT
  ϕ is NOT directly measurable
  e.g. $|\phi(\underline{x}, t)|^2 \neq$ Probability density
  It is a TOOL



Blob ≃ particle (at best)

e.g. we work with $\hat{\phi}$ an operator in QFT not a $\mathbb{R}$ valued function ϕ

Expectation values $\langle \hat{\phi} \rangle \in \mathbb{R}$ ($\mathbb{C}$) closer to classical ϕ
* There are

e.g. $A^{\mu}(\underline{x}, t)$ is the classical field used to represent photons in QED
$\underset{\sim}{E}(\underline{x}, t)$ & $\underline{B}(\underline{x}, t)$ are familiar electric & magnetic

e.g. $A^{\mu}(\underset{\sim}{x}, t)$ is four-vector potential used as the classical field for photons in QED
$\quad \shortparallel$
$(\phi, \underset{\sim}{A})$

N.B. ϕ Electrostatic pot$^{\text{e}}$ NOT QFT scalar field!

$\underset{\sim}{E}(\underset{\sim}{x}, t)$ is the Electric field
$\underset{\sim}{B}(\underset{\sim}{x}, t)$ " " Magnetic "

$$\underset{\sim}{E} = -\underset{\sim}{\nabla} A^0 - \frac{\partial}{\partial t}\underset{\sim}{A} \; ; \quad \underset{\sim}{B} = \underset{\sim}{\nabla} \wedge \underset{\sim}{A}$$

eo L1, 10/10/14
L1, 13/10/15
L1, 6/10/16