USING STATISTICAL PHYSICS TO UNDERSTAND RELATIONAL SPACE: 
A CASE STUDY FROM MEDITERRANEAN PREHISTORY 

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SYNOPSIS 

In this paper we investigate features of ‘sociophysical’ systems from an interdisciplinary perspective that combines insights from graph theory, social network analysis, statistical physics, geography and archaeology. The system on which we focus is the southern Aegean archipelago during the Bronze Age, consisting of settlement sites and their interconnections (as inferred from archaeological evidence). We treat the interactions between sites in geographical space in terms of a dynamic network; the network evolves so as to minimise the ‘costs’ of its maintenance. Sites are network vertices and their connections are network edges. By bringing together the social and the physical dimensions of this archipelago during the Bronze Age, we hope to shed new light on interaction networks more broadly. 

I. INTRODUCTION 

Spatial relationships between entities, across a range of scales, are fundamental in many of the social sciences, not least in human geography, physical geography and archaeology. However, space has received a surprisingly uneven treatment; in archaeology, for example, spatial analysis only really came to the fore in the 1960s and 70s, through the influence of the ‘New Geography’ (Haggett 1965; Chorley and Haggett 1967). David Clarke (1968, 1977), one of the principal exponents of spatial analysis in New Archaeology, described three levels of resolution in spatial archaeology: the micro level, the semi-micro or meso level and the macro level, the last of these being constituted by relationships between sites (Clarke 1977, 13). Yet despite the clear implication that these levels should articulate, many studies have tended since to aim at just one level. World-systems theory, for example, forms the basis for core-periphery models that examine macro-level spatial relationships (e.g. Schortman and Urban 1992; Peregrine 1996; McGuire 1996; Stein 1998; Chase-Dunn and Hall 1997; Kardulias 1999). 

While Clarke’s emphasis on different spatial scales has the advantage of clarity, his general approach, and indeed that of much spatial analysis of this kind, has been criticised for its overly deterministic approach to space. The idea that space has absolute geometric properties has been increasingly challenged by scholars arguing that space is a relative construct, a process that emerges out of social practices. In geography this critique already has a long history (e.g. Harvey 1973), but has been finding increasingly diverse and
influential voice (e.g. Lefebvre 1991; Harvey 1996; Thrift 1996; Hetherington 1997; Soja 1996; Murdoch 2006). This ‘spatial turn’ has also been experienced in archaeology, with approaches to space ‘relationalised’ through the influence of phenomenology in landscape studies (Bender 1993; Tilley 1994; Knapp and Ashmore 1999; Smith 2003; Blake 2002, 2004).

In both disciplines, however, and perhaps across the social sciences more broadly, the move towards relational conceptions of space and away from geometric determinism has arguably created a dualism between relational and physical space. It is our aim in this paper to develop a methodology that can go some way toward bridging the gap that has opened up between them (cf. Hillier 2005). What is required is an approach that incorporates the fundamental notion that humans create space through social practices, while also acknowledging that physical parameters are not entirely redundant in this process. One of the misconceptions hindering this rapprochement has been that spatial analysis is bound to Euclidean geometry; however, recent advances in complexity science, and in the study of complex networks in particular, give the lie to this idea (e.g. Batty 2005, on networks in geography). What these advances also allow for is the evolution of spatial dynamics from the bottom-up, in ways seemingly unimaginable to central place theory or core-periphery models. This then also fulfils the ideal of being able to link together different scales, as in the micro, meso and macro levels mentioned above in relation to the work of Clarke (1977).

While complexity science has certainly had a major influence on our approach, we believe that some of the problems with spatial analysis can actually be worked through at a more basic level. A fundamental problem is one of emphasis – in much spatial analysis, even in the more sophisticated forms of GIS, interactions between points are seen as secondary to the existence of those points. It is what Batty has described as ‘the geography of locations, not relations’ (Batty 2005, 149). The same criticism has been levelled in very similar terms by Doel, who bemoans the fact that “in geography the fundamental illusion is the autonomy and primacy of the point” (Doel 1999, 32). The equivalent to this in the archaeological analysis of regional systems is that the sites are thought to emerge and gain their character on largely local grounds, and any interactions with other communities in the region follow on from that. The connections between sites are simply drawn as lines, without weight or orientation. Such ‘site-centrism’ makes it difficult to entertain the thought that site interactions might themselves contribute to the size and status of the sites in question.

How, then, might we turn the tables, and treat interactions as primary and sites as secondary? How can we achieve what we might dub, borrowing from Batty, an ‘archaeology of relations’? This is precisely what we will now explore through our case study area, the Bronze Age Aegean. We have chosen this area partly because of the specialism of one of the authors (CK), but also because it provides an excellent example of just the kind of interactionist perspective that we seek to develop: the work of Broodbank on the Early Bronze Age Cyclades (Broodbank 2000). It is the only systematic attempt thus far, for any period of the prehistoric Aegean, to explain the growth of certain sites in terms of their interactions. This approach was perhaps encouraged by the fact that some important sites in the area and period in question - the
Cycladic islands during the Early Bronze Age (c. 3000-2000 BC) - are very hard to explain in terms of local resources, occurring on small rocky islands with limited agricultural or mineral resources. Indeed, some are only inhabited for the first time in the Late Neolithic. It thus seemed likely that relative regional location had a substantial role to play in a site’s importance.

II. FROM EBA TO MBA NETWORKS

It should be emphasised that our main thrust in this project began not with the Early Bronze Age (EBA) Cyclades, but with a larger area in a later period – the whole of the southern Aegean in the later Middle Bronze Age (MBA). This is well bounded in time as the record shows significant gaps at the boundaries of this period. Furthermore, the sail appears c. 2000 BC, which facilitates new levels of inter-regional interaction and a metamorphosis in the character of regional exchange networks.

One key question we wanted to answer was why some sites, like Knossos on Crete, grew to be so large and influential. The size of such sites is usually explained in local terms of surplus and growth, with these local conditions then enabling exchange with other sites. We were interested in reversing this equation, exploring the possibility that some characteristics of the larger interaction networks contributed to the growth of such sites (see also Rihll and Wilson 1991, in relation to the growth of the city-states of Geometric Greece, c. 800-700 BC).

Before we can do this we need to step back, and ask, ‘what are the fundamental characteristics of the EBA Cycladic network?’ and ‘how might the more complex networks of the MBA differ?’ Broodbank’s treatment of the EBA Cyclades as a network is straightforward in principle, taking sites as the vertices and their connections as the edges, transforming the Cyclades into a simple mathematical graph. He then adopts a basic technique from graph theory known as ‘Proximal Point Analysis’ (PPA), already used effectively in archaeology and anthropology for interaction studies in other archipelagos, notably in Oceania (Terrell 1977; Irwin 1983; Hage and Harary 1991, 1996). In this, edges are drawn from each hypothetical site to its three nearest neighbours in geographical space. Some sites emerge as more connected than others, with five or six edges to other sites. These sites possess greater ‘centrality’ in the network, meaning that they might be expected to have a more prominent role in regional interactions. When certain parameters such as site density are altered, simulating population increase over time, the texture of the network changes and other sites can emerge as central. When Broodbank compared the results of his PPA with the archaeological data, he found that it did indeed predict that a site on Keros, for example, would possess centrality in such a network. Of the five major Early Cycladic sites, three were ‘central’ in the PPA. Of course, Broodbank also had to suggest some motivation for these interactions – communities do not just interact without motives or goals. The EBA Cyclades are agriculturally marginal and not self-sufficient and he cited basic demographic processes and the need for

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1 See Evans 2005 for a review of basic graph theory and bibliography of exemplary applications in a variety of fields.
social storage networks (Broodbank 2000, 81-96), with power and prestige emerging consequentially out of network interactions.

Whilst inspired by Broodbank’s approach, we realised that his networks, while appropriate for the EBA Cyclades, could not be translated to the MBA Aegean for reasons that can be summarised as geographical, technological and organisational.

‘Sails’ change behavioural scales
We are familiar with the fact that changes in transport technology, which expand the distance scales over which individuals can easily travel, lead to new behavioural patterns. In our case, there appear to have been substantial changes in transport technology between the EBA and the later MBA, with the advent of the sail c. 2000 BC replacing, or supplementing, rowing technology. As a result, the distances travelled could easily increase by an order of magnitude. Even if such long trips were still not the norm, sail technology may make them just significant enough that they form the basis of important, if weak, links in the sense of Granovetter (1973; 1983). Whereas Broodbank has argued that the EBA Cyclades can form a consistent network within themselves (with some external linkages to the mainland) as in Figure 1, the innovation of the sail renders feasible MBA interaction networks spanning the whole Aegean (Figure 2), including not only the Cyclades but also the Dodecanese, Crete, and the landmasses of Asia Minor and mainland Greece.

Figure 1. The Cyclades
In the EBA Cyclades it was plausible for Broodbank to allocate equal site size in his analysis, and also equal connections, in terms of weight and directionality. When we come to the interaction networks emergent in the late MBA appropriate to the Aegean of Figure 2, a very different picture confronts us:

1. **vertices**: we know that there are sites of substantially differing sizes and roles, quite unlike the situation in the EBA. Note the assumption that large sites developed due to local internal processes (e.g. access to agricultural surplus).
2. **edges**: we can also see that there are very different kinds of links existing simultaneously, varying in orientation, length and weight.

With this scalar change the main dynamic to concern us is the emergence of ‘Minoanisation’ at the end of the MBA (see Broodbank 2004 for a recent review). In this process a number of sites across the south Aegean, on both islands and mainland, develop increasingly complex exchange links and shared cultural traits. The driving force behind this is the large island of Crete, with certain central sites, and Knossos in particular, seemingly most involved. The similarities in material culture between sites on and off Crete are so pronounced that some have been led to speak of colonisation. This interpretation is connected with the idea of a Minoan sea-empire (‘thalassocracy’). There is no direct evidence that the fleet needed to maintain such an empire actually existed; the source of the thalassocracy idea can actually be traced back to Thucydides, who was of course commenting more than 1000 years later than the period described.
Whether through direct colonisation or indirect acculturation, the Cretan palaces capitalised on their regional dominance and extended their influence beyond the island. Essentially this represents the earliest ever occurrence of state-led expansionism in the prehistoric Aegean (and by extension, Europe). Present interpretations are, however, inadequate, at many levels, not least the general tendency to explain first the growth of individual sites in local terms (good land, resources, etc), and then to extrapolate connections between sites from there. In other words, the ‘vertices’ (sites) always precede the ‘edges’ (links). There are, naturally, some exceptions to this, with Davis’ work on the ‘Western String’ route through the Cyclades linking Crete to the mainland (Davis 1979), and Berg’s assessment, using world-systems theory, of Southern Aegean interactions in the Middle to early Late Bronze Age (Berg 1999). However, these and other studies, while focussing on interactions, have tended not to use explicit network models composed of nodes and links (in these cases, the nodes are undefined).

The two key aspects of the Minoan networks briefly described above are:

1. an evolution from exchange to affiliation – initially the connections between islands involve exchange of goods, but eventually these are supplemented by actual imitation of artefact styles and technologies, suggestive of some process of cultural affiliation (Knappett and Nikolakopoulou 2005, in press). It is interesting that this latter process appears to correspond in time with the probable emergence of a single political centre on Crete – i.e. Knossos (although this is debated – see Adams 2006). This central site can be regarded as a hub, and so we intend to investigate the possible link between hubs and strong ties in networks of this kind.

2. a relatively rapid emergence and collapse. Interaction networks only endure for a mere two hundred years or so. The EBA networks are followed by a gap in occupation at many sites, if not total abandonment; some of the most important ‘vertices’, such as Chalandriani on Syros or Dhaskaleio-Kavos on Keros, are never again occupied (Broodbank 2000). This does not sound like a particularly resilient system. Furthermore, the MBA network is followed by others in the Late Bronze Age (LBA), based on the Mycenaean mainland, especially the Argolid (Mycenae). Like the Minoan networks, each only lasts a few hundred years, ending cataclysmically with the onset of the so-called ‘Dark Ages’.

With these observations in mind we shall argue that the way in which the meso-level of intra-island site activity is accommodated in the macro-level of the inter-island network as a whole plays a crucial role in how it functions. This has profound consequences for the way in which we make and understand our models.

III. INCORPORATING THE ARCHAEOLOGICAL RECORD IN NETWORK MODELS

Meso- and Macro-levels: the EBA Cyclades: Broodbank revisited
The main characteristics of the EBA Cyclades are that they are agriculturally marginal, with small populations that are not self-sufficient. Broodbank’s
model is one of exogenous evolution, in which population growth within an island leads to new communities budding off from the old, maintaining an approximately common size. See Figure 3, taken from Broodbank, in which the increasing number of vertices corresponds to an increasing population.

In Figure 3 each vertex corresponds to a definite population/unit of resource (e.g. 50 people). On attaching each vertex to its three nearest neighbours, we see that, as population increases, the islands become more self-sufficient and contacts between them become less necessary. In this model for the EBA Cyclades it is plainly the meso-level that drives the macro-level.

The Broodbank analysis brings a major question to the fore which has to be addressed in any network model; what is the role of the archaeological record? The significance of this question is that the EBA sites are not chosen by Broodbank through direct archaeological evidence, but are assigned hypothetically in a simple geometrical way, more or less equidistantly throughout each island on the basis of population estimates derived from archaeological survey data. As such, the vertices serve as a proxy for the archaeological record, but any attempts to relate them to significant sites are largely doomed to failure on two counts. First, the archaeological record is extremely patchy, and not all significant sites are identified. In fact, one of the main purposes of model-making is to help us anticipate where such sites may be. Secondly, even when we have reasonable knowledge of sites, the geometric algorithm is too rigid to replicate them, as we see from Figure 4, also taken from Broodbank, where significant EBA sites are displayed. For
this reason we shall be careful to distinguish between the ‘site’ (with a significant archaeological record) and the ‘effective site’ which is labelled by the network vertex or node.

Robustness
This obliges us to address the question of robustness. At its simplest this is a question as to whether we think the situation we are describing has generic features i.e. can be thought of as one outcome in a continuum of plausible outcomes, or whether, by its nature, it is intrinsically unique. Of course, the evolution of the Aegean, or anywhere, is unique. The question here is whether different reruns of history, with essentially the same starting conditions, could give similar structural outcomes. In model-making this is, in the first instance, a question of morphology. Two prescriptions that have the same functional form, but which differ only in quantitative details can be construed as adjacent in some abstract space of models. Models are robust if, for the same inputs, adjacent models lead to adjacent results (outputs).

We shall examine the question of morphology later, when we present an explicit model. For the moment we return to the question of the archaeological record. In this we are concerned with a separate aspect of robustness: that of robustness with regard to different initial conditions. For this, Chaos Theory has provided a cautionary tale in that, for the same model for some of the time, marginally different initial conditions can give very different outcomes. Our problem is not the same, in that we do not expect chaotic behaviour, but no less important. As will be seen, an important input to our MBA model is the carrying capacity for a site, with corresponding output the occupation index (or detrended population) for the same site. Even a small island will have several significant meso-level communities, only some
of which will correspond to archaeologically significant sites, because of the incompleteness of the record. The question is, does it matter how the population is distributed between the occupied sites on, say, a given island? If it does, then, given our lack of knowledge about them, our conclusions can depend on unknown data to an extent that the usefulness of the model is limited to very general statements. From this viewpoint, robustness can be understood as minimising the consequences of our ignorance about the meso-level of the network.

Returning to Broodbank’s algorithm, it does not look very robust in this latter sense. To take one example, let us suppose that, instead of allocating equal populations to each vertex, we allocate population in a probabilistic way, keeping only the total per island fixed. For example, a population of 200 would have different probabilities for breaking up into four communities of 50, two of 100, etc., equally defensible *ab initio*. That should be compounded with a probabilistic distribution of vertex positions on each island, and a probabilistic number of links (centred on three, say). We have not had time to implement such a programme, but we would be surprised if we ended up with the same ranking of centrality as Broodbank. This is not to disparage Broodbank’s groundbreaking work, since the importance of inter-island links is an essential ingredient in the development of intra-island communities, and vice-versa. However, in our attempts to develop more sophisticated measures of influence (e.g. frequency of interactions, cultural transmission, in-between-ness, etc.) we need robustness if we are to make progress. It is then a consequence of our results to decide whether robustness can be justified, post-hoc.

Meso- and Macro-levels: the MBA Aegean
As we have already indicated, the situation for the MBA southern Aegean differs from the EBA Cyclades. Geographically, our area of intended study of Figure 2 is very heterogeneous, with many small and some large islands, such as Crete and Rhodes, and areas of mainland. In contrast, Broodbank’s study area (Figure 1) consisted solely of islands of roughly equivalent size. In fact, if we were to lay a regular grid over the network of the EBA Cyclades with grid size that of the largest islands, we would see that it can in fact be conceived of as a lattice whose symmetry has been distorted. Although the islands are irregularly spaced, with some squares in the grid more heavily populated and others less so, it should be emphasised that edge length does not vary that strongly and very few squares are empty. This means that clustering is unlikely to be all that pronounced. Without being quantitative, we see in Figure 2 that when we increase the scale of the network to include Crete, the Dodecanese and parts of the Greek mainland and Asia Minor the topology is quite different. There are many more open squares if a lattice of the same size is overlain on this space, and much greater scope for clustering. In the earlier Cycladic cluster, one island can connect with any other through a series of relatively short hops. This is not the case for the larger network, in which some long-distance edges are unavoidable if the network is to remain fully connected.

Returning to the larger southern Aegean network, what might it take to overcome the clustering that comes with such an asymmetrical topology? Surely these clusters will remain separated unless there is a strong incentive
to connect. In a patchy resource environment there might very well be such an incentive, with interconnectivity providing a safety net against annual fluctuations in resource provision from region to region. Another parameter to consider is the expense of maintaining a long-distance edge. Presumably a large site with more resources has a greater chance of maintaining such an edge than does a small site. And this forces us to realise that if a network is indeed created over a large asymmetrical grid of this kind, then large sites are likely to feature. Furthermore, large sites searching for information about resource availability are much more likely to target other large sites in that quest. This means that there is gravitational pull that needs to be taken into account when we examine such networks – the tendency of like to seek out like. We shall see that, as a beneficial by-product, this does lead to a robust description in the sense above.

Let us do this back-to-front. We can enforce robustness by minimising our ignorance and insisting that the meso-level does not determine the macro-level for the MBA Aegean. Given our earlier discussion of the MBA period, this is not unreasonable. Then, as a consequence, we shall see the tendency for like to seek out like. To explain this further, consider two maps of the southern Aegean, one at a large enough scale to show all major (known) sites of MBA habitation, from which Figure 6 (showing Naxos and Mykonos) is a part. Known, or hypothesised, sites are listed but, by definition, unknown sites cannot be. However, the carrying capacity (overall resources available to the island’s inhabitants) can be estimated.

If the primary dynamic is the affiliation between islands and major centres, and not the detailed interactions within each island, this suggests that we can replace all the individual sites of Naxos in Figure 5 by the single effective supersite (represented by the single vertex of the large circle) of Figure 6, whose attributes are the sum of those of the individual sites. When later we attach a single coordinate to this supersite we in effect adopt a ‘centre-of-mass’ approach. This aggregation can then be applied island by island, or local region of influence by local region.
Figure 6: A supersite on Naxos: There is some ambiguity in the positioning of this vertex, when its spatial coordinates are needed later, but if a natural harbour exists this provides an obvious bias. (after Broodbank 2000, figure 109)

In the large scale this then permits us to ignore the mesoscopic details and just incorporate the supersites, one per small island, and more for Crete and the mainland. This process is often performed in physics where it is known as ‘coarse graining’.

This assumption, that the whole is equal to the sum of its parts, is not all. The assumption of affiliation being accountable as an inter-island process not only suggests that local sites can be aggregated into supersites, but the multiple links between individual sites can be replaced by superlinks between supersites (see Figure 7). That is, we would infer the same relative affiliation strengths irrespective of the scale of the map. To pursue the analogies with Newtonian mechanics further, this forces us to adopt ‘gravitational’ models, since the attribute of gravitational energy is that it is the same, whether it is calculated from the centres of mass, or from the individual constituents of those masses.²

Of course, for this to be exactly the case we need a definite power-law behaviour describing the fall-off of the linkage-‘potential’ with distance. We

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² However, we do not assign any type of inverse square law of force between sites which is another aspect of Newtonian gravity.
could not justify such a specific behaviour, and some dependence on the cost of a community breaking into two will be present. Nonetheless, on changing the scale of the map to a larger scale (coarse-graining) so that the meso-level detail becomes indistinct as it merges into a macroscopic network across the Aegean, the broad patterns of influence remain the same. Another way of saying this, in analogy with condensed matter physics, is to say that the model is approximately ‘block-renormalisation’ invariant. When we turn to model-making that incorporates both this centre-of-mass and gravitational effects, we shall see that this enthusiasm for like-to-like in establishing links has general implications, such as a tendency to instability in the networks.

Henceforth all our sites are supersites and our links are superlinks, and we shall revert to calling them ‘sites’ and ‘links’ respectively. With the positions of the vertices determined by ‘centre-of-mass’ they are not directly correlated to a single site from the archaeological record, just as the vertices of Broodbank’s networks were not, although in each case they are informed by it.

IV. MODEL-MAKING FOR THE MBA AEGEAN

While there are models within social network analysis which use graph theory in increasingly complex ways (de Nooy et al. 2005; Carrington et al. 2005), we have decided to go a step further and combine some of these insights with techniques from statistical physics. Statistical physics shows how large numbers of interacting entities often have relatively simple generic behaviour on large scales regardless of the details of their interactions. Network theory shows how specific behaviour is embedded within this. This approach can help us develop an explicit focus on the dynamics of interaction in complex networks and on the interface in such networks between local and global behaviours. Such a recourse to ‘harder science’ than that of Broodbank may at first seem reminiscent of the application of systems theory to archaeology in the late 60s and early 70s, particularly with the idea of small change in one subsystem leading to substantial change at the overall system level. However, this endeavour was hamstrung by a number of factors, not least the tendency to prescribe the character of the subsystems and their interactions mechanistically. With the new generation of network analysis it is possible to conceive of order emerging from the bottom-up, in a far more fluid and contingent manner.

To be specific, our analysis is performed on 34 site vertices in the southern Aegean, as shown in Figure 8. The overall approach we have chosen to adopt portrays the MBA network based on these vertices, with its complicated constraints and interactions, as explicable in terms of an ‘energy landscape’ through which the system moves (hence conceiving of the system as having agency or behaviour of some kind). In a contemporary context we would think of this as a ‘cost-benefit’ analysis in which the ‘energy’ or ‘benefit’ is a function $H$ of the state of the network. A system with low energy $H$ is close to some optimal solution in which all the different constraints and interactions are balanced.\(^3\)

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\(^3\) In his seminal work on spatial analysis in archaeology, Clarke identifies 4 general theories underlying most of the detailed spatial studies in archaeology that have attempted to move beyond description.
Figure 8: The Aegean with the location of the 34 sites used for initial investigations. Site 1 is Knossos, site 2 is Malia, site 10 is on Thera, and site 27 is Mycenae. The key to the remaining sites is in the table below. In the following figures all the sites are given equal weight ($S_i=1$) and distances ($d_{ij}$) are as the crow flies. Both aspects would need to be improved in a real study but these figures illustrate the concepts and are not to be taken as the basis for detailed archaeological discussions.

Table 1: Key to sites used in the Monte Carlo simulations

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Rather like the stock market, evolution has both long-term and short-term characteristics that, most simply, can be thought of as a smooth general trend upon which is superimposed volatile short-term fluctuations. Although the optimal solution is rarely if ever reached, there may exist numerous different

These are anthropological spatial theory; economic spatial theory; social physics theory; and statistical mechanics theory (Clarke 1977, 18). Considering just the last two for present purposes, Clarke notes that the physical analogies from particle physics have proved helpful in formulating models, as in gravity models for example, but that they have been conceptually unsatisfactory, able only to describe rather than explain. As for statistical mechanics theory, it “represents an interesting elaboration of the missing statistical and stochastic background behind the social physics approach” (1977, 20). Its basis is that “the most probable state of any system at a given time is the one which satisfies the known constraints and which maximises its entropy” (1977, 20).
solutions that approach the optimal. So, one of these solutions may have Knossos as a key central place. Some small changes in certain parameters might then jog the system and cause it to fall into another configuration, equally close to optimal, but in which Knossos is no longer central. Or perhaps it might transpire that Knossos is again central, but for different reasons and with a different set of connections. It is in this sense that our conclusions are statistical. This is an ideal approach for systems in quasi-equilibrium over long periods of time in which evolution is smooth. However, it can also indicate the onset of rapid change due to a shift in external circumstances. That there are dramatic ‘jogs’ to Aegean interaction systems seems quite clear – the innovation of the sail at the beginning of the MBA could be one, and the destruction caused by the Theran eruption might be another. This latter most likely corresponds to a major change in the nature of the ‘landscape’. In such circumstances, what were stable site exploitations in the valleys of this ‘landscape’ can become unstable configurations on its hills, which lead to a major readjustment in site use as the system migrates to the new ‘valleys’.

Generalities
The energy landscape that we wish to describe has two types of coordinates; site vertex variables and link edge/link variables between (different) sites (generalisations of latitude and longitude). The energy of the landscape is denoted by its altitude. To be concrete, we think of these landscapes statistically, adopting a Boltzmann-Gibbs distribution, whereby the likelihood of achieving a particular value of $H$ is

$$H = -T \log[\text{likelihood}].$$

The assumption is that the system will evolve from the unlikely ‘peaks’ to the more likely ‘valley bottoms’. Pursuing the simile further, the long-term evolution of a network can be thought of as a slow buckling of the terrain (plate tectonics). The short-term volatility, controlled by the ‘temperature’ $T$, whereby high volatility is ‘hot’, low volatility ‘cold’, corresponds to shaking it (earthquakes). The parameters that control the contours of the landscape are measures of site independence or self-sufficiency, and constraints on population size, etc. Thus, for example, as populations grow or total trade volume increases, the landscape changes, and the positions of the valleys into which the system wishes to fall changes. This is rather like Broodbank’s increase in the number of links per island as population increases. Volatility here would correspond to short periods of drought, or unexpected changes in local population.

In all models, the ‘Hamiltonian’ energy function $H$ which characterises each configuration of the system separates into four terms:

$$H = -\kappa S - \lambda E + (jP + \mu T). \quad (1)$$

In some roughly defined way, $H$ measures the 'cost' (in manpower, resources, etc.) of organising the system of sites and their trading links. The aim is to find the configuration of the network that makes $H$ as small as possible, for fixed values of $\kappa$, $\lambda$, $j$ and $\mu$. Earlier, we raised the issue of robust morphology. This
is less a problem than one might think, a partial resolution given by the notion of a universality class. By this we mean that, rather than try to prescribe ‘fuzzy’ functions to accommodate our uncertainty in $H$, we can hope for a family of ‘crisp’ functions that, provided we ask the right questions, will all give us the ‘same’ answer. The notion of topological congruence, taken from population biology, is most helpful. Functions which can be deformed into one another by stretching and squeezing are topologically congruent.

The individual terms that constitute $H$ are understood as follows:

**Sites:** $S$ only depends on the properties of the site vertices (usually islands) in isolation. As such, it is a sum of terms, one term for each site, which describes the exploitation of the site as a function of its de-trended population or occupation index (i.e. the fraction $v_i$ of its total resources that have been exploited). Each site $i$ is given a physical location, a fixed characteristic carrying capacity $S_i$ (its effective size) and a variable occupation index $v_i$ to be determined. One possible representation is that the active population at a site is $(S_i v_i)$ with $S_i$ setting the maximum self-sustainable population at a site. Small rocky islands will have small $S_i$, yet they might have a large population, $v_i >> 1$, if they play a pivotal role in the global network. We will denote the total number of sites by $N$. We have tested our model using the list of 34 known MBA sites shown in Figure 8. Initially, we assume that all sites are equally easy (or difficult) to exploit and have assigned equal relative sizes $S_i = 1$ to all. At a later stage we could adopt a systematic approach to site location, such as the cultivatable land/population density method used by Broodbank (2000), at the loss of simplicity but at no cost to the numerical work.

By itself, $v_i$ takes a minimum at some intermediate value$^4$. As a simple example of morphology, congruence here means little more than the observation that over-exploitation of resources incurs an increasingly non-linear cost, whereas under-exploitation permits growth.

**Edges:** $E$ is the edge/exchange/trade term which shows how the sites interact with one another (trade, influence) in a way that depends on both the properties of the sites and the network and weight of their interactions. Most simply, it is a sum of terms, one term for every pair of sites that is linked by trade or for other reasons. We associate an edge variable $e_{ij}$ to each link between sites $i$ and $j$. One interpretation is that $e_{ij}$ represents the trade from site $i$ to site $j$ and need not equal $e_{ji}$. We also define an effective distance $d_{ij}$ from site $i$ to site $j$ which here is just the distance between the two sites. In later work we will modify $d_{ij}$ to take account of difference between land and sea transportation, prevailing winds and currents and so forth.

**Constraints:** The final terms (in brackets) enable us to impose constraints on population size ($P$), and on total trading links (and/or journeys made) in $T$.

**Parameters:** The parameters $\kappa, \lambda, j, \mu$ that control the contours of the landscape are measures of site independence or self-sufficiency, and constraints on population size, etc. Thus, for example, as populations grow or total trade volume increases, the optimal network (lowest energy

$^4$ It is not the value of S that is important but its derivative (slope).
configuration) changes. All other things being equal, increasing $\lambda$ increases the importance of inter-site interaction, whereas increasing $\kappa$ increases the importance of single site behaviour. On the other hand, increasing $j$ effectively corresponds to reducing population, and increasing $\mu$ reduces exchange.

**Transformation Properties:** To further constrain $H$ we demand that it behaves appropriately under special transformations. One such principle is the symmetry of the form of $H$ under the interchange of labels of any two sites. That is, every site is governed by the same type of interactions as any other. This does not mean that every site is identical; we break this homogeneity when we incorporate different resources, $S_i$, and unequal distances $d_{ij}$ between sites. Finally, we impose ‘block renormalisation’. That is, if we were to split a single site into two sites close together of the same total size, then we wish the energy of the configuration to be invariant. In this way the precise determination of what was the centre of any one site should be unimportant.

**A ‘gravitational’ Hamiltonian:** The example we have proposed in our initial proof-of-concept studies that embodies the above is

$$H = -\kappa \sum_i S_i v_i (1 - v_i) - \lambda \sum_{i,j} V(d_{ij} / D) (S_i v_i) e_{ij} (S_j v_j)$$

$$+ j \sum_i S_i v_i + \mu \sum_{i,j} S_i v_i e_{ij}$$

The sums are over the different sites or over all pairs of sites, labelled by $i$ or $j$. The first term proportional to a constant $\kappa$ controls the size of sites as if there were no outside contacts. It is the logistic map as used for simple models of population dynamics. Sites have negative energy for $0 < v_i < 1$, while for values larger than 1 the cost is positive. Note this term is invariant if we split a site into two by dividing $S_i$ between the two new sites but keep the occupation fraction $v_i$ the same for both new sites – our centre-of-mass principle.

The second term allows for interactions, ‘trade’. It is proportional to the total ‘populations’ at both ends of a link $(S_i v_i)$ and to an edge weight variable $e_{ij}$. This ensures block renormalisation provided we ignore any new edge between the two new sites, since the number of possible edges involved also doubles, so that the total energy remains the same. For such models it is advantageous, in cultural exchange, or trade, for both a site and its exchange partner to have large resources. We realise that the cultural exchange/transmission that we are considering here is by no means simply economic but, in contemporary economic parlance, we would say that this model embodies the advantages of a large consumer market and producer power.

It is through the interaction term that the effects of distance are included. This is done through a ‘potential’ term that is essentially zero for long range distances and unity for short distances. Thru direct long distance interactions give virtually no benefit and are unlikely to appear in our simulations; they are
deemed to carry prohibitively high overheads. We introduce another parameter \( D \) which defines the boundary between short and long distances. It is \( D \) that is set to 100km in all the Monte Carlo simulations shown below as this is taken to be the distance scale appropriate for sailing in the MBA. By way of contrast we might imagine that \( D \) should be 10km for a rowing-based EBA simulation. The shape of the potential function used ought not to be too important but so far we have worked only with the form \( V(x) = \frac{1}{(1+x^-4)} \) which gives the desired behaviour. In principle we also need to introduce a very short distance scale. This is the minimum separation required before we consider two sites to be separate entities. This is needed for our block renormalisation analysis to work appropriately. In practice, all our input sites are already deemed to be independent entities so that is not needed for the archaeological data.

V. ANALYTIC (MEAN-FIELD) SOLUTIONS

Before attempting any numerical modelling with the real island parameters, it is useful to see some of the behaviour that might arise, using simple analytic approximations for an idealised network of sites. We work at zero temperature and make a mean field approximation in which we replace every value of \( v_i \) and every value of \( e_{ij} \) in \( H[v_i, e_{ij}] \) by their average values \( v \) and \( e \) respectively. We then look for minima of \( H(v, e) \), a two-dimensional energy landscape in which, for simplicity, we restrict ourselves to \( 0 \leq v \leq 1 \) and that \( 0 \leq e \leq 1 \). In some cases the lowest values will be at one of the boundaries and indeed the energy landscape will force the system to move to extreme values in one or both parameters.

As we have suggested, increasing \( \lambda \) increases the importance of inter-site interaction, whereas increasing \( \kappa \) increases the importance of single site behaviour. If \( (\lambda/\kappa) \) is relatively small the latter effect may overwhelm and we expect a stable energy minimum. That is the advantage of being close to the optimal population \( \nu = 0.5 \) is too great for trade to matter very much. The plot in Figure 9 for small \( \lambda \) does indeed show a valley near this value.
Figure 9: The energy landscape for small \((\lambda/\kappa)\) with the vertical axis \(H\) and horizontal axes \(v\) and \(e\). In this regime sites appear to be close to their optimal size and edges can have non-trivial values.

Figure 10: The energy for large \((\lambda/\kappa)\) with the vertical axis \(H\) and horizontal axes \(v\) and \(e\). Now the network is forced to extreme values.

On the other hand, when islands are not self-sufficient and \((\lambda/\kappa)\) is relatively large the latter effect may not be enough to inhibit runaway growth as trade brings benefits that outweigh local overpopulation effects and this is seen in Figure 10.

However, in this situation we have a saddle point and there are two possible outcomes: the runaway growth or collapse of the system. That is, it may be better to reduce the population to reduce the penalty of having large...
populations and suffer the loss of advantageous trade. Iterating this brings us to collapse. Which wins depends on which side of the saddle point leads to the lower valley bottom. In general this will not be a blanket collapse. There will be a mixture of valleys and cols in this multidimensional landscape and not all of the latter will be traversed in the direction of local collapse. Nonetheless, this shows the ease with which many sites in the network can either disappear ($v_i = 0$) or cease to communicate ($e_{ij} = 0$).

Roughly, provided $\lambda$ is large enough then, as $\lambda$ increases from zero for fixed $\kappa$, there is a monotonic growth in average site exploitation from under-exploitation to full exploitation. Provided $\lambda$ is large enough then, if $\lambda$ is held fixed and we increase $\kappa$, all sites undergo medium exploitation as trading links become unimportant. The major difference occurs when $\lambda$ (trading strength) decreases for small fixed $\kappa$ (low self-sufficiency). Then, for only a small reduction in trading strength, exploitation of resources can collapse from full exploitation to no exploitation which, naively, we might infer as site abandonment. This is shown in Figure 11 in which we see the mean field average of $v$ for varying $\lambda$ and $j$, for fixed $\kappa$ and $\mu$.

![Figure 11: The energy landscape for fixed $\kappa$ and $\mu$. The mean field average of $v$ (vertical axis) is shown against varying $\lambda$ and $j$. Vanishing $v$ denotes collapse.](image)

This behaviour is not specific to the particular form of $H$ given above, but is a consequence of $H$ having positive eigenvalues which become negative, leading to instability. Gravitational models guarantee negative eigenvalues because of the way in which large sites interact preferentially with large sites so that, for a prosperous network, the effect is non-linearly beneficial. This non-linearity has to compete against the non-linear costs of over-exploiting resources. Negative eigenvalues will arise once the system becomes less insular, with less stress on individual island self-sufficiency. In this regard we note the following observation by Broodbank et al. (2005):

“For the southern Aegean islands in the late Second and Third Palace periods, an age of intensifying trans-Mediterranean linkage and expanding political units, there may often have been precariously little
middle ground to hold between the two poles of (i) high profile connectivity, wealth and population, or (ii) an obscurity and relative poverty in terms of population and access to wealth that did not carry with it even the compensation of safety from external groups”.

We note that these rapid collapses are not induced by volatility but correspond to a smooth buckling of the landscape.

**Non-gravity models**

Despite their virtues, models that minimise our ignorance like gravity models are unlikely to be more than roughly correct and may be seriously at fault if applied across the whole network, even if applicable to its more homogeneous regions. As we have stressed, if this is the case we need more detailed archaeological data than the aggregate data of the ‘gravity’ models, and the answers will depend on the island meso-scale population distributions. To take the other extreme, in non-gravitational models it may be advantageous to connect to bigger sites, without any further advantage if one is big oneself. More simply, it could be that an exchange/trade term at a site only depends on the existence of links to other sites, and is insensitive to the resources/population available on the site itself. In contemporary economic parlance, we might term this a ‘supply-side’ model which ignores consumer demand. Conversely, the exchange/trade term at a site might be determined by the sites to which it is linked, insensitive to the resources/population available on those sites. In contemporary parlance, we might term this a ‘demand-side’ model. Because there is not the positive feedback in the virtues of becoming large, it is difficult to see how negative eigenvalues can arise. As a result, global collapse lessens. This is not to say that links or sites do not become abandoned, but they do it smoothly as a consequence of shifts in the external population and maintenance pressures.

Taken together with the structural differences in the role of intra-island interactions, the existence of these further differences is important in that it shows no universal network structure exists for island archipelagos. Instead, perhaps contrary to the hopes of some network theorists, the nature of the networks is strongly conditioned by geography and society.

**VI. NUMERICAL SIMULATIONS**

We are beginning to apply our models described above to the Aegean network of Figure 8, using Monte Carlo methods. Before displaying our results, it is useful to contrast our approach with PPA as described earlier. Figure 12 shows PPA applied to our sites. In general PPA emphasises the closest links and tends to produce tightly connected groups of sites. In Figure 12 we see this as four strongly connected cores in the PPA network, that is, regions where every site can reach every site when we take account of the direction of the PPA edges. In fact only by ignoring the arrows are the sites of the Cyclades linked to those of Crete and vice versa.

Our optimal MC model is not as unambiguous. A priori it is difficult to make sensible estimates for the model parameters and we have to search for robust ranges where features are visible, much as we have to choose the right scale and coverage when choosing a map for a problem in real life. Figure 13
shows a typical result from our Monte Carlo model for a similar density of edges as in Figure 12. One of the first things to note is that in the PPA model there is no direct or indirect link between Crete and the Cyclades, if one retains the directionality of the PPA links. For the PPA the shortest possible Cyclades-Crete link would be from Thera (10) to Knossos (1) or Malia (2), which is around 100km long. However they all have several closer neighbours within a 50km radius and the long distance Thera-Knossos/Malia links are never selected in PPA. On the other hand, while the Thera-Knossos/Malia links are at the distance scale D=100km used in the MC simulation of Figure 13, such links are not a priori excluded in the MC model. In this case the link is used in the model, reflecting the advantage such a direct link might have had in any MBA network.

Figure 12: PPA analysis with three outgoing edges to nearest three sites. The darker and larger the vertex, the larger the in-degree: Miletus (18) and Myndus (32) have largest in-degree of 6, Knossos (1) has in-degree 4, Malia has in-degree 2, Kastri (12) has in-degree one while Rhodes (16) has no incoming connections. However using betweenness as a measure we find that Knossos (1), Malia (2) and Kastri (12) are the most central. Note that the PPA does not assign a link between Crete and the Cyclades. Further if the sense of direction given by the arrows is respected then there are four strongly connected cores: Crete, the Cyclades, the Dodecanese and mainland Greece.

In terms of analysis the increased complexity of our networks provides several challenges. For instance, the degree of a vertex is no longer a useful measure as edges are likely to carry a non-zero weight. For visualisation we have used a cut-off, and in our figures we do not show edges or vertices which are below 10% the size of the largest in that network. We could use a similar threshold method to map our network onto a simple graph such as that constructed by PPA. However the raison d’être of our work is precisely to exploit this as a feature. Thus we have to introduce new methods to the networks of island archaeology.
Figure 13: Monte Carlo analysis for $\kappa = 2.0, \lambda = 1.0, \mu = 0.35, j = 0.7, D = 100km$. The size of the vertices is proportional to their strength, the total weight of the in and outgoing edges. The largest are Gournia (8), Mallia (2) and Knossos (1) followed closely by the rest of central Crete. The Dodecanese are about half the strength and the Cyclades are a third of the strength. The darker the vertex is coloured, the larger the betweenness and this shows a very different story with sites on the edges of clusters scoring highly. This includes Malia and Knossos but now the Cyclades scores even higher than these indicating their central role if all vertices are treated equally. However all vertices are clearly not equal in our networks and many standard measures of network properties, such as betweenness, are of little use in our work.

There are numerous areas where improvements are already being made in our models. We have to adapt our input distances $d_{ij}$ to reflect actual transport times rather than physical distances. The list and size of sites can be fitted to archaeological data, both adapting Broodbank’s method of assigning sites on the basis of cultivatable area and exploiting modern GIS-based techniques. Within the model we have variations where we use network distances rather than pure physical distances $d_{ij}$, both within the Hamiltonian and in the analysis. Once we have constructed our networks there are numerous ways to analyse them. We are developing the use of random walkers to rank sites (an eigenvalue problem used in sports rankings), but there are many other methods. In particular, limiting the walks that such agents can make leads to simple models of cultural transmission, somewhat similar to those which have been applied to both ancient and modern contexts (see for instance Neiman 1995, Bentley et al. 2003, 2004 and references therein).

Finally, we are beginning to study problems of time evolution. This can occur in the form of slow ‘adiabatic’ changes, such as population build up or quick
'quenches’. We can simulate, for example, the change that occurs between Late Minoan IA and IB (c. 1600 BC), when the volcanic eruption of Thera removes that island (the site of Akrotiri in particular) from the system (this is shown in figure 14, before and after). Whatever the scenario, it is quite possible that the system gets stuck for a time in a meta-stable state with the instability only apparent much later. This might be a good model for the transition that occurs after Late Minoan IB (c.1500 BC), when sites across Crete are destroyed and the balance of power shifts to the Greek mainland (i.e. Mycenae) for the following three centuries. It is longer-term dynamics of this kind that we hope to move towards in future work.
Figure 14: The same values are used but in network B Thera has been removed (marked by a cross) while it remains in place in the network A (which is the network of Figure 13 displayed in a different manner). The sizes of sites (size of circles) are similar but now the Dodecanese is ranked far higher.
VII. CONCLUSIONS

An important part of our model-making has been our assumption about the way in which the meso-scale of the individual island societies is accommodated within the macro-scale of the overarching MBA network. This introduction of what we might call ‘social’ or ‘behavioural’ aspects into the physical system shows that, rather than assuming diffusion or gravitational pull to be intrinsic physical properties of the system, they are only relevant given certain (social) conditions, as exemplified by the ‘gravitational’ MBA Aegean and the very different EBA Cyclades. We can then alter other ‘social’ conditions of the network by adjusting, for example, the degree of commitment to local resources or trade, in order to achieve better fits with perceived or actual past scenarios.

Both the analytical and numerical approaches outlined in the previous sections furnish insights into the articulation of the physical and relational dimensions of regional interaction networks. This articulation is expressed through the gravity models discussed in the analytical section. In terms of the results arising from this investigation there is clearly still much to do. There are, at this stage, interesting indications that the development in the later MBA of affiliation networks linking certain larger sites (Knappett and Nikolakopoulou 2005, in press) might be amenable to further exploration using gravity models. The Monte Carlo analyses run thus far do seem to testify to the importance of the link between north-central Crete and Thera under certain conditions, a pattern that is very clearly seen in the archaeological evidence. Our goal is to explore these conditions much more fully. The same goes for the consistently central role of Knossos, or of other sites in north-central Crete, and the apparent robustness of this pattern to changes in network conditions. There is much we need to do to produce more results, and to compare the results with the archaeological data. Our work indicates that these techniques open up numerous possibilities, and offer new means of assessing the different kinds of networks that have long preoccupied the social sciences in various manifestations. Post-Watts and Strogatz new approaches have also emerged. Rather than use these tools for their own sake, however, we must seek to ensure that our techniques are commensurate with the complexity of the archaeological data, and ensure any conclusions are robust against changes in the details of our models.

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