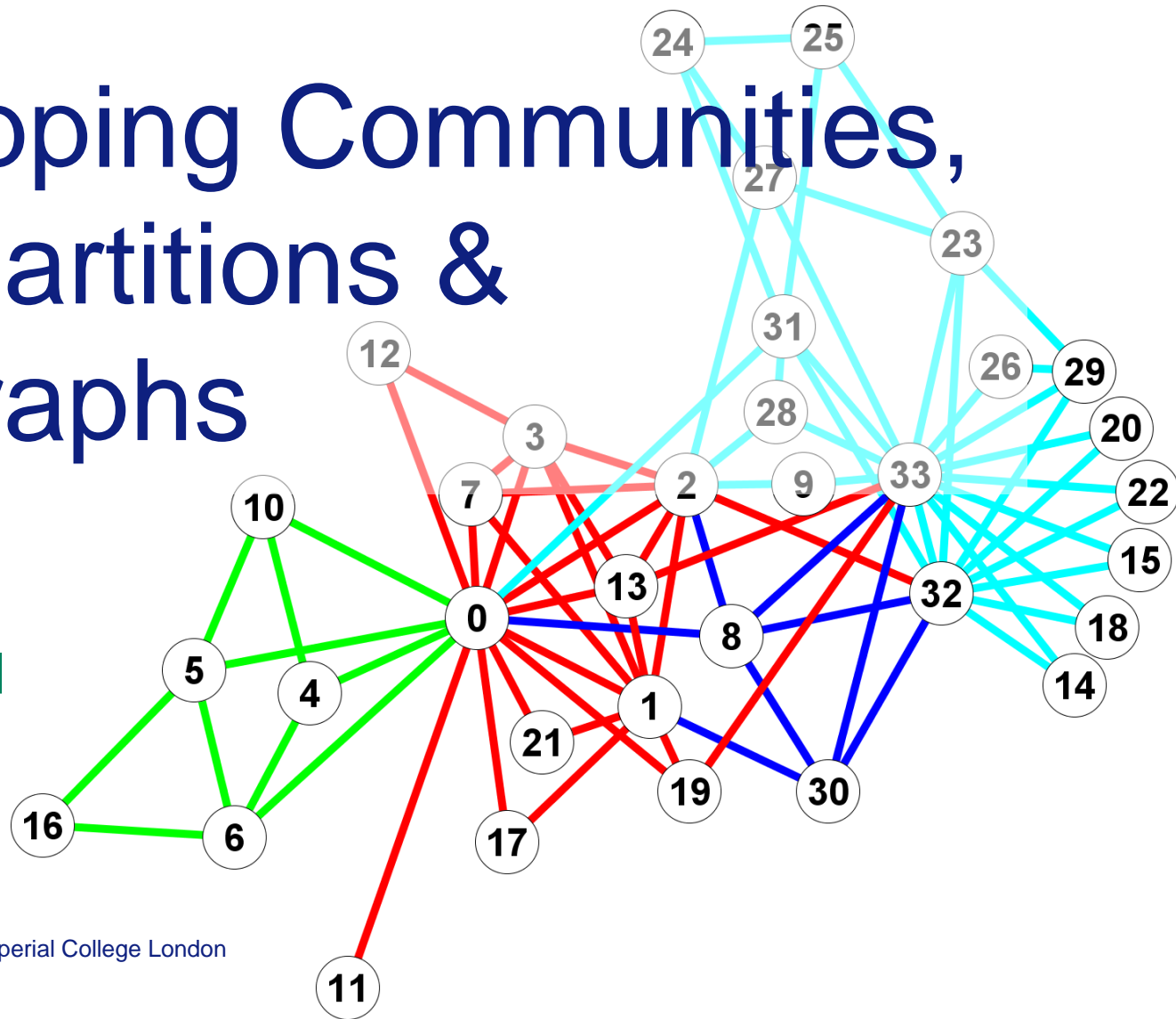**Imperial College London**

Overlapping Communities, Edge Partitions & Line Graphs

Tim Evans,
IMS and Theoretical Physics,
Imperial College London

- **Our Node Centric Viewpoint**

- Line Graphs

- Application to Community Detection

- Conclusions

Node Centric Viewpoint
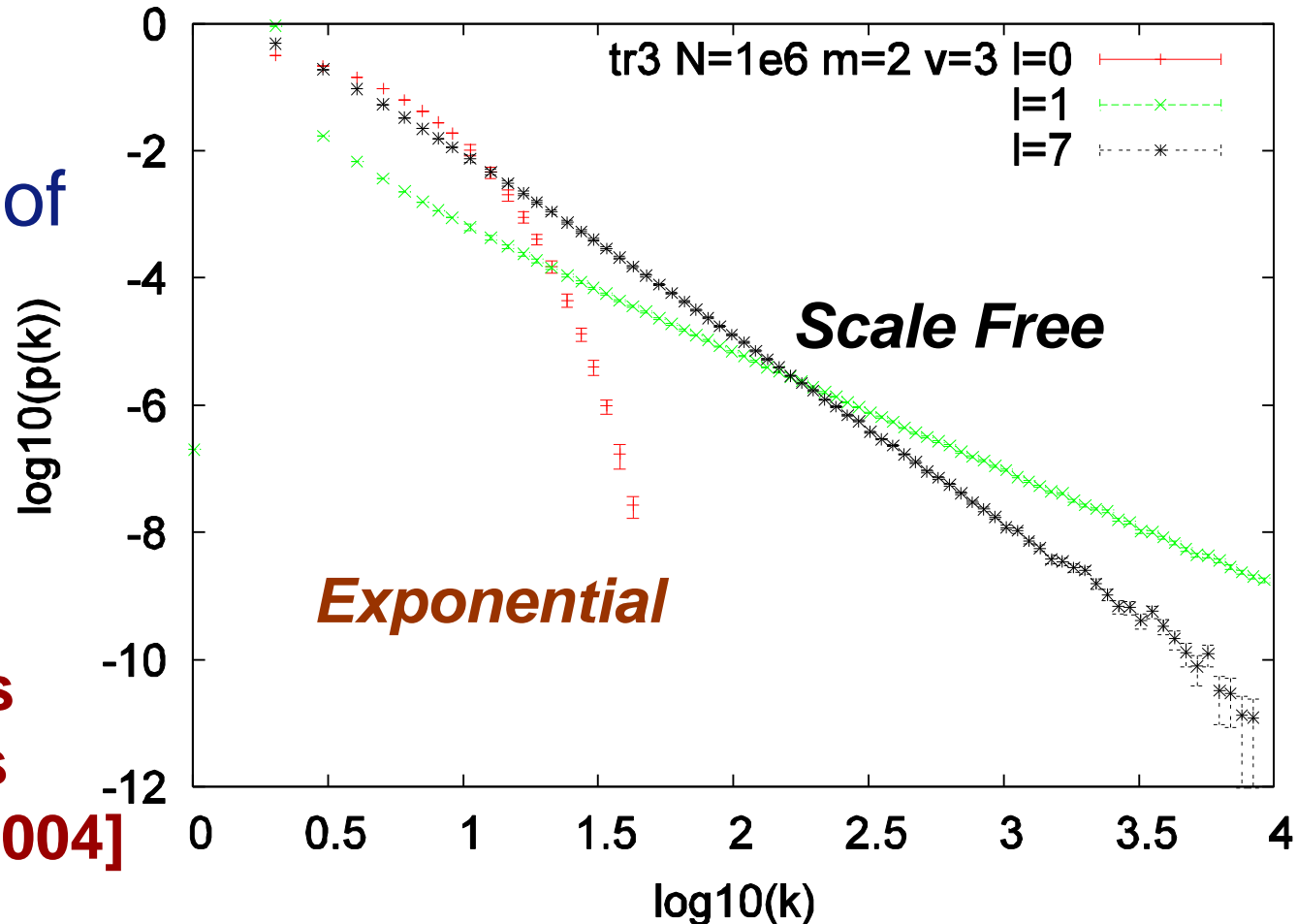
A network is

1. a set of nodes

AND

2. a set of vertices

We tend to have a very NODE centred viewpoint

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Node Centric - Scale Free Networks

Power law degree distributions of NODES

Random walk on NODES produces scale free graphs [TSE, Saramäki 2004]

Plot axes: x-axis log10(k), y-axis log10(p(k))

Legend:
- tr3 N=1e6 m=2 v=3 l=0
- l=1
- l=7

Scale Free

Exponential

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Node Centric – Node Partitions

Community detection via partition of NODE set
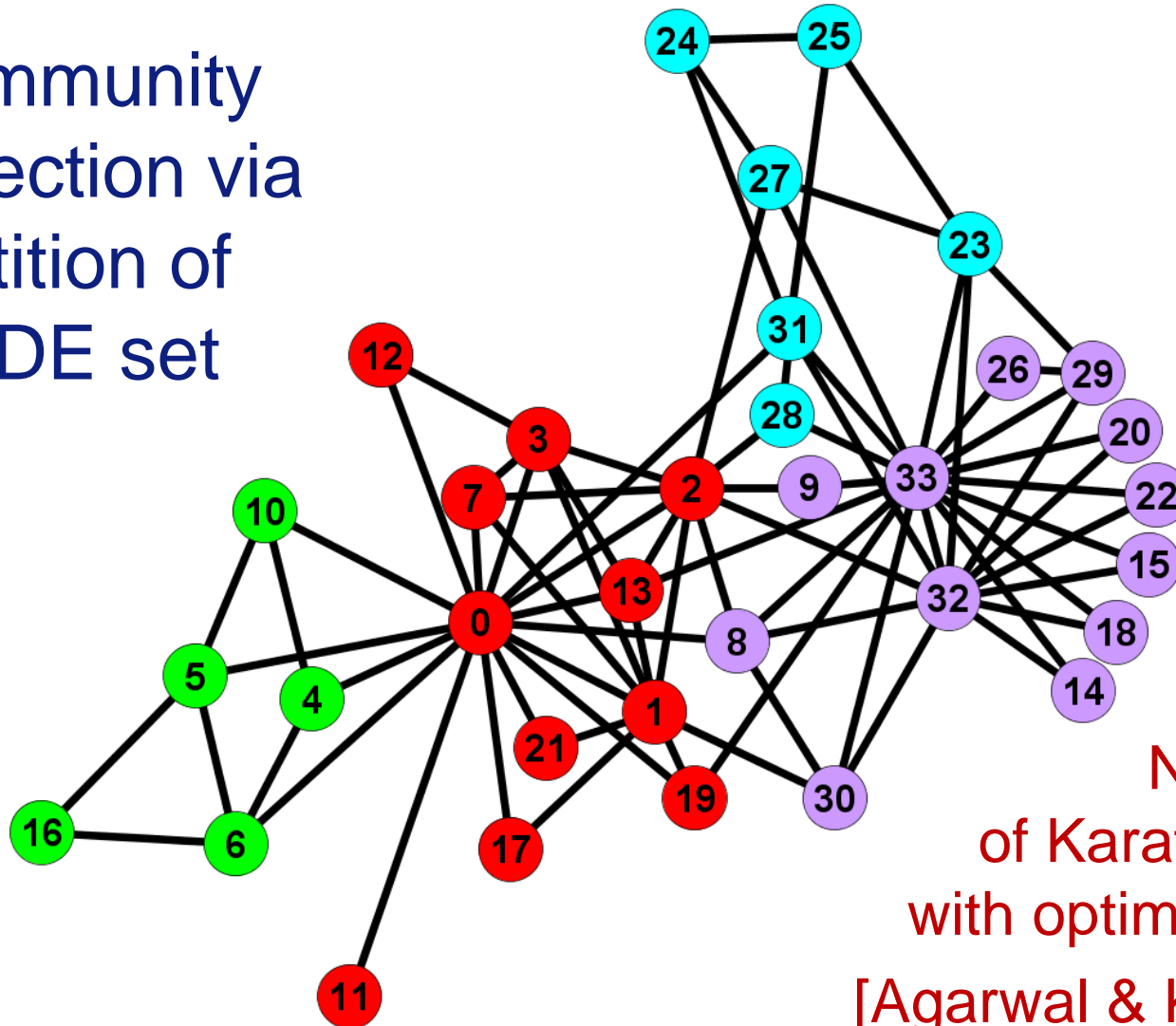
Node partition of Karate club graph with optimal modularity

[Agarwal & Kempe 2007]

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Edge Centric Viewpoint?

- A graph is both a set of nodes AND a set of edges

- However we naturally focus on nodes not edges

- Can we find a trick to help us shift our viewpoint from nodes to edges?

- Our Node Centric Viewpoint

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Nodes of a Line Graph
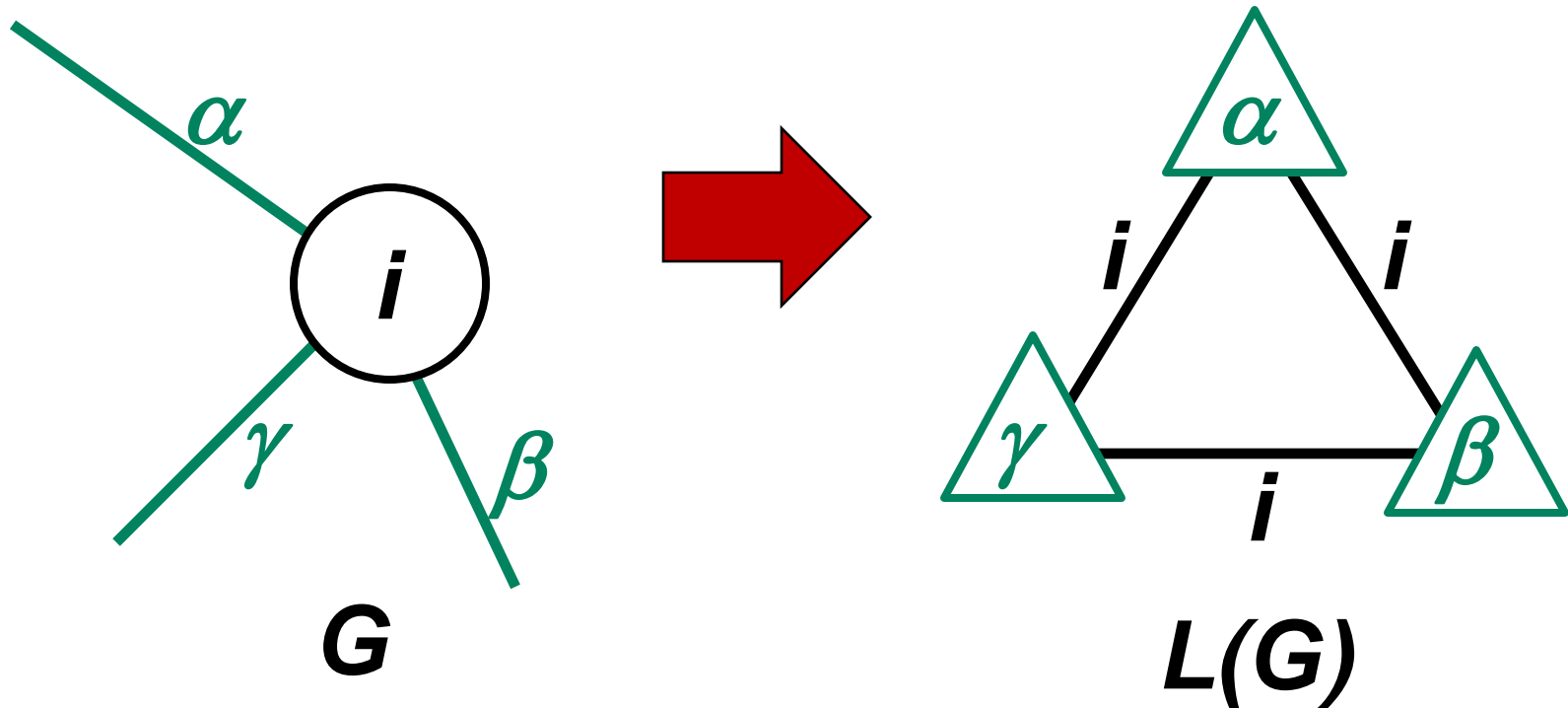
1. For every edge α in original graph **G** create a node α in the line graph **L(G)**

G

L(G)

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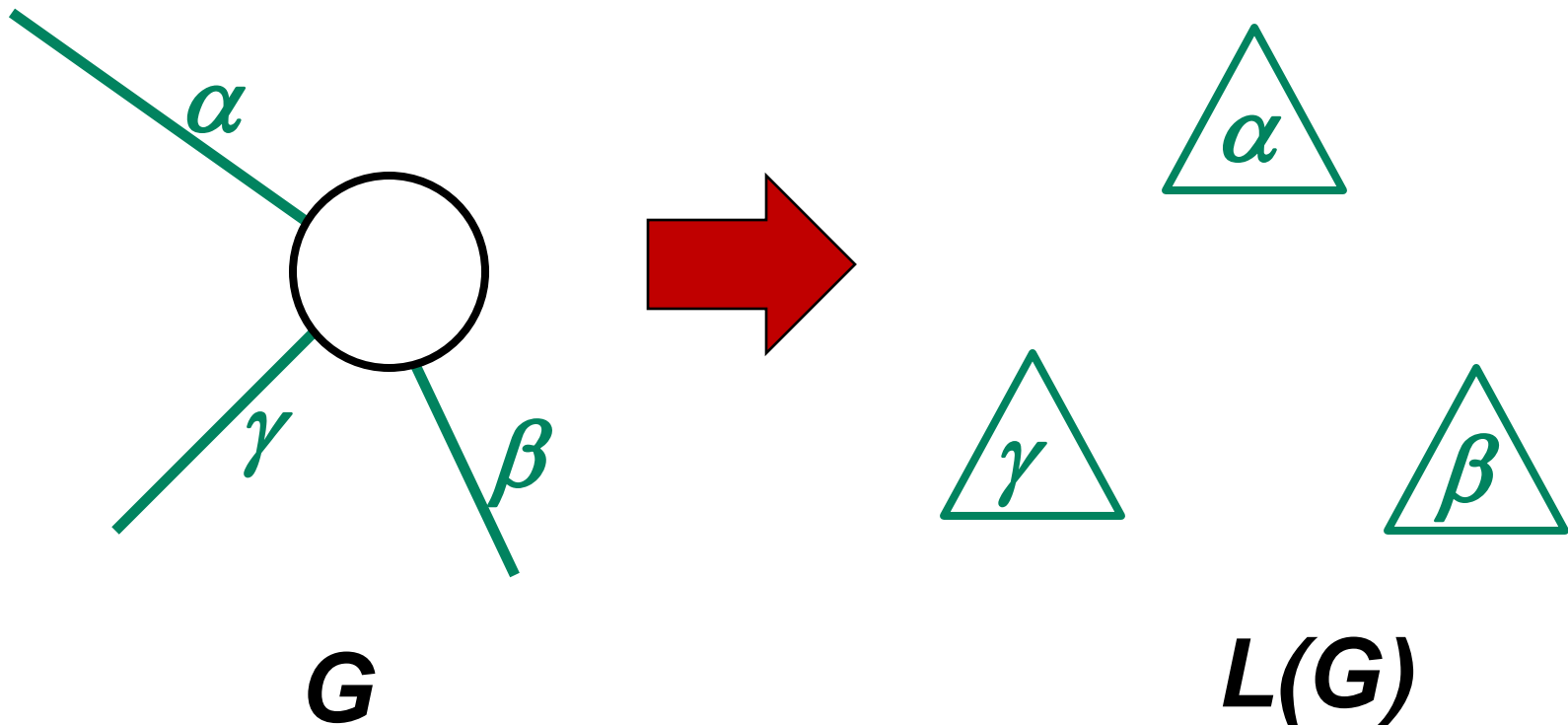
Edges of a Line Graph

2. Connect the nodes α and β in the line graph **L(G)** if the corresponding edges in original graph **G** were coincident

G
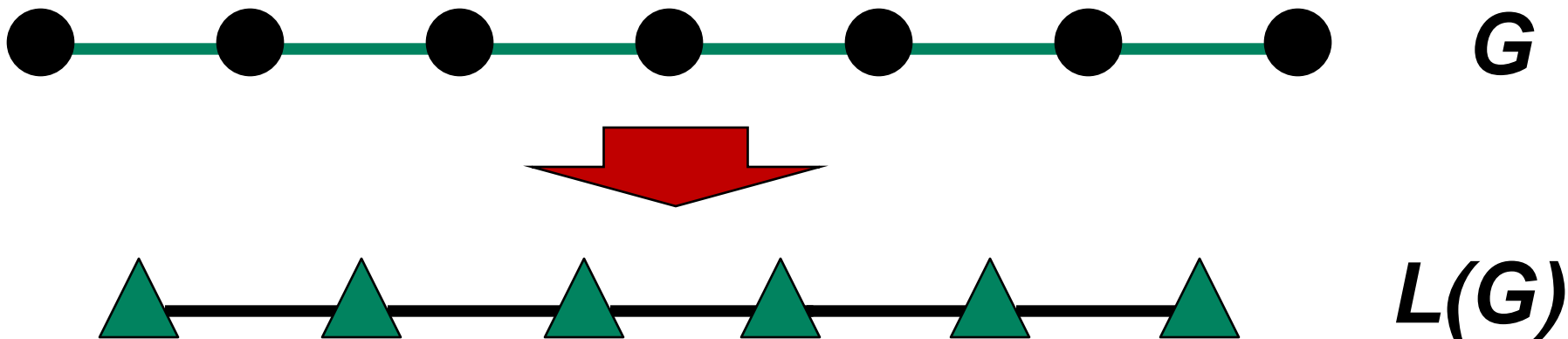
L(G)

© Imperial College London

Properties of a Line Graph

- Not usually a duality transformation

$$L(L(G)) \neq G$$

- (Almost) always reversible [Whitney 1932]

$$L(G) \rightarrow G$$
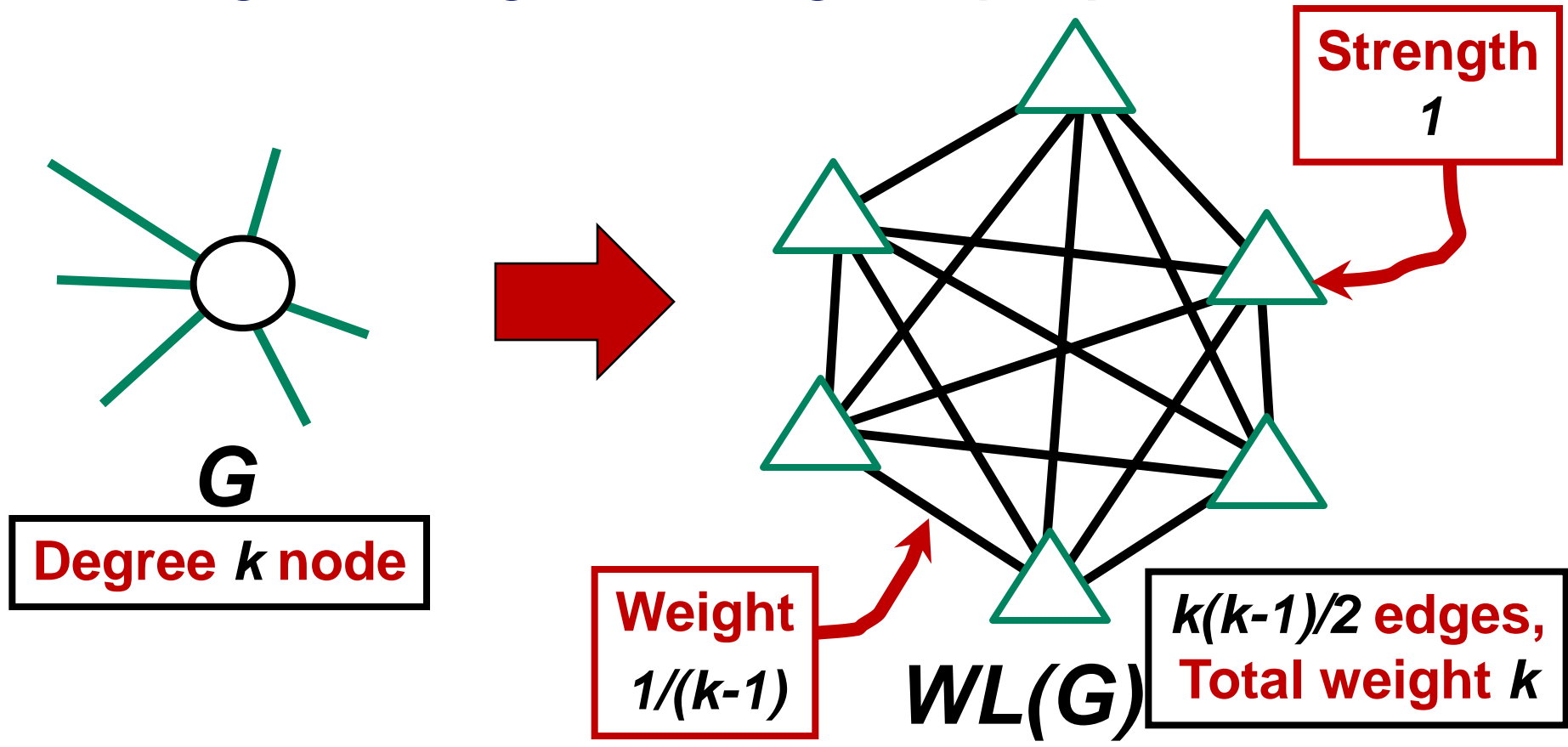

G

L(G)

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The Problem with a Standard Line Graph

High degree nodes in original graph **G** over represented by edges in Line Graph **L(G)**.

G

Degree **k** **node**

L(G)

k(k-1)/2 **edges**
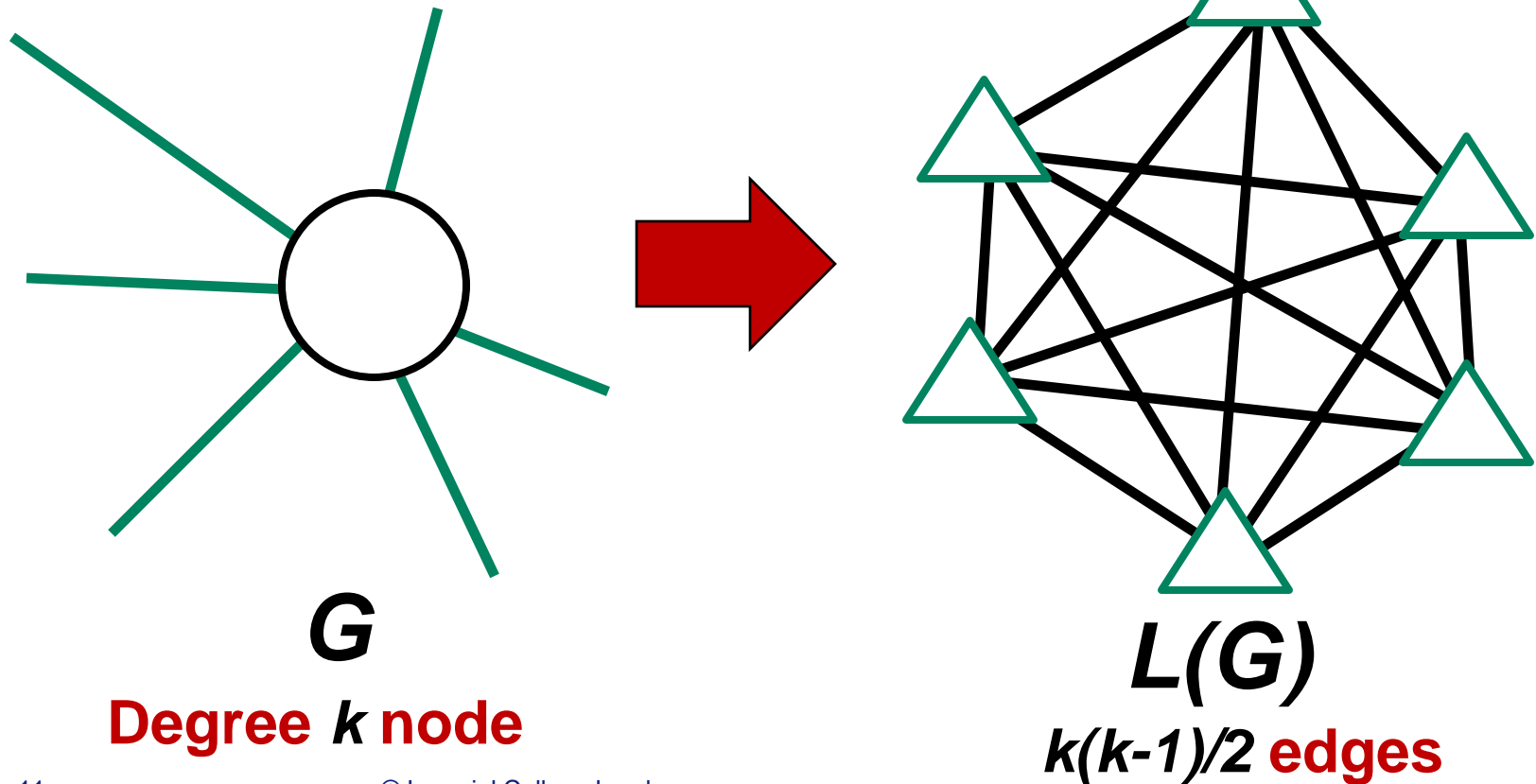
Solution – Weighted Line Graphs

- Original graph node of degree **k** produces line graph edges of weight **1/(k-1)**

Strength 1

G

Degree k node

Weight 1/(k-1)

WL(G)

k(k-1)/2 edges, Total weight k

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Community Detection - Node Centric version

- Partition nodes into communities

- Perform random walk on nodes

- Compare number of random walkers which stay within community after one step against number which remain within communities after infinite number of steps

= Optimisation of Modularity

[Girvan & Newman 2002;
Lambiotte, Delvenne & Barahona 2008]

Community Detection - Edge Centric version

- Partition edges into communities

- Perform random walk on edges
  = Random walk on line graph vertices

- Compare number of random walkers which stay within community after one step against number which remain within communities after infinite number of steps

  = Optimisation of Modularity of Weighted Line Graph [Evans & Lambiotte 2009]

Karate Club

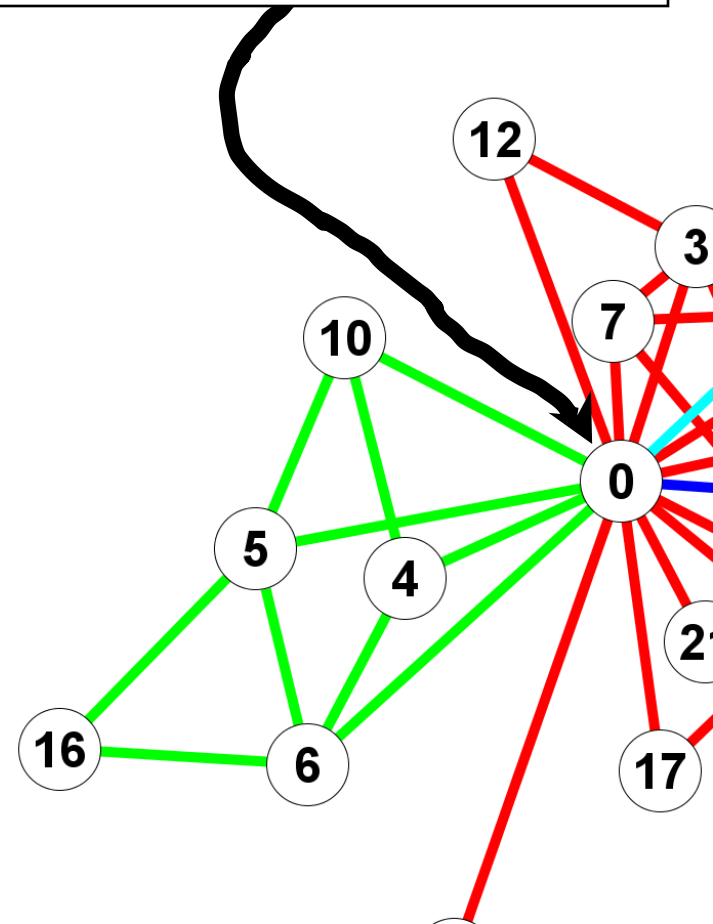Edge partition means individuals nodes can be members of more than one community

Ford-Fulkerson partition of Zachary

© Imperial College London
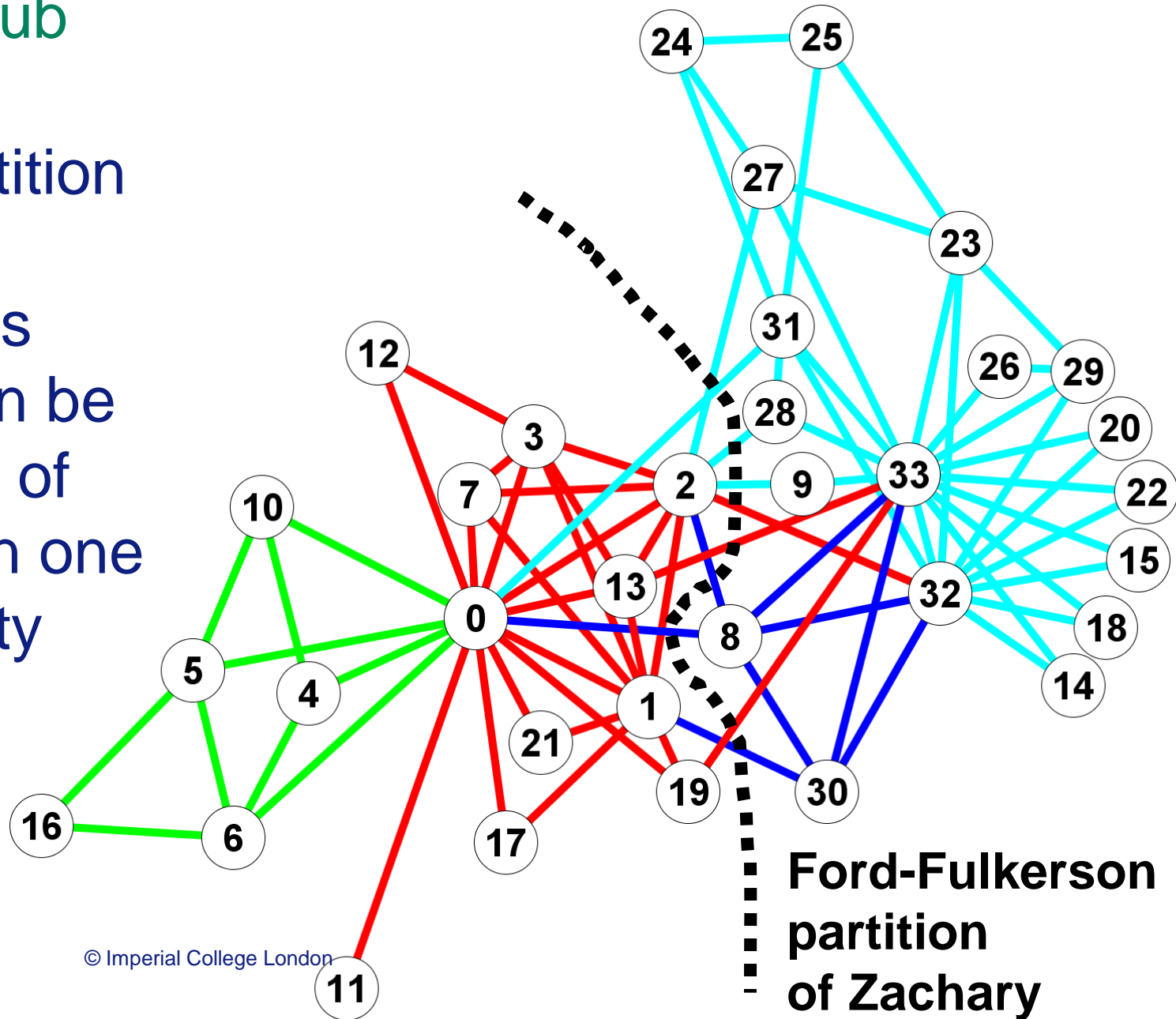
Karate Club Analysis

Nodes in One Edge Community

#	k	Fraction k In Green C
5	4	100%
6	4	100%
10	3	100%
4	3	100%
16	2	100%
0 (Mr_Hi)	16	25%

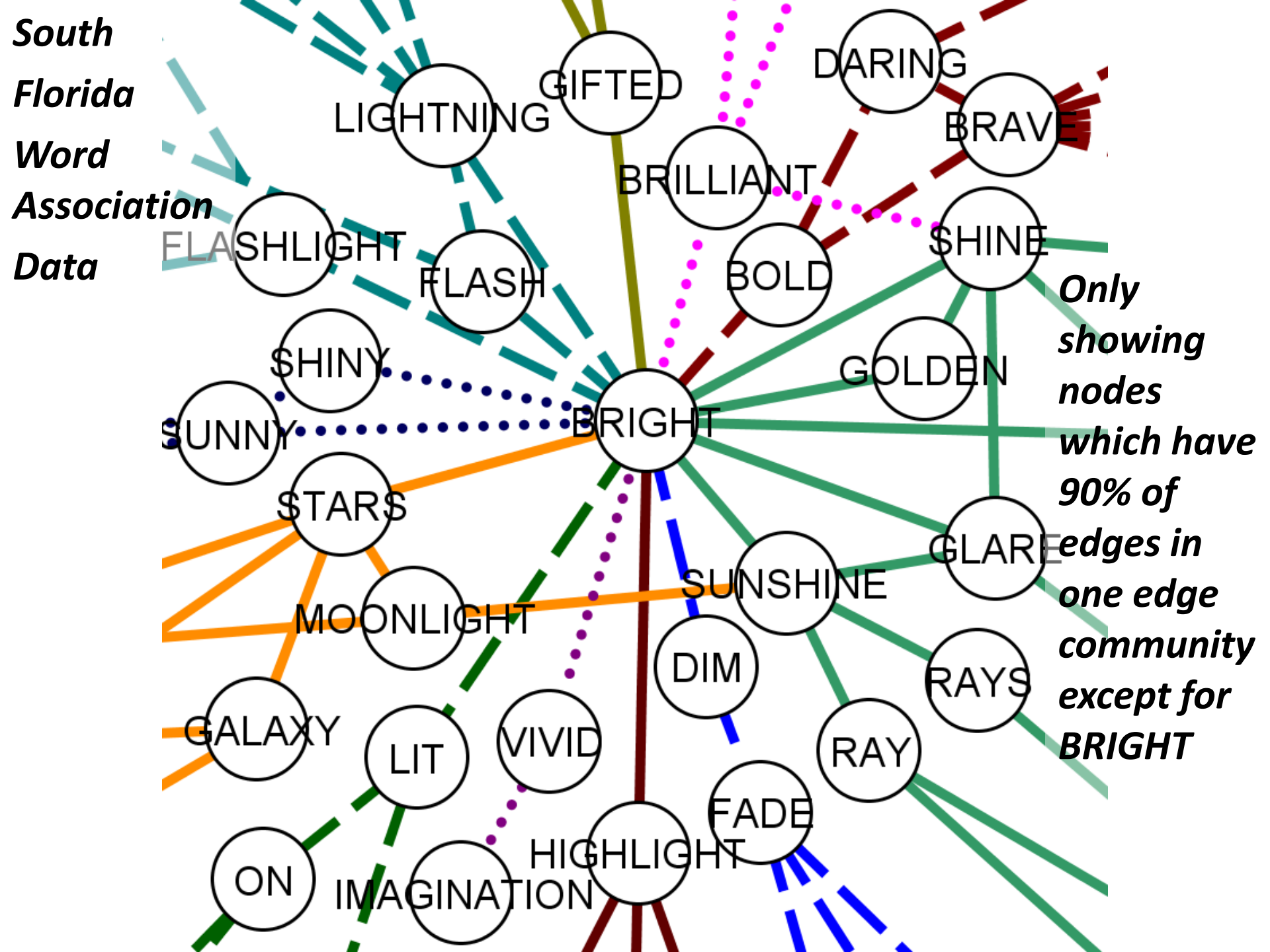**Mr Hi (the Instructor) bridges several groups**

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Karate Club Edge Partition

Nodes can be members of many communities

An overlapping community structure for nodes

Name	Community	Total k	k in C
0 Mr Hi	0	16	10
	1		4
	2		1
	3		1
33 John A	3	17	12
	0		3
	2		2

South Florida Word Association Data

Only showing nodes which have 90% of edges in one edge community except for BRIGHT

Edge Partition of Word in Paper Titles

- ## Some words have all edges in one partition
  - they define these communities
    e.g. cassini
- ## Other words have edges in several communities
  - stop words
    e.g. signature

Stem	Total k	k in C
interplanetari	78	78
cassini	62	62
heliospher	59	59
magnetopaus	53	53
spacecraft	52	52
signatur	91	32
solitari	30	10
radar	21	7
mhd	18	6

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Conclusions

- Line graphs move focus from nodes to edges with minimal effort

- Weighted line graphs avoid problem of over representation of high degree nodes

- Community detection on line graph produces overlapping node communities for original graph
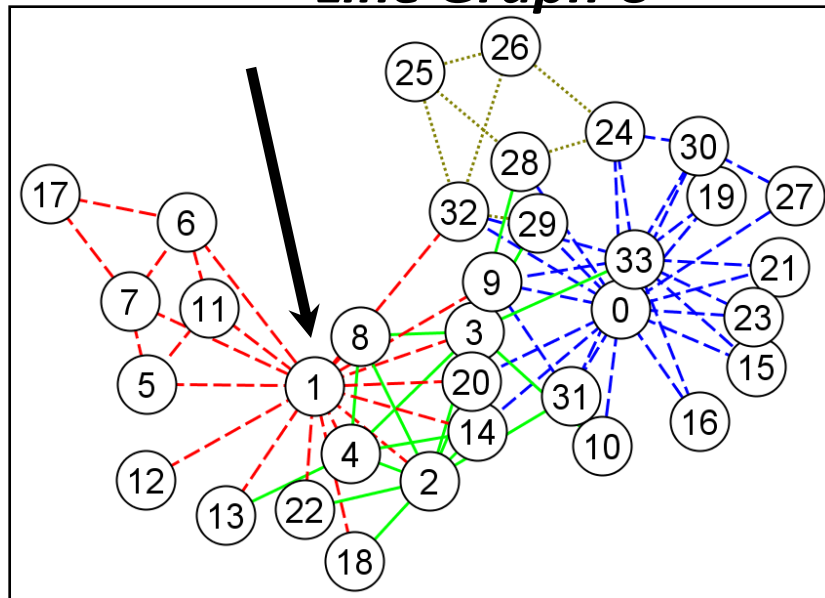
Evans & Lambiotte, Phys.Rev.E 80 (2009) 016105

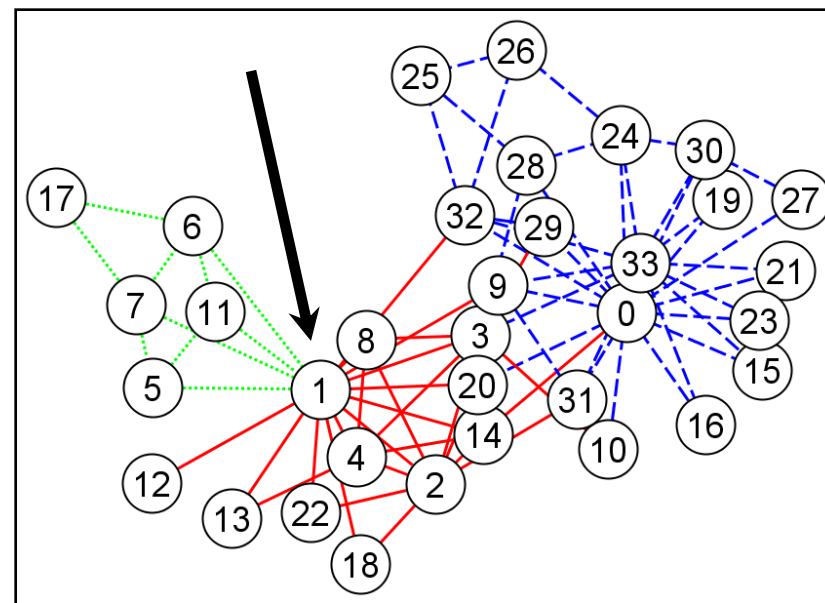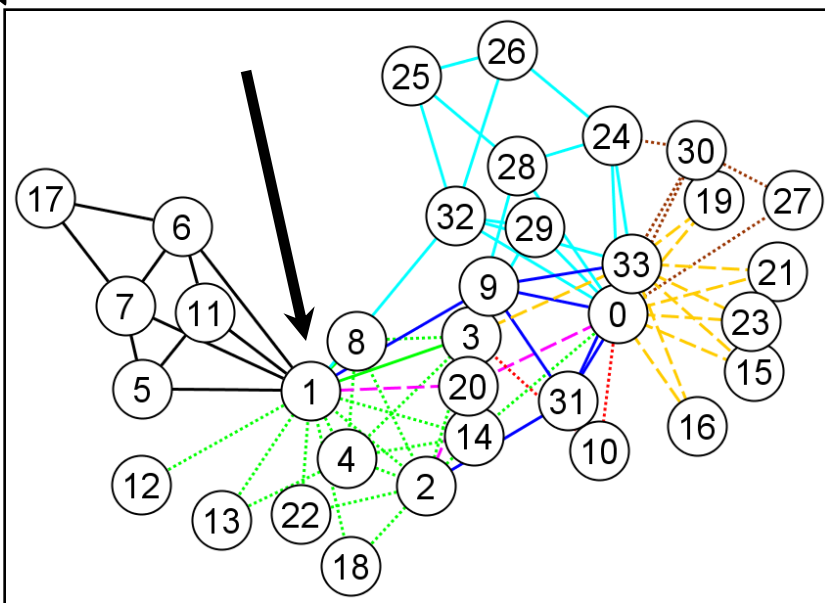**http://theory.ic.ac.uk/~time/networks/**

Additional Material

Node Partition A
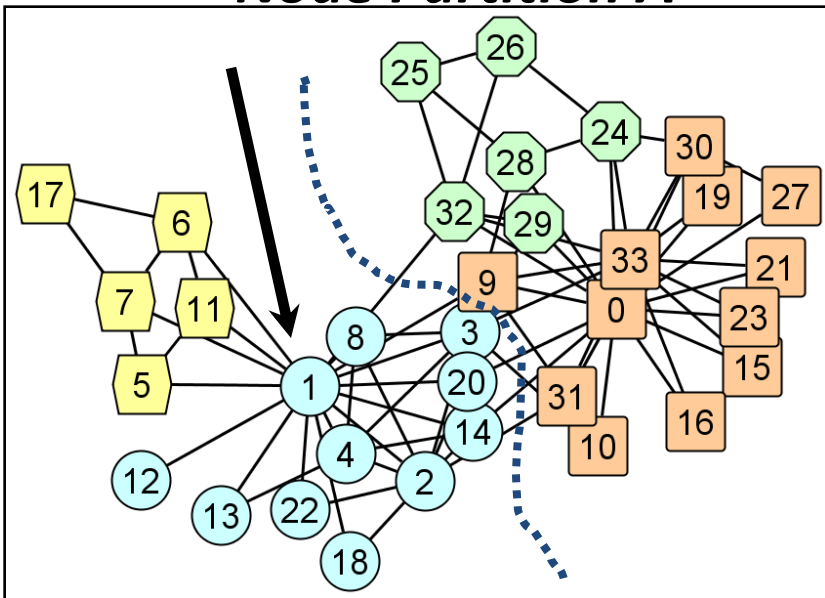
Line Graph C

Zachary Karate Club

Weighted Line graph D

Weighted Line Graph E1