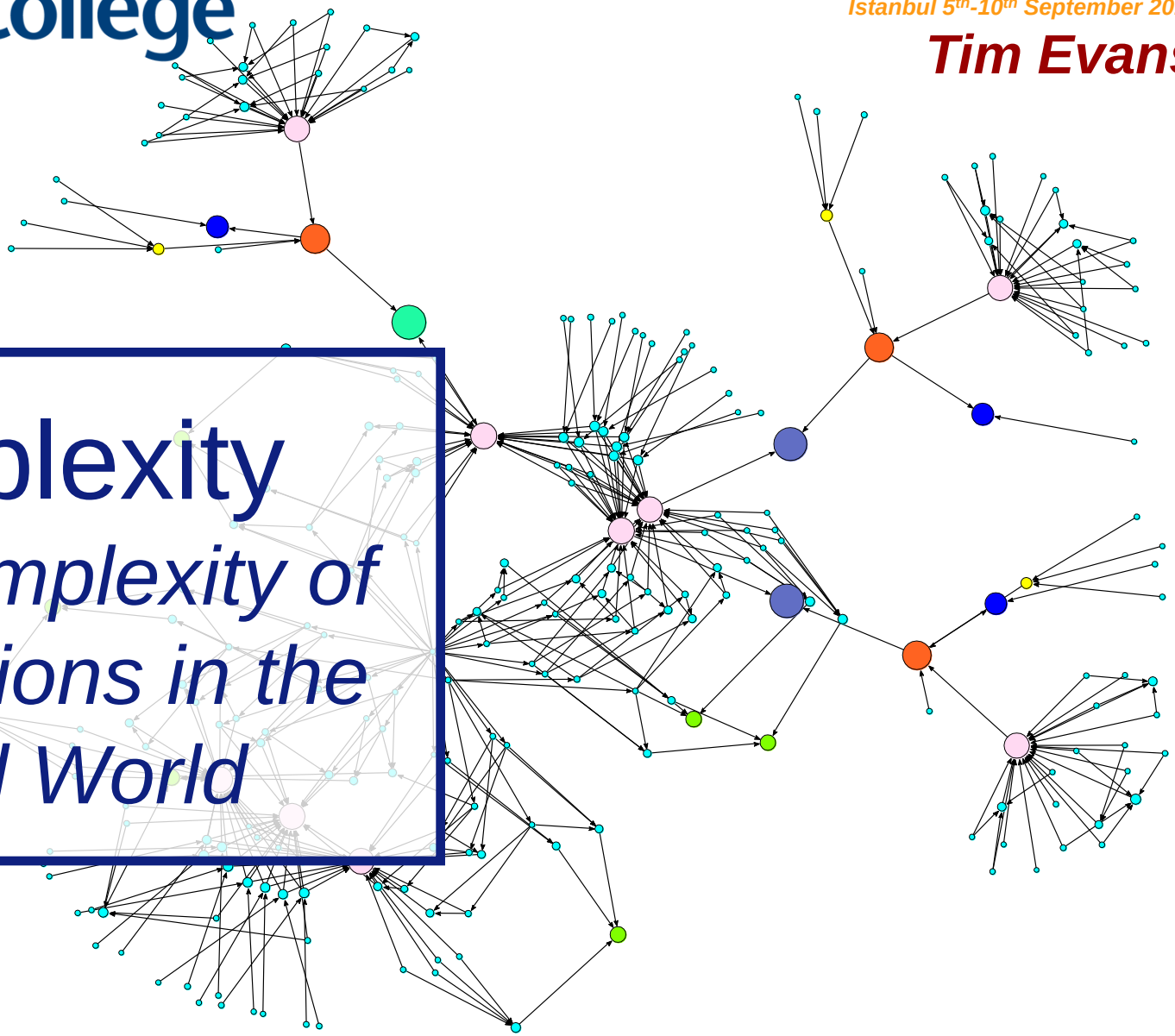
Imperial College London

Feza Gürsey Institute - Imperial College International Summer School and Research Workshop on Complexity, Istanbul 5th-10th September 2011

Tim Evans

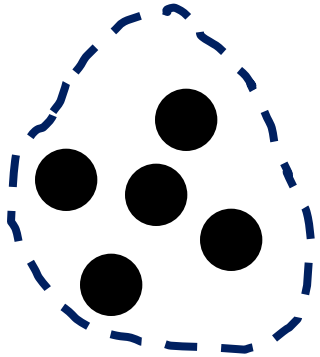


Netplexity

The Complexity of Interactions in the Real World

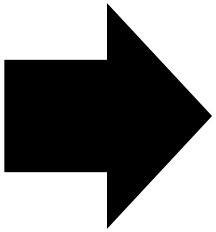
What is a Network?

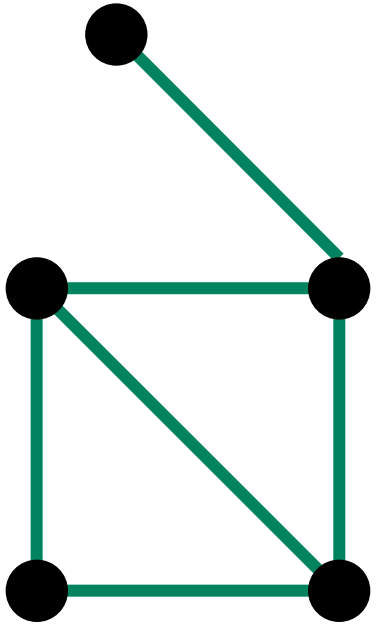
Mathematically, a ***network*** is a ***graph***

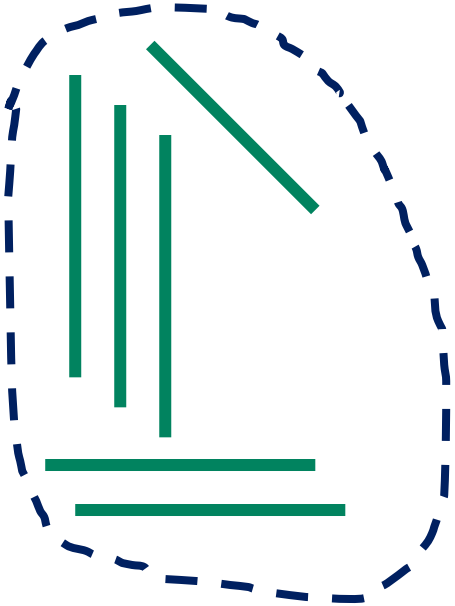


a set of ***vertices***
(or *nodes*)

and



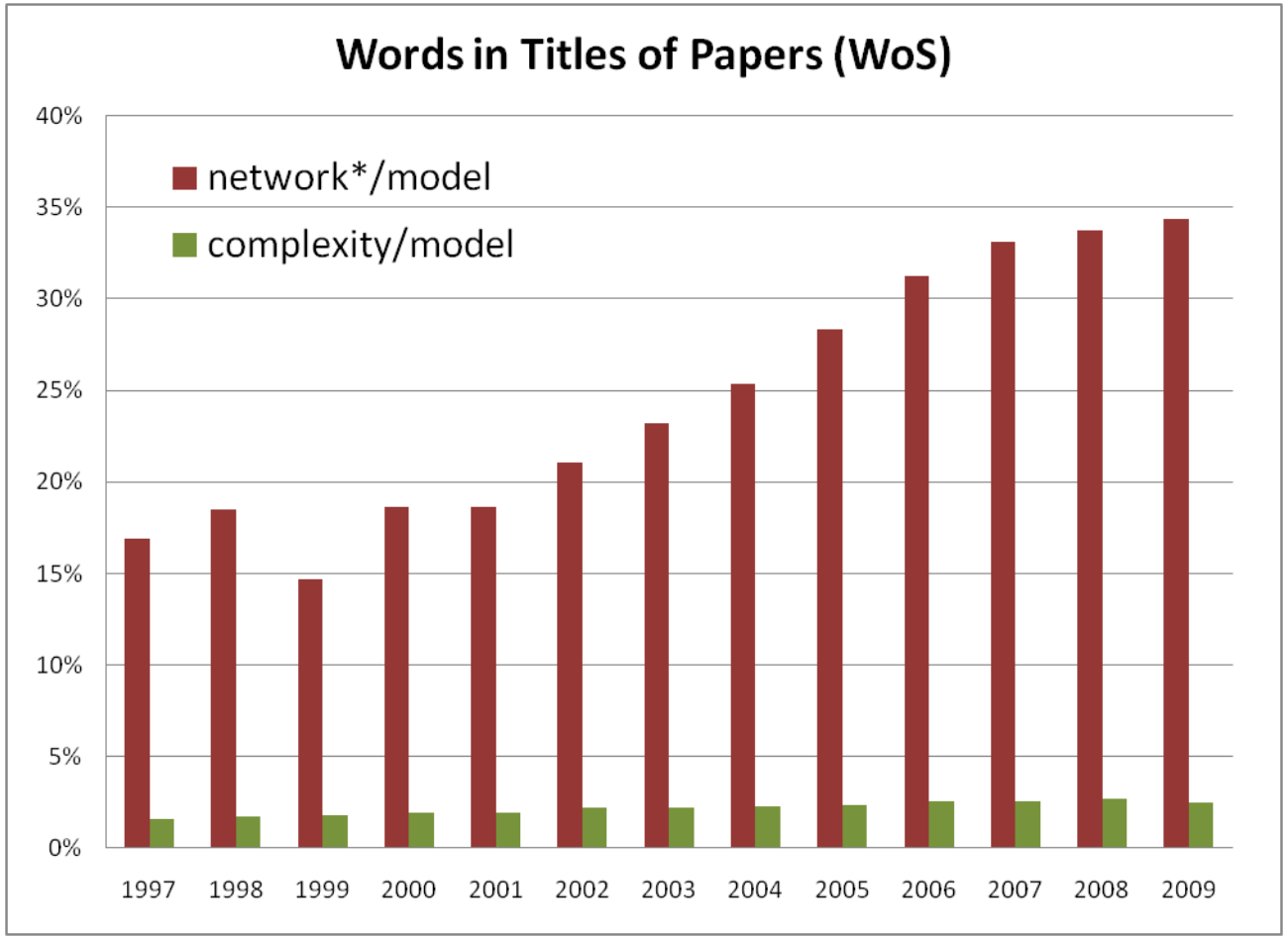




a set of ***edges***
(or *links*)

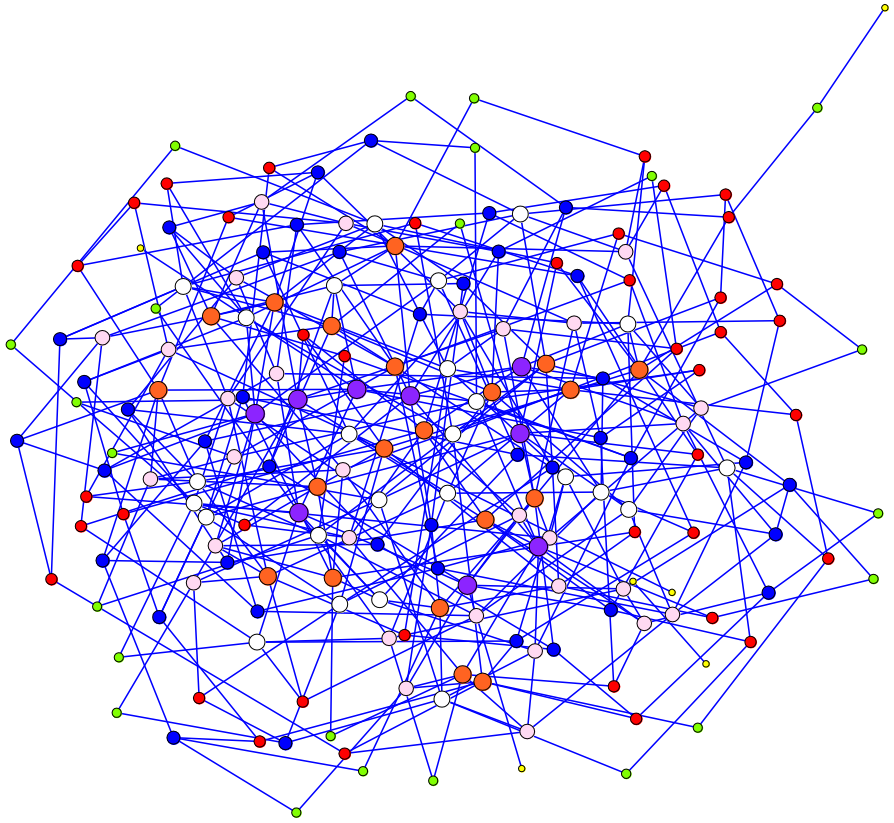
[Watts & Strogatz, Nature 1998]

Explosion of interest since 1998



Fraction of papers with word starting "NETWORK" in title compared to number of papers with word "MODEL"

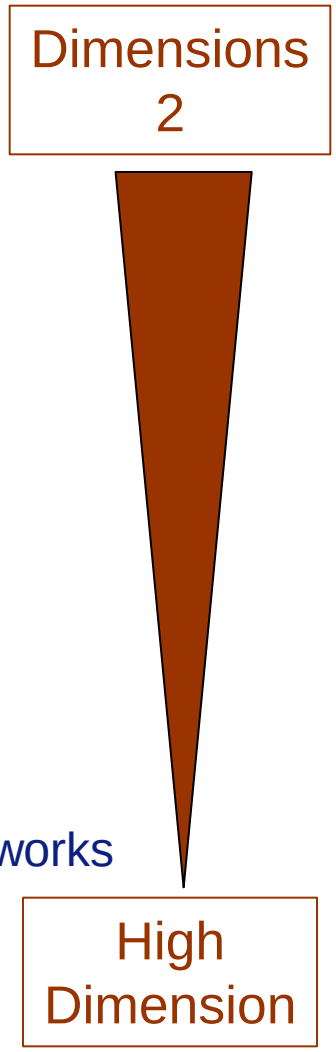
➡ WHY?



EXAMPLES

Types of Network – by application

- Physical links/Hardware based
  - telephone links, internet hardware, power lines, transport
- Biological Networks
  - neural, biochemical, protein, ecological
- Social Networks
  - Questionaires, observation, electronic social networks
- Information Networks
  - academic papers, patents, keywords, web pages, artefact networks



Death of Distance?

[Cairncross 1997]

Type of Network – by features

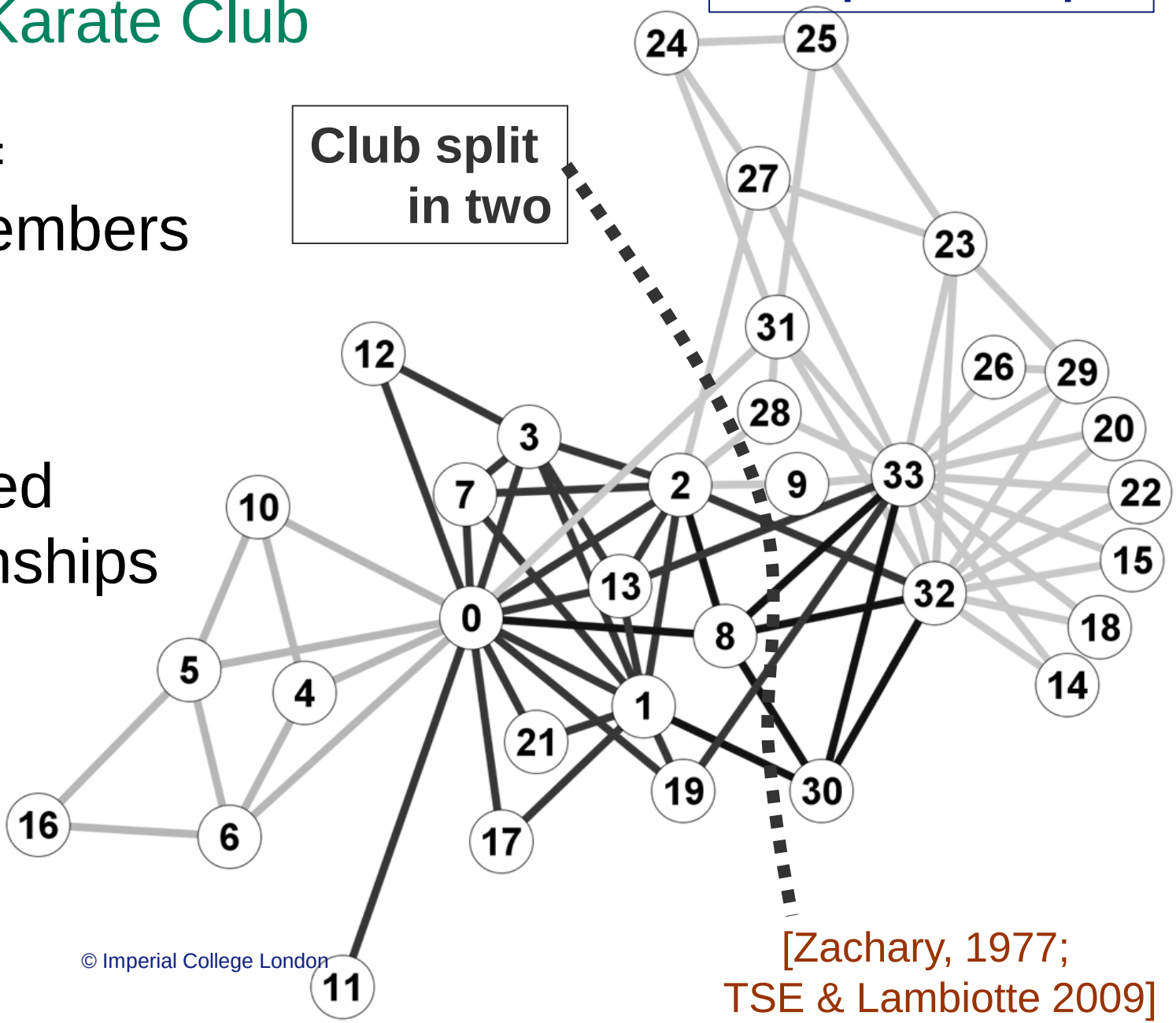
- Simple
- Weighted – edges carry numbers
- Directed – edges point in one direction
- Acyclic – no loops
- Bipartite – two types of vertex

Zachary Karate Club

Simple Graph

Vertices =
- Club Members

Edges =
- Observed Relationships
- ON or OFF



[Zachary, 1977; TSE & Lambiotte 2009]

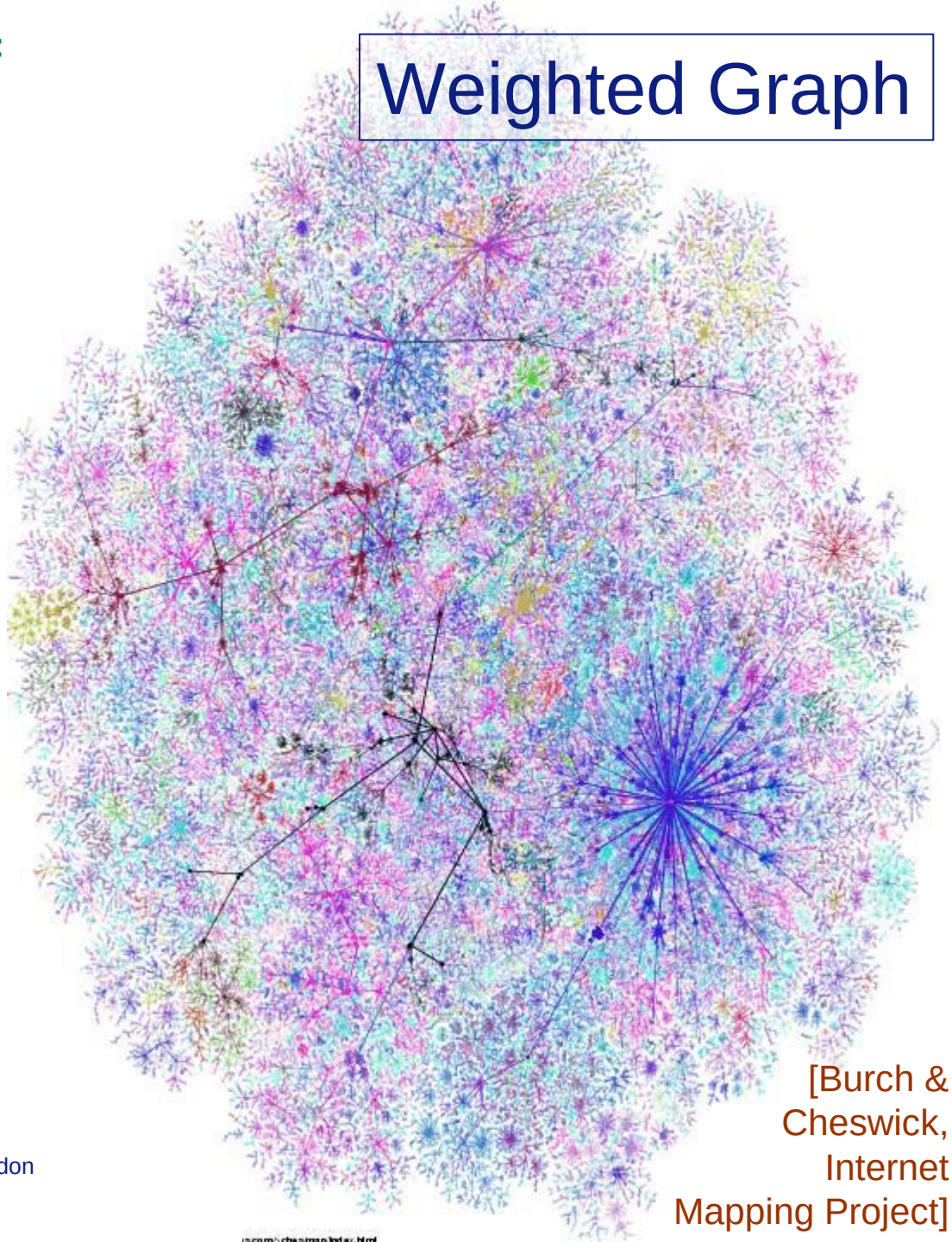
Router Level Map of Internet

Weighted Graph

Vertices = routers

Edges =
number of packets between router pairs

Understanding network properties important for design



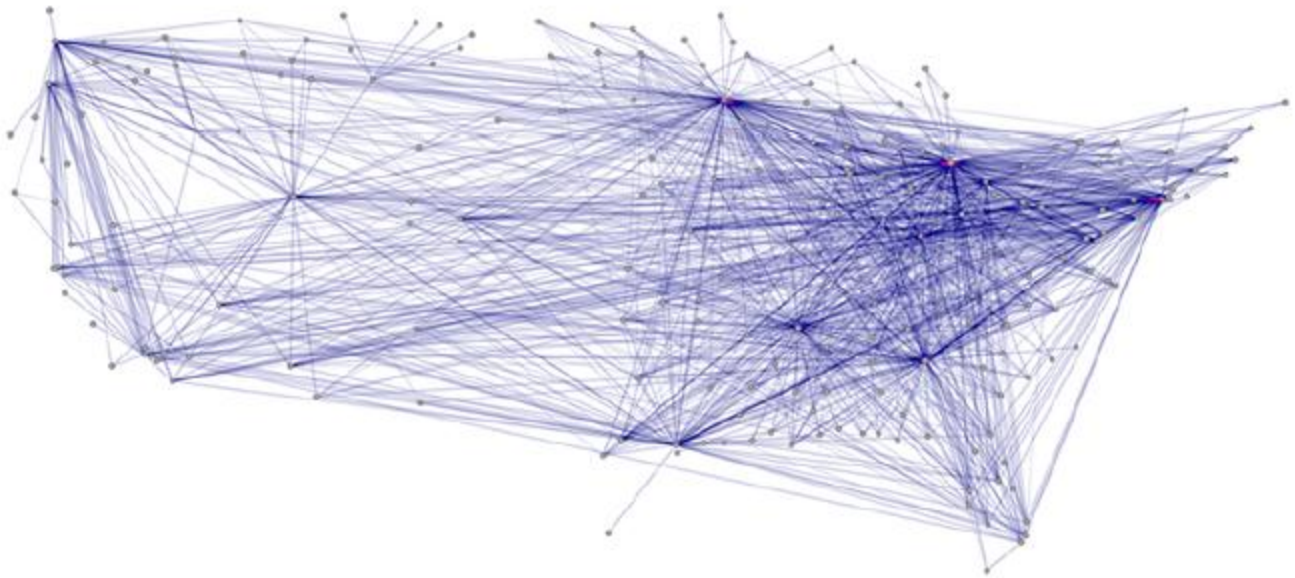
[Burch & Cheswick, Internet Mapping Project]

Transport – Airline Map

Weighted Graph

Vertices = airports, geographical location

Edges= flights from/to, thickness~passengers

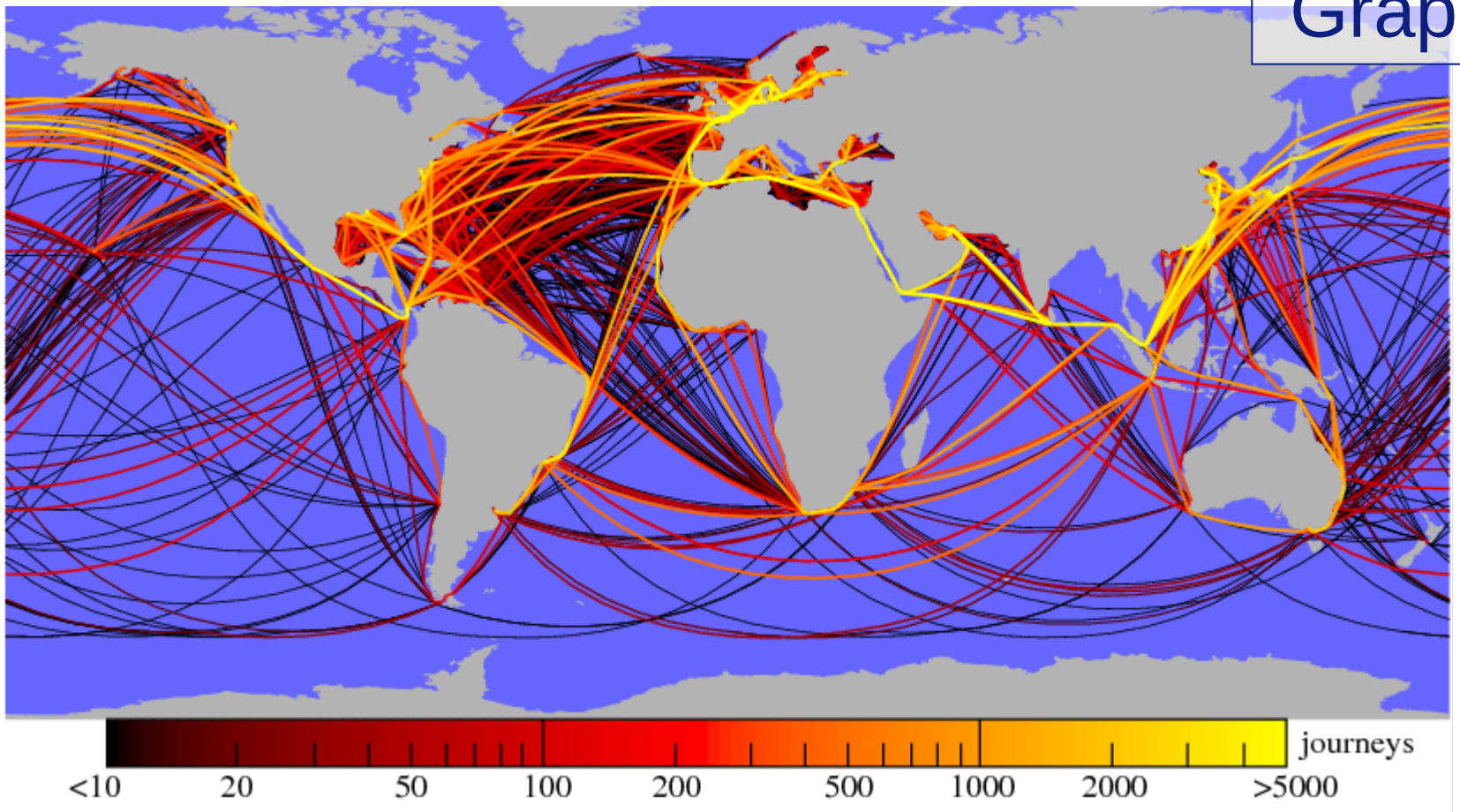


[Holten & van Wijk 2009]

Transport - Cargo Ship Movements

Weighted Directed Graph

Vertices = Ports, Edges = Trips From/To



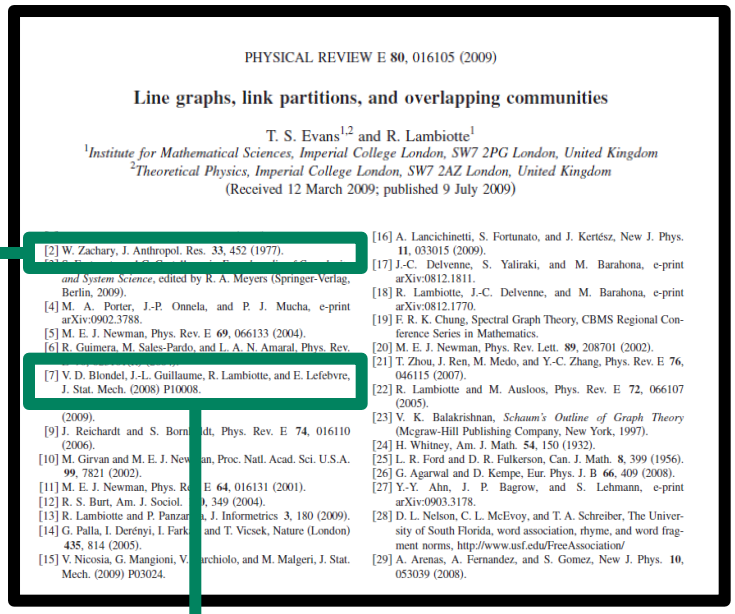
⇒Invasive Species [Kaluza et al, 2009]

Citation Networks

Vertices = papers

Edges = citations *from* bibliography *to* paper

[TSE & Lambiotte, 2009]



PHYSICAL REVIEW E **80**, 016105 (2009)

Line graphs, link partitions, and overlapping communities

T. S. Evans[1,2] and R. Lambiotte[1]
[1]*Institute for Mathematical Sciences, Imperial College London, SW7 2PG London, United Kingdom*
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[2] W. Zachary, J. Anthropol. Res. **33**, 452 (1977).
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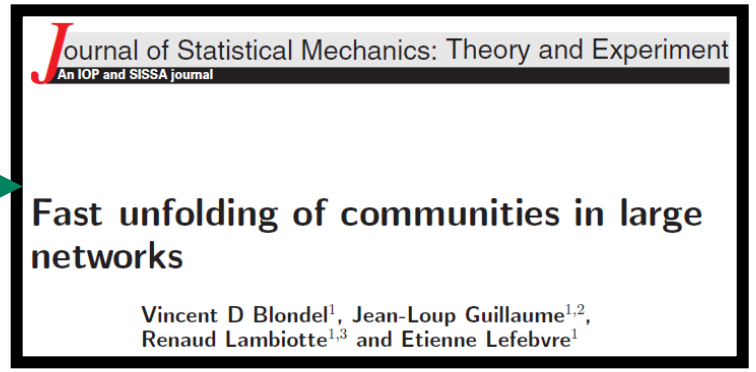
Directed Acyclic Graph

[Blondel et al, 2008]

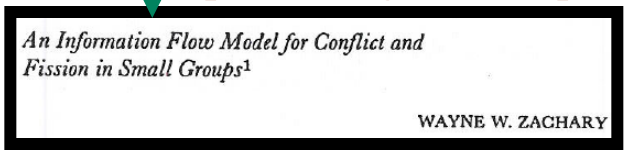


Journal of Statistical Mechanics: Theory and Experiment
An IOP and SISSA journal

Fast unfolding of communities in large networks

Vincent D Blondel[1], Jean-Loup Guillaume[1,2], Renaud Lambiotte[1,3] and Etienne Lefebvre[1]

[Zachary, 1977]



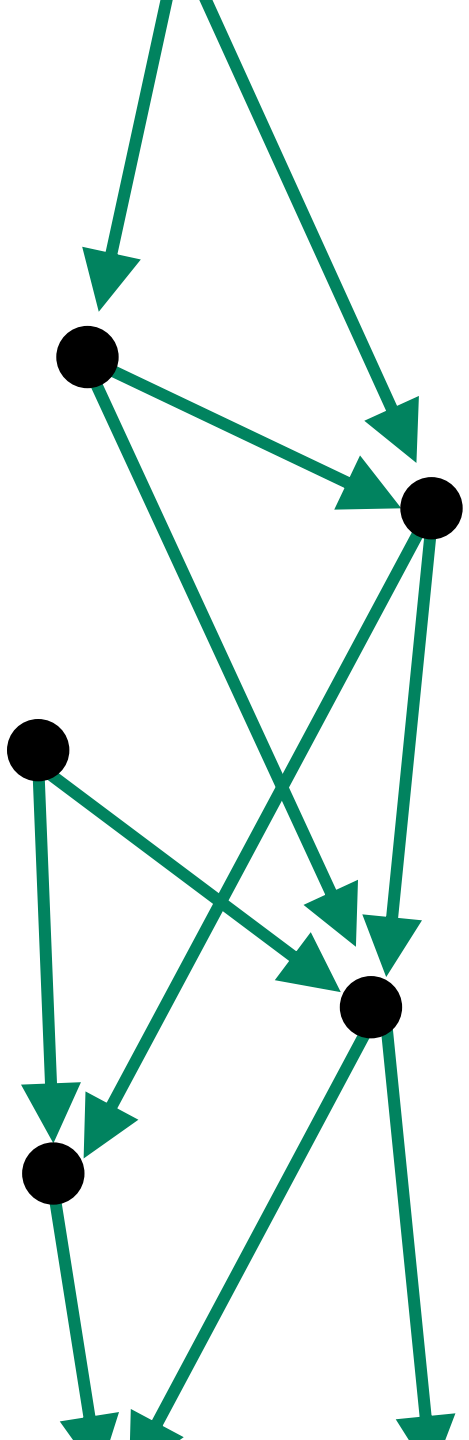
An Information Flow Model for Conflict and Fission in Small Groups[1]

WAYNE W. ZACHARY

Citation Networks

Vertices =
papers

Edges =
citations
from
bibliography
to paper
back in time



TIME

Directed Acyclic Graph

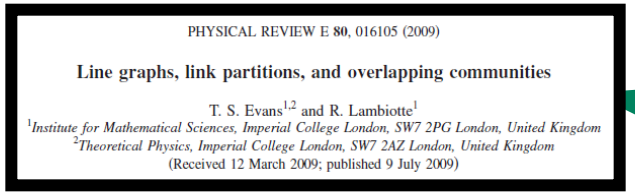
Also used in *Hasse Diagrams*, *Causal Sets*

Coauthorship networks

Vertices =
- papers ***or***
- authors

Edges =
- link authors
- to papers

[TSE & Lambiotte, 2009]



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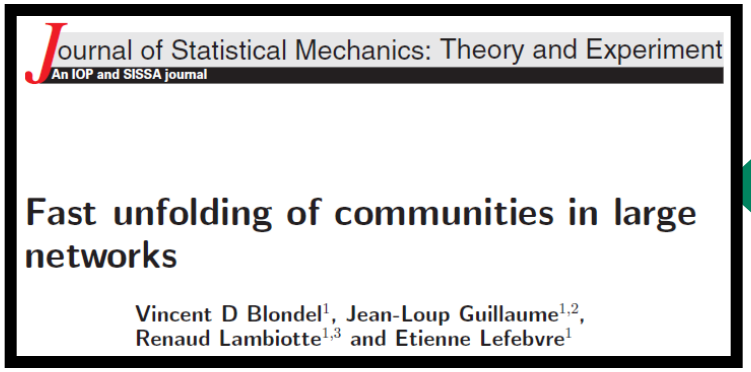
[Blondel et al, 2008]



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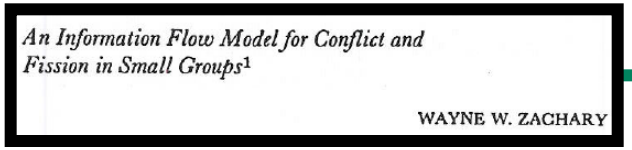

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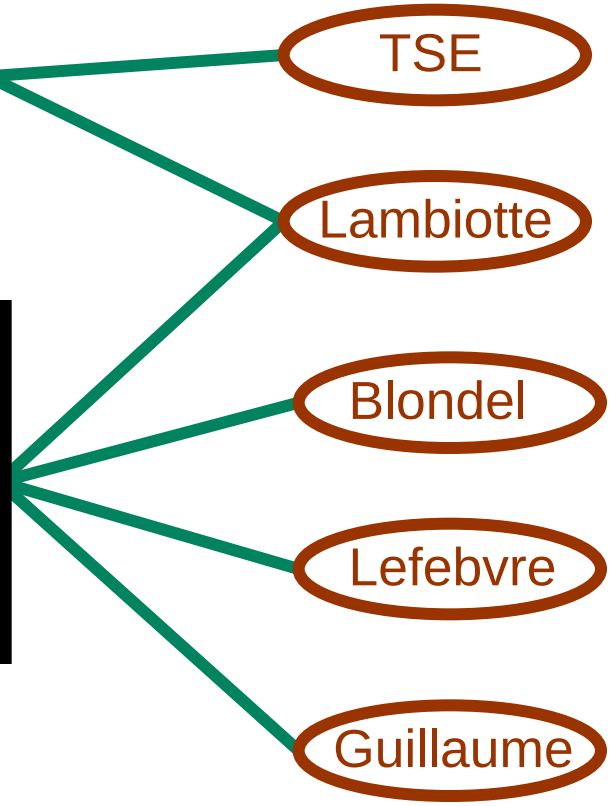
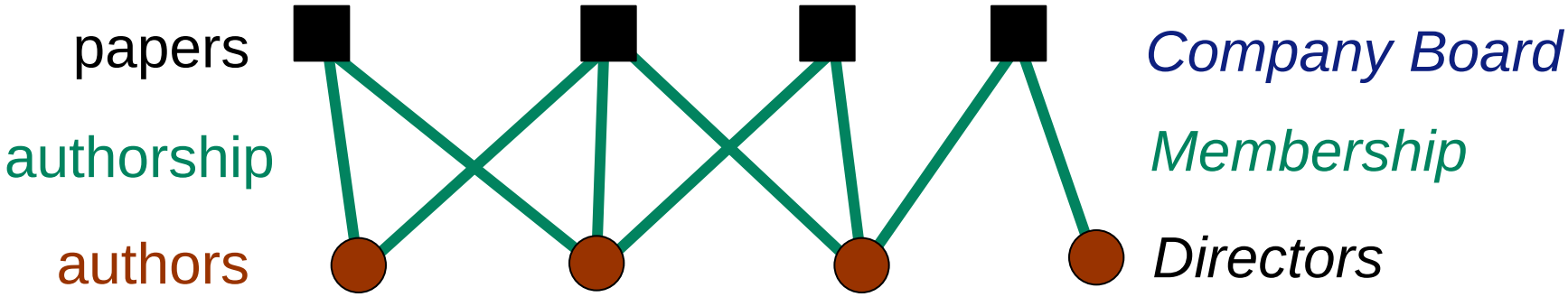
WAYNE W. ZACHARY

Bipartite Graph

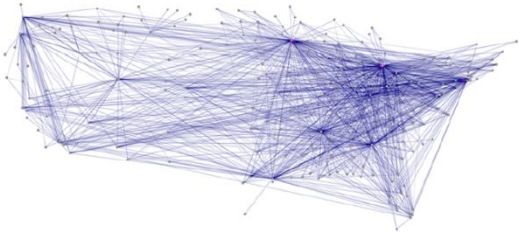


Bipartite Graph

Coauthorship networks



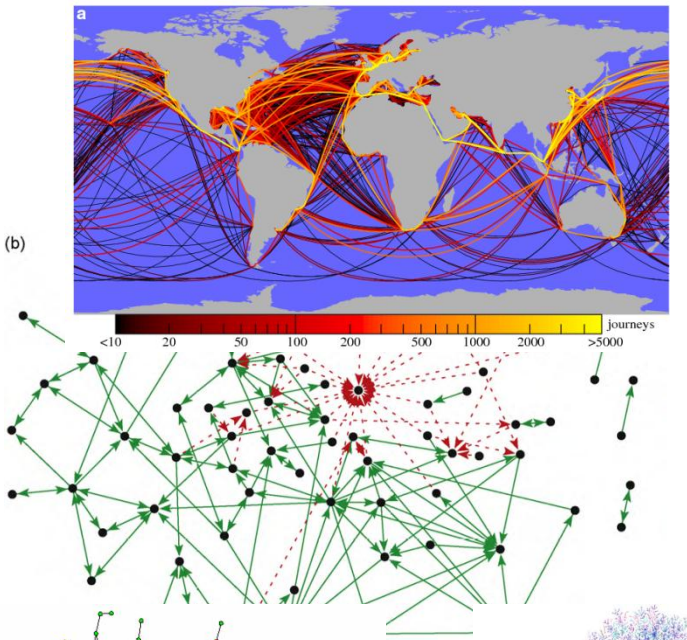
⇒ Reveal relationships between people

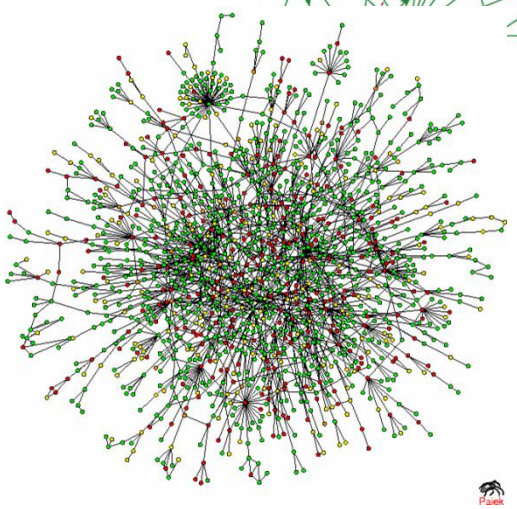


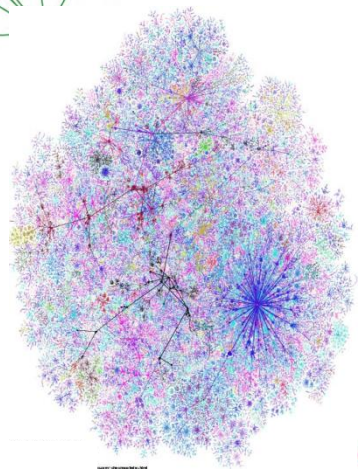
So many networks

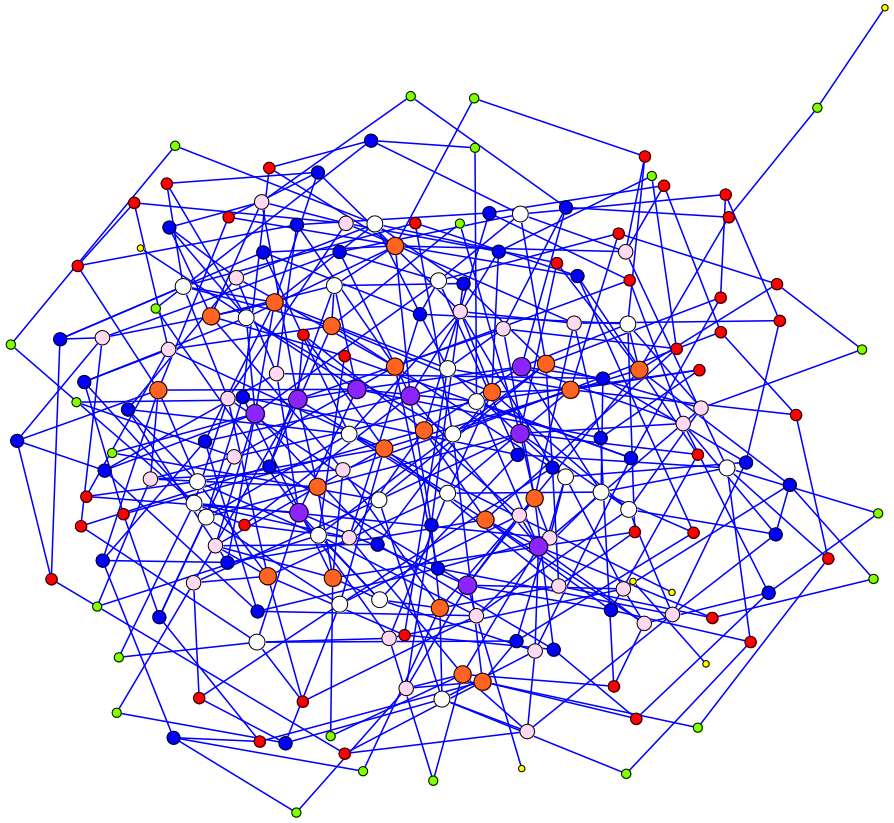
Networks are a useful way to describe many different data sets

- Physical links/Hardware based
- Biological Networks
- Social Networks
- Information Networks









REPRESENTATIONS

Representations

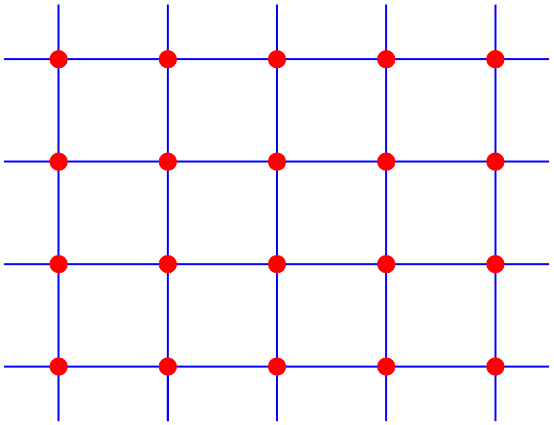
- Data often has a `natural' network
- There is no one way to view this natural network $\Rightarrow$ ***visualisation***
- There are always many different networks representing the data

Visualisation

[Evans, 2004]

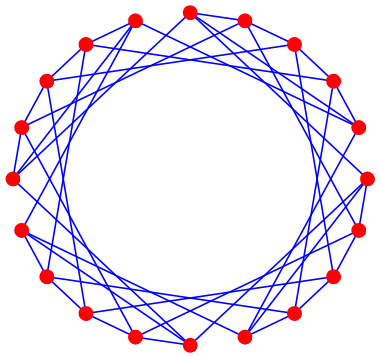
In a network the location of a vertex is defined only by its neighbours

➡ Many Networks are not embedded in a space

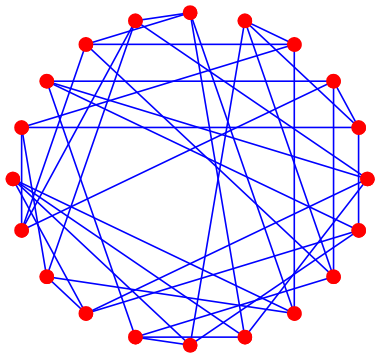


Periodic Lattice

N=20, E=40



Same network with vertices arranged in regular order.

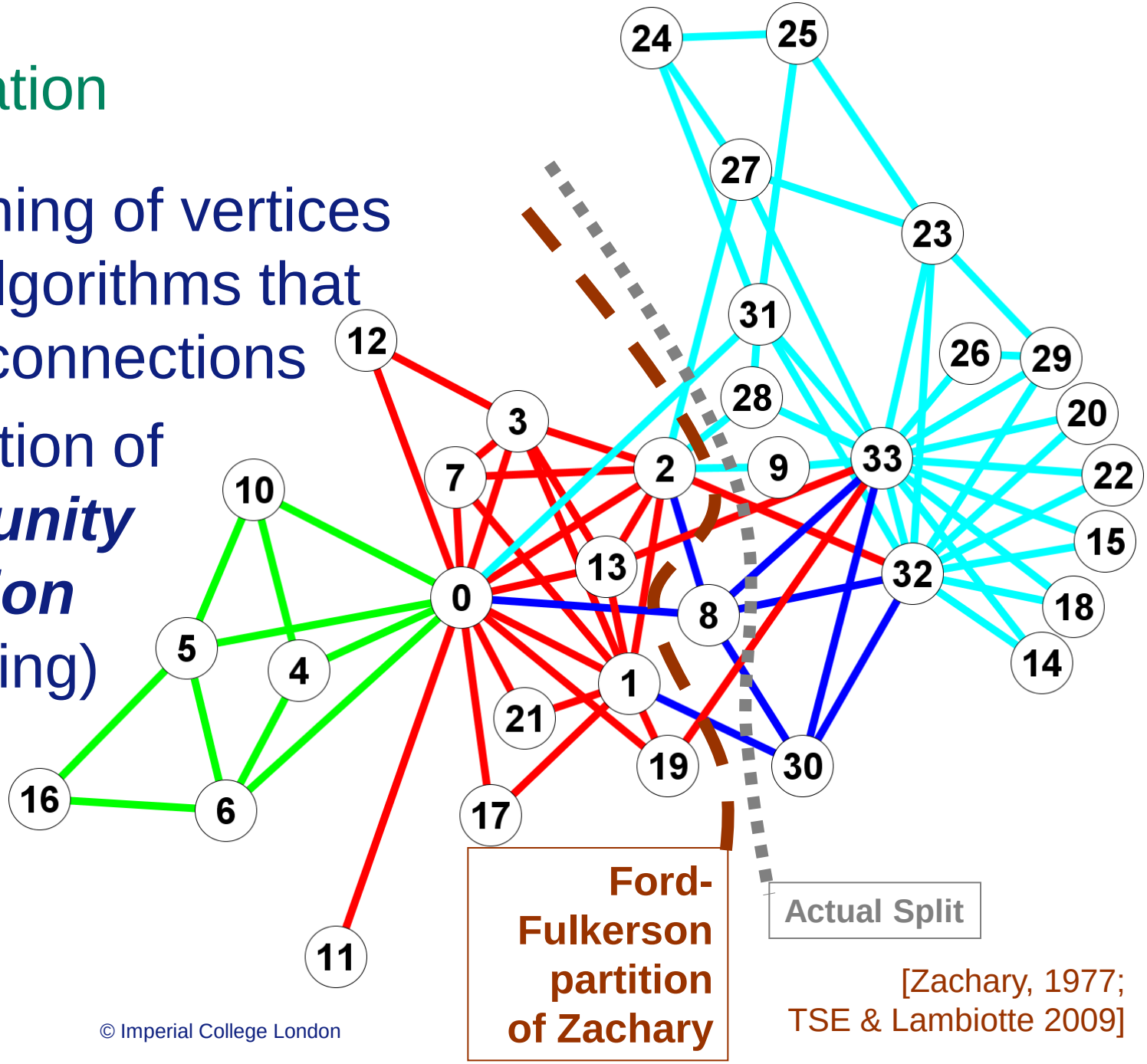


Same network with vertices arranged in random order

Identical Networks

Visualisation

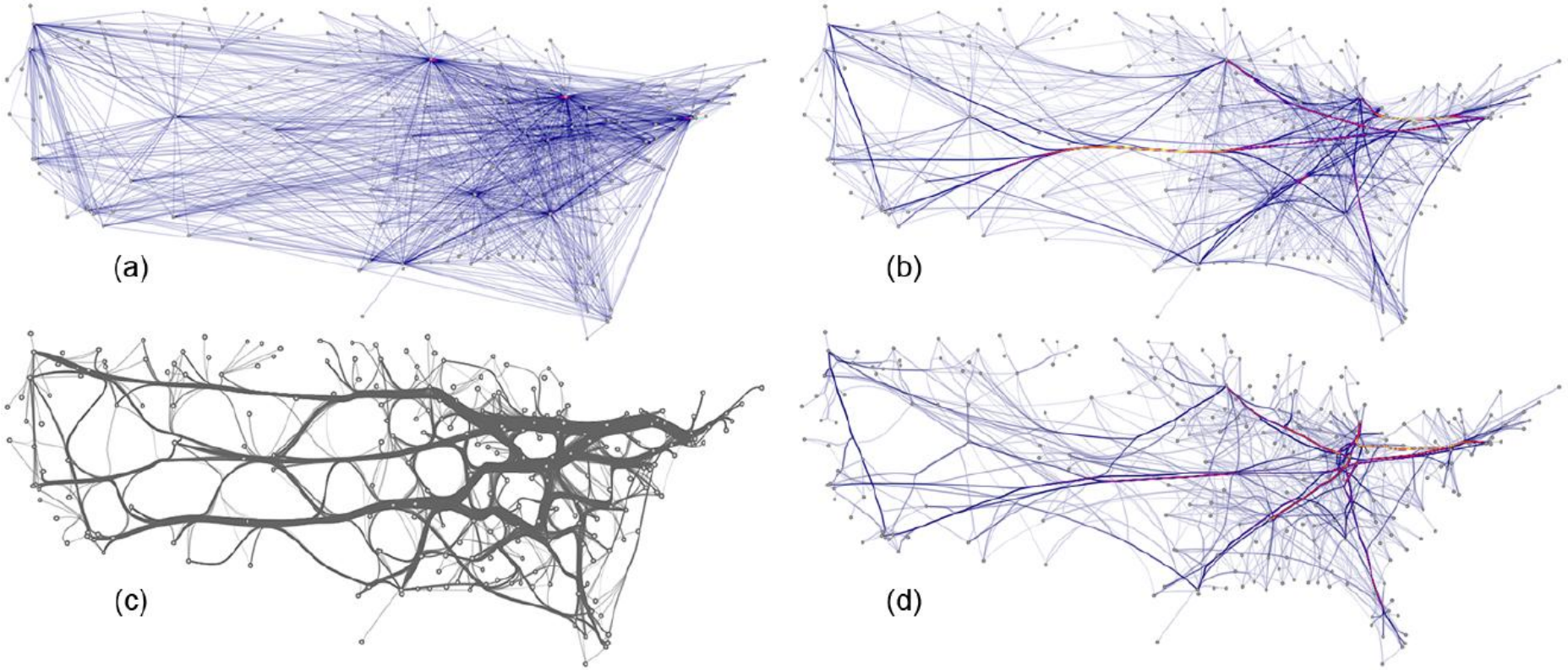
- Positioning of vertices using algorithms that reflect connections
- Application of ***Community Detection*** (clustering)
- Edge colour also



[Zachary, 1977; TSE & Lambiotte 2009]

Visualisation

Choosing the right visualisation is a powerful practical tool, and its not just the vertices ...



e.g. Edge bundling for air network [Holten & van Wijk, 2009]

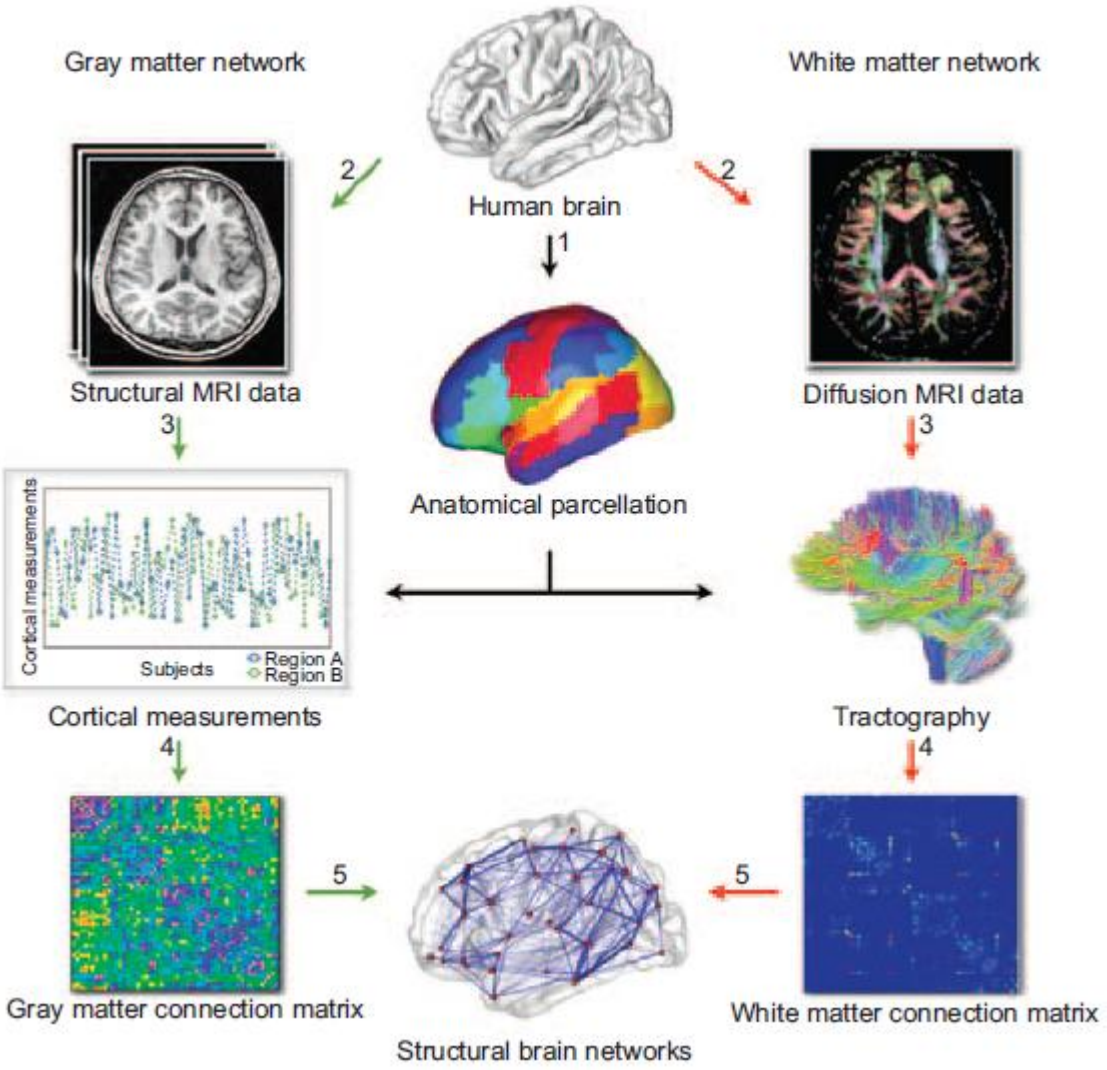
Correlation Matrices

e.g. Neuroscience Networks

Vertices =
brain regions

Matrix =
Correlated
Activity

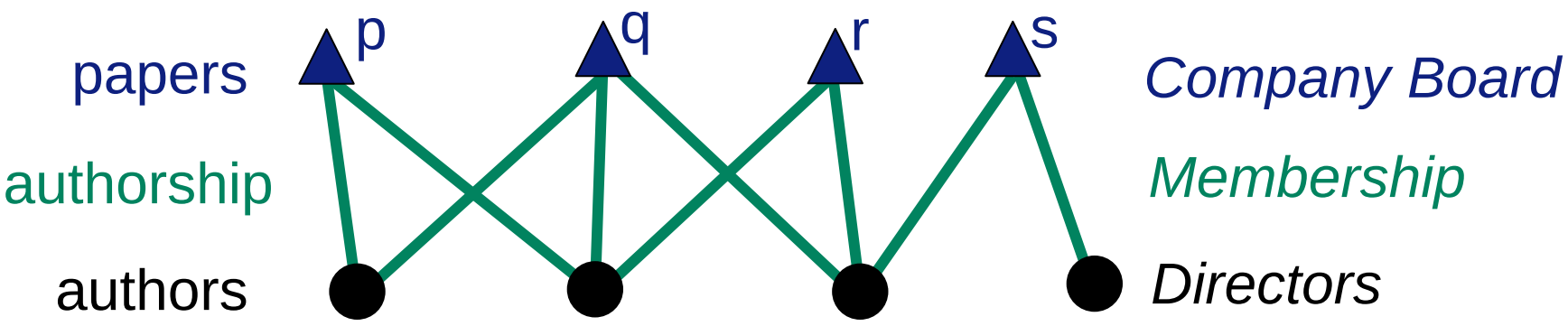
Edges =
Minimum
Correlation



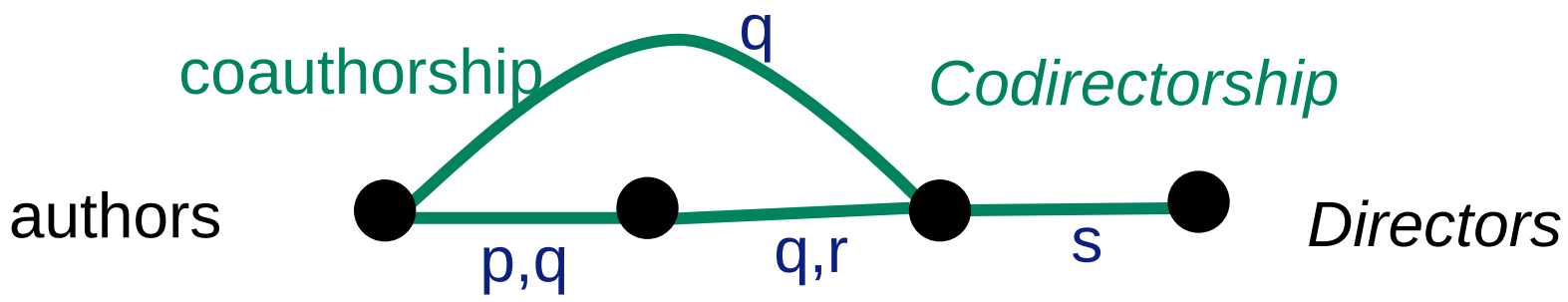
[Lo et al, 2011]

Network Projections

e.g. Coauthorship networks



bipartite network



unipartite network

Line Graphs

Vertices = Intersections

Edges = Streets

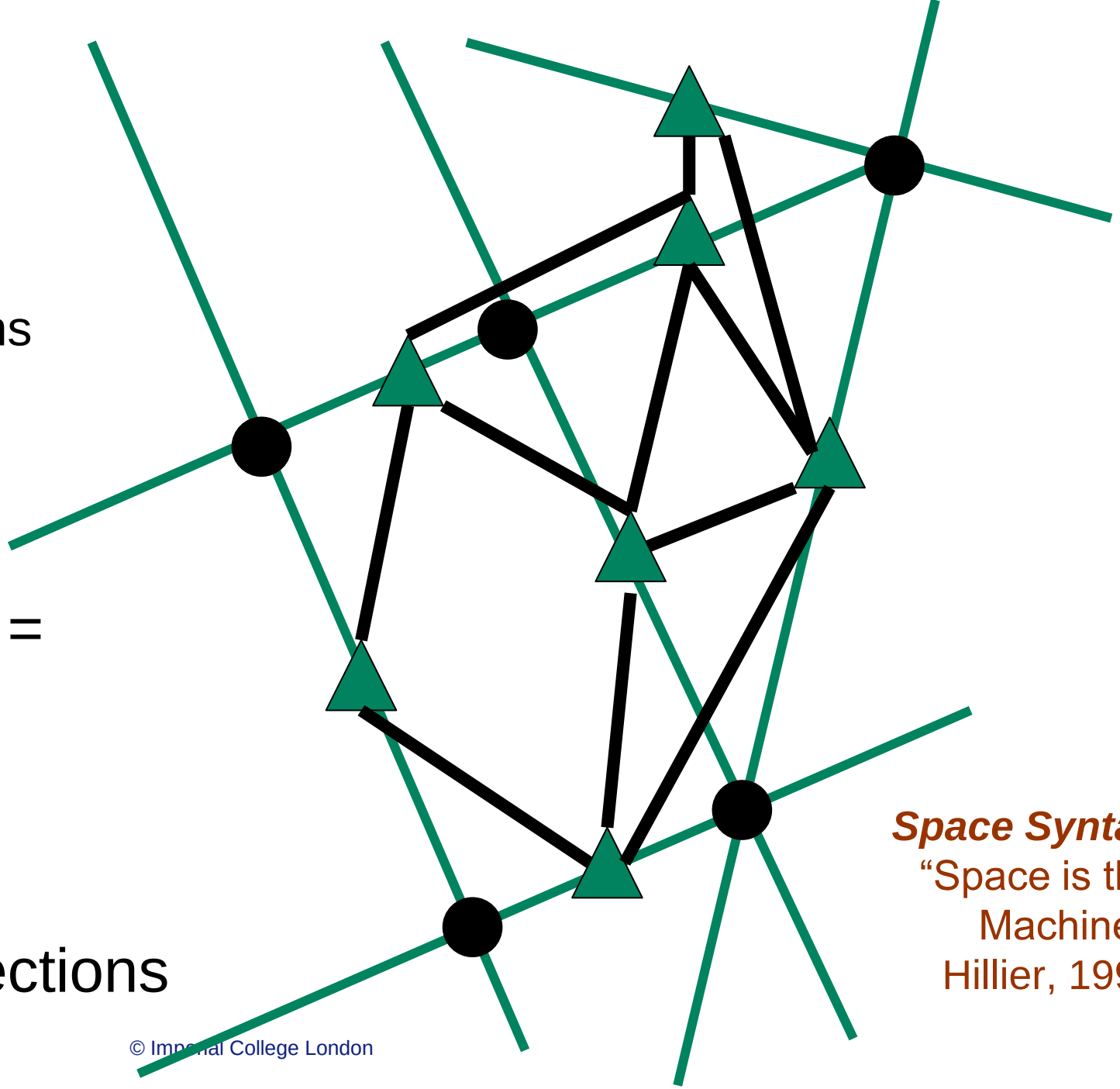
Line Graphs

Vertices = Intersections

Edges = Streets

Vertices = Streets

Edges = Intersections



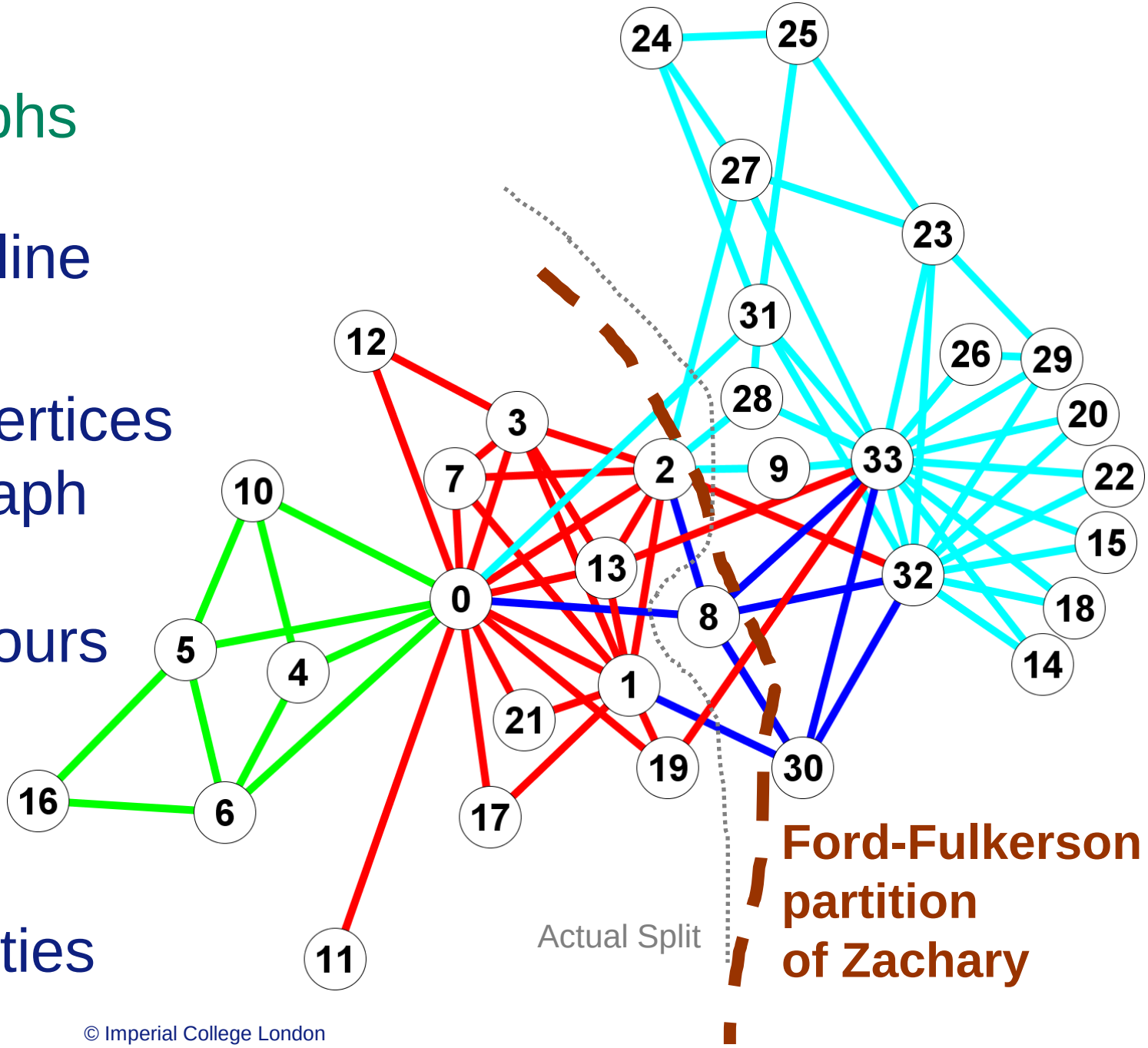
Space Syntax
"Space is the Machine", Hillier, 1996

Line Graphs

- Produce line graph
- Cluster vertices of line graph

  =

  Edge colours
- Vertices in several communities



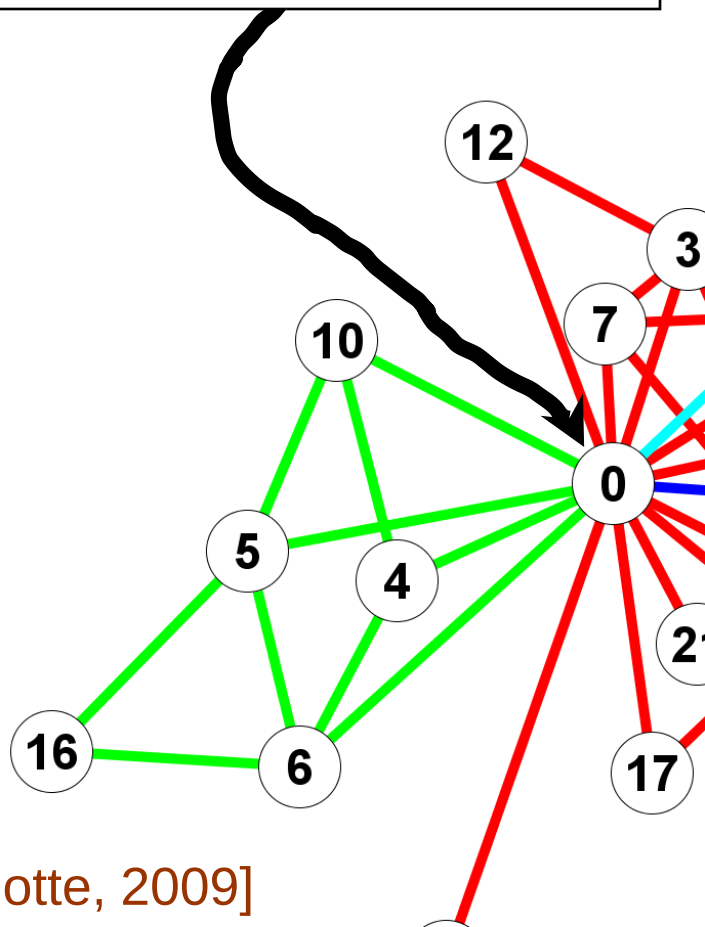
[TSE & Lambiotte, 2009]

Karate Club Analysis

Vertices in One Edge Community

#	k	Fraction k In Green C
5	4	100%
6	4	100%
10	3	100%
4	3	100%
16	2	100%
0 (Mr_Hi)	16	25%

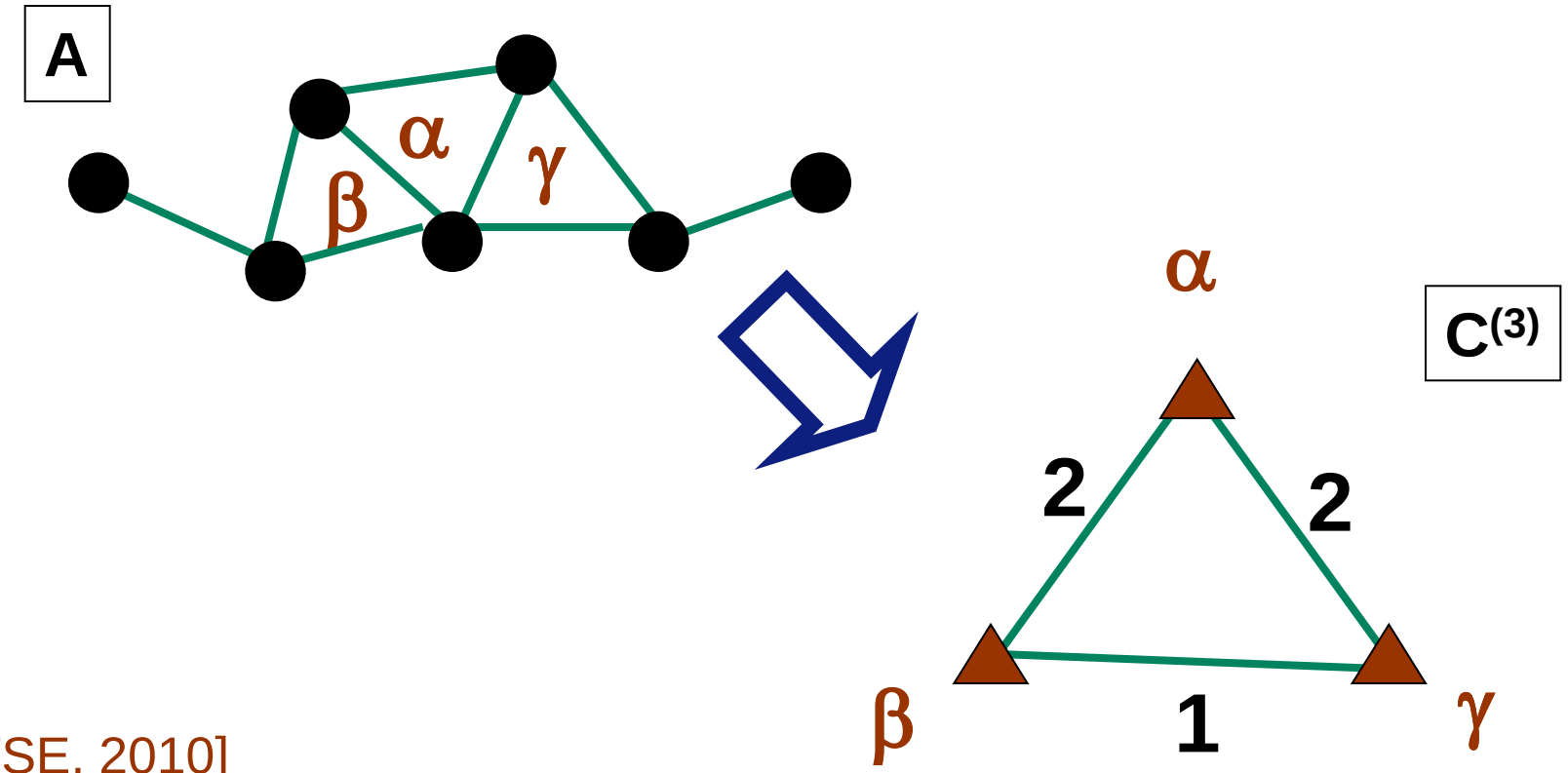
Mr Hi (the Instructor) bridges several groups



[TSE & Lambiotte, 2009]

Clique Graphs

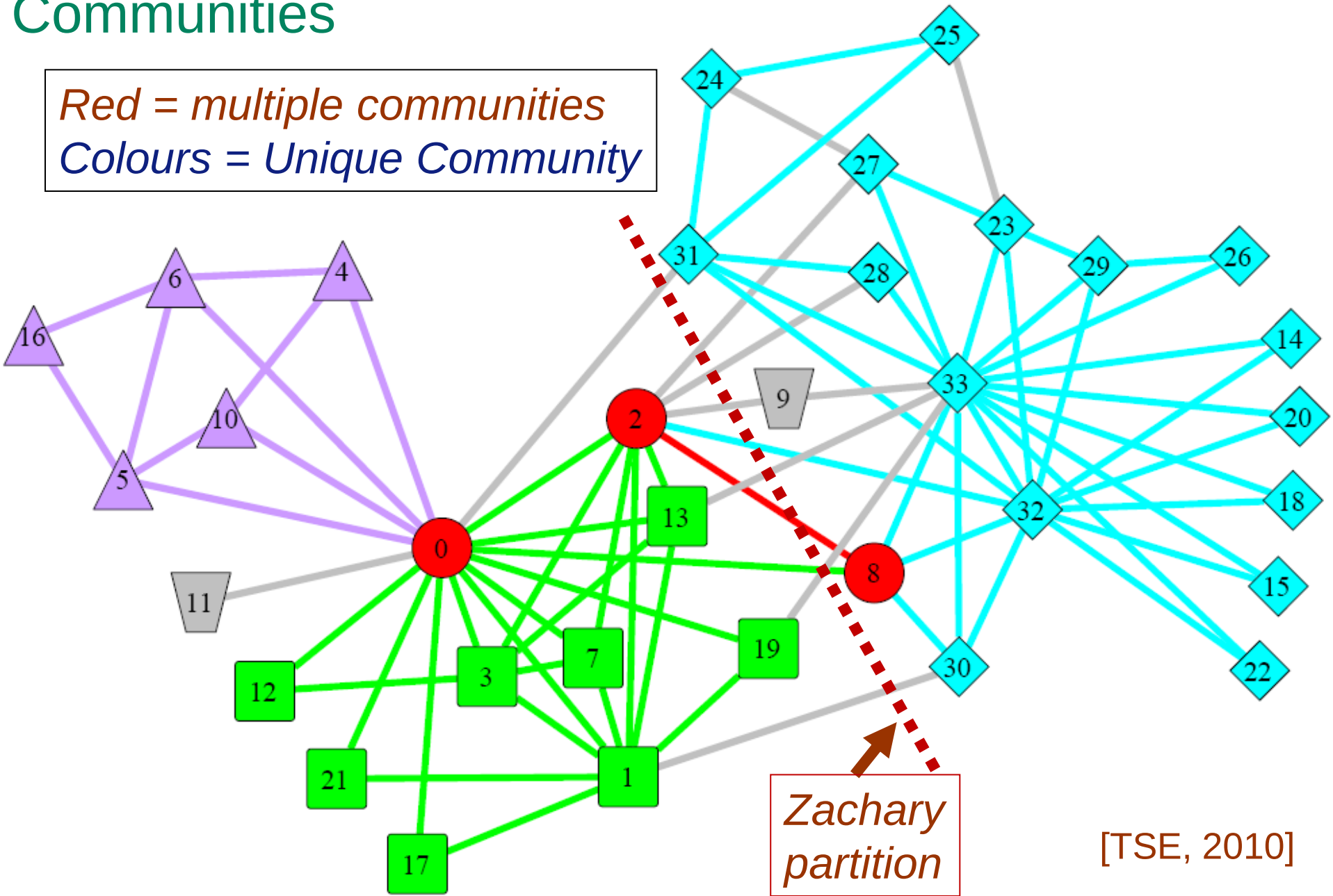
Clique Graphs record the number of vertices common to two cliques in in the original graph ***A*.**



[TSE, 2010]

Karate Club Clique Graph Communities

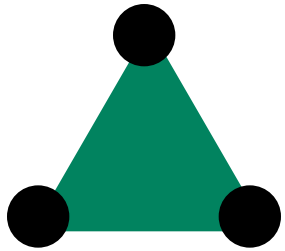
Red = multiple communities
Colours = Unique Community



[TSE, 2010]

Hypergraphs

- A collection of vertices
- A collection of ***hyperedges*** subsets of vertices of any size
  - Subsets of different sizes ***n***
  - These indicate ***n***-times relationships between people
  - No longer just bilateral relationships
- A graph is just a 2-regular hypergraph i.e. n=2 for all hyperedges.



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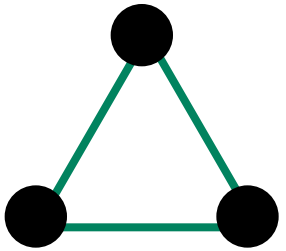
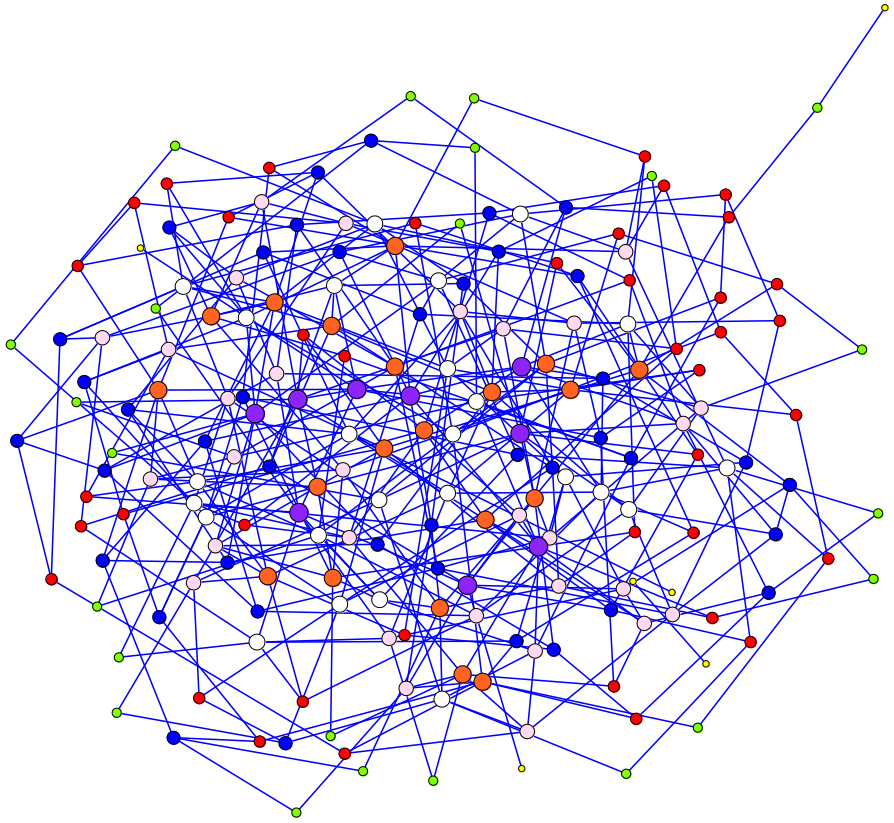
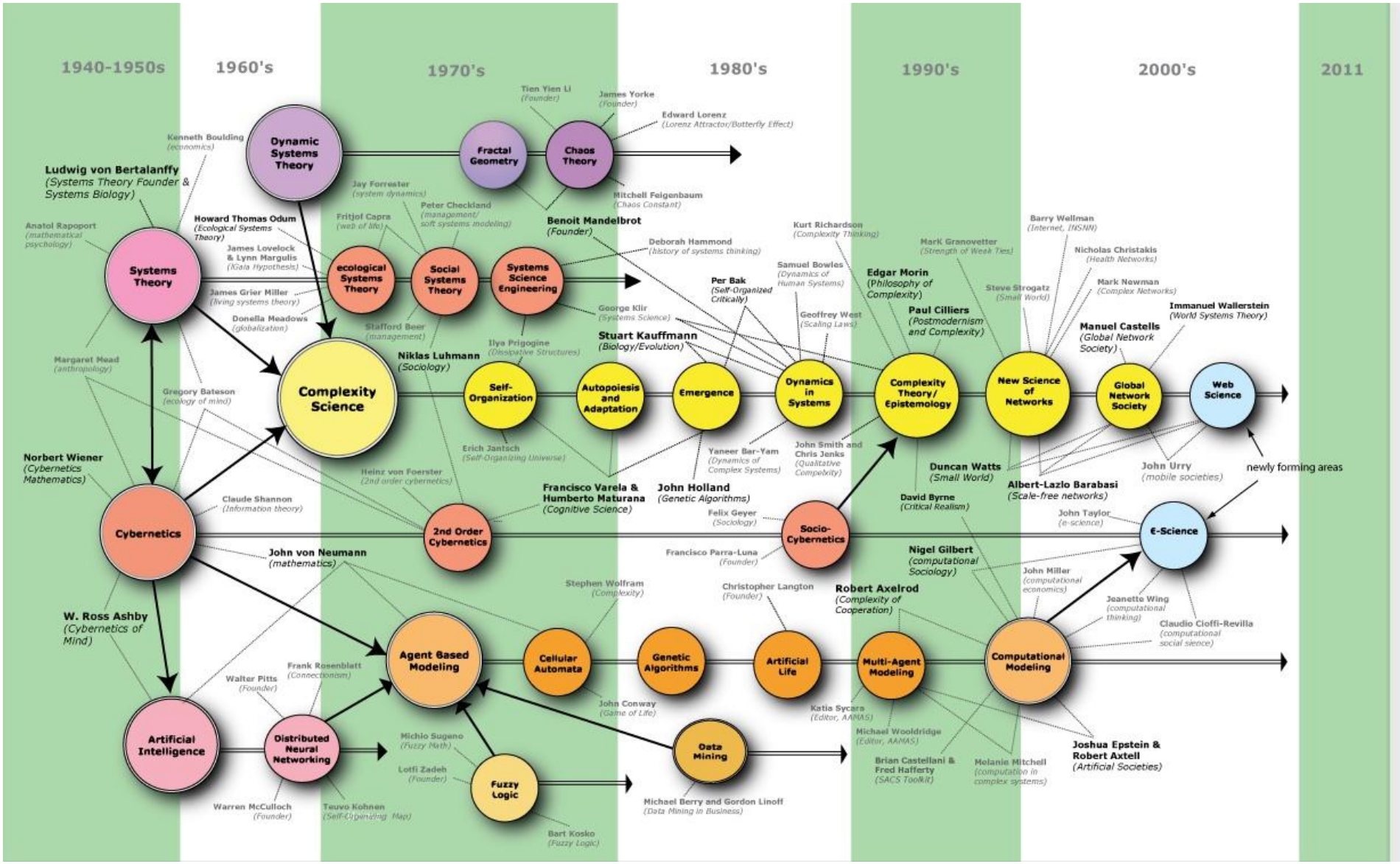




COMPLEXITY

Map of Complexity



["Complexity", Wikipedia, Castellani]

Complexity is

- Difficult
- Interactions occur defined at small, local scales
- Emergence of large scale phenomena

"Life? Don't talk to me about life!"

[Marvin in "The Hitch Hikers Guide to the Galaxy", Douglas Adams]

Just statistical mechanics applied to new problems?

Complexity and Networks

Networks are a natural part of Complexity

- Real networks are difficult
  - Mathematical proofs only for random graphs
  - Computational algorithms often NP-complete
- Edges represent local interactions
- Emergent Phenomena
  - Communities/Clusters
  - Small Worlds “Six degrees of separation”

$\Rightarrow$ Netplexity = Complex Networks

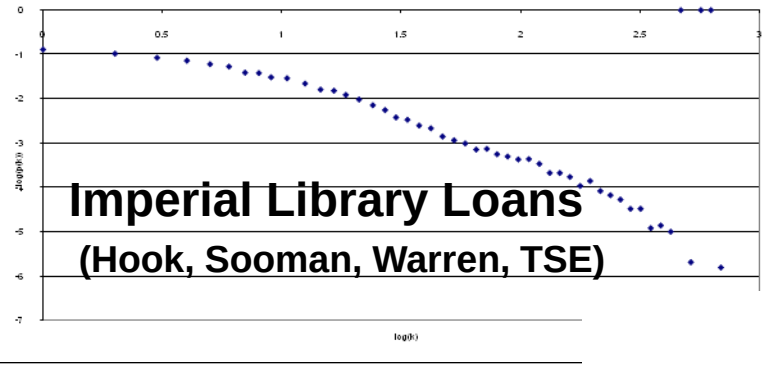
Critical Phenomena and Networks

- Large fluctuations
- Detailed microscopic rules irrelevant
- Universality classes

Models of self-organised criticality often mix ideas from complexity with critical phenomena concepts e.g. Sand pile model

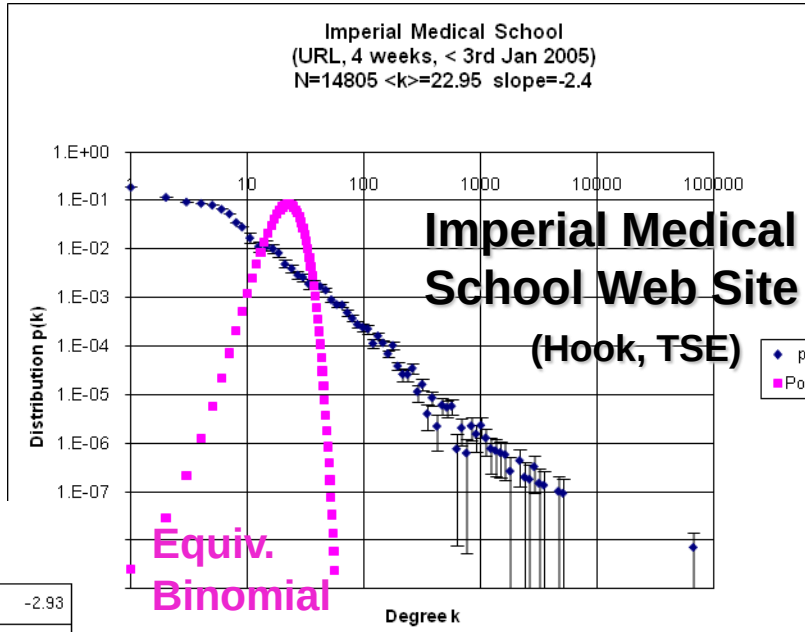
? Are critical phenomena relevant to networks?

Large Fluctuations in Networks



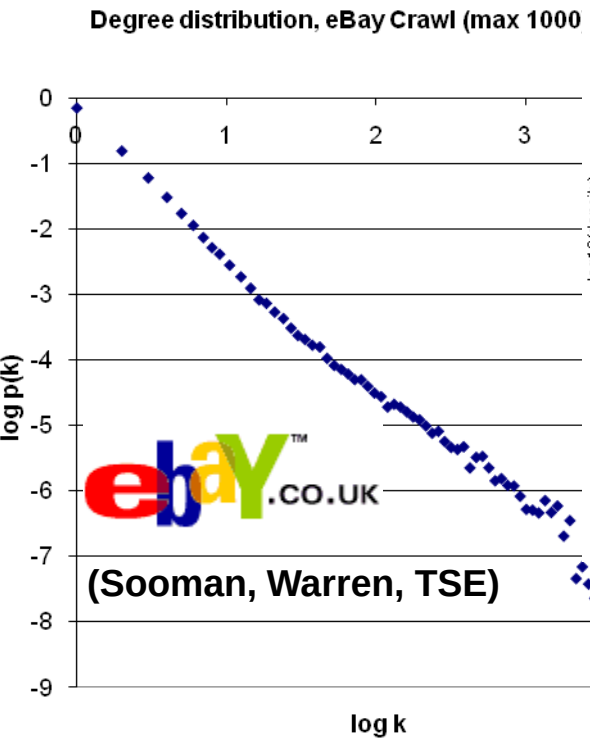
Imperial Library Loans

(Hook, Sooman, Warren, TSE)

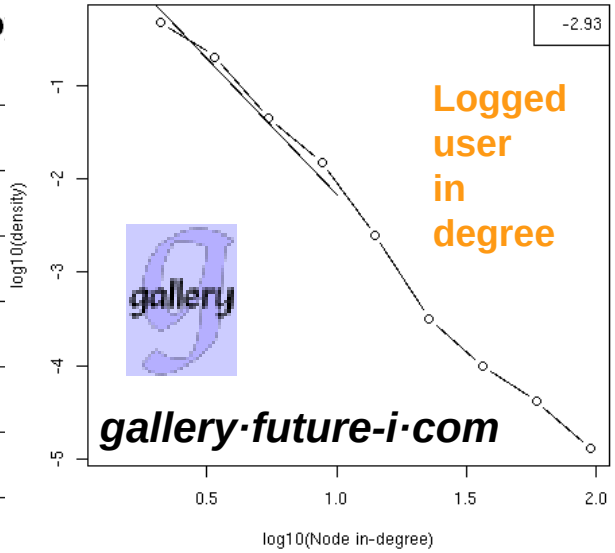


Imperial Medical School Web Site

(Hook, TSE)



(Sooman, Warren, TSE)



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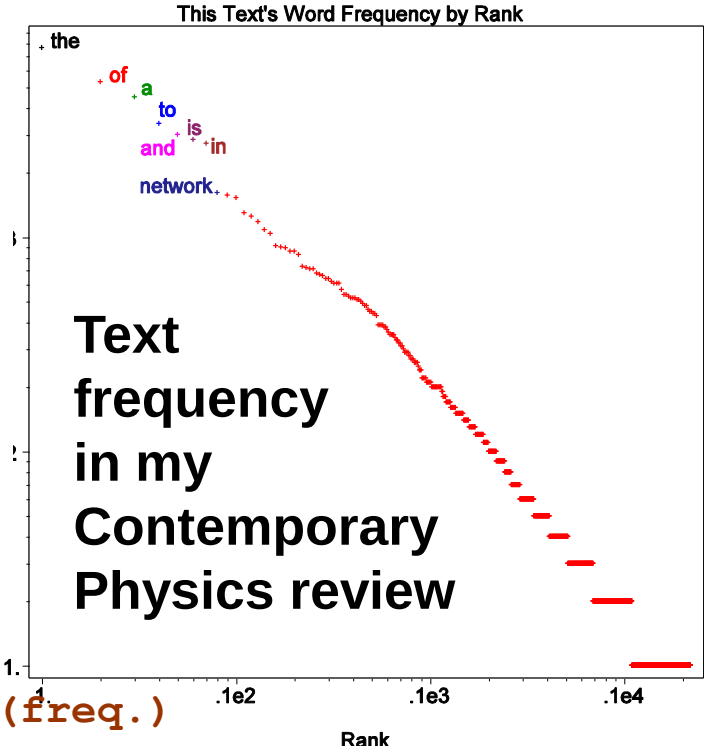


Text frequency in my Contemporary Physics review

[Barabási, Albert 1999]

ege London

All `log(k)` vs. `log(p(k))` except text `log(rank)` vs. `log(freq.)`

Networks are not Critical Systems

No universal behaviour

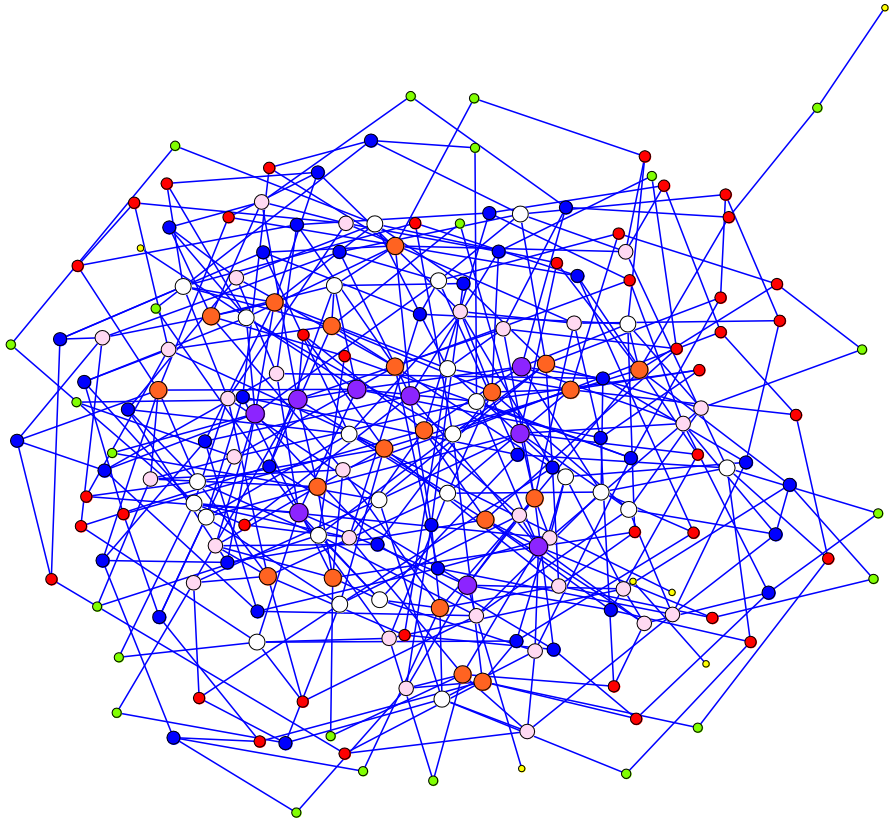
- Power law or other long tails?
  Finite size effects cause uncertainty
  Networks are **mesoscopic systems**

- Value of power in power laws ***not***
  universal even if power law behaviour is.

Criticality, Complexity and Networks

- Networks are usually Complex
- Networks are not usually critical systems

$\Rightarrow$ Netplexity = Complex Networks



NETWORK SCIENCE

What is “Network Science”?

- Based on analysis through networks
  - Graphs, hypergraphs
- Part of wider studies in complexity
  - Local interactions produce emergent phenomena
- Not new
  - Social Network Analysis since 50’s
  - Mathematical graph theory since Euler in 1735
- New aspect is Information Age
  - Large data sets and their analysis now possible
- Multidisciplinary
  - Communication difficult between fields

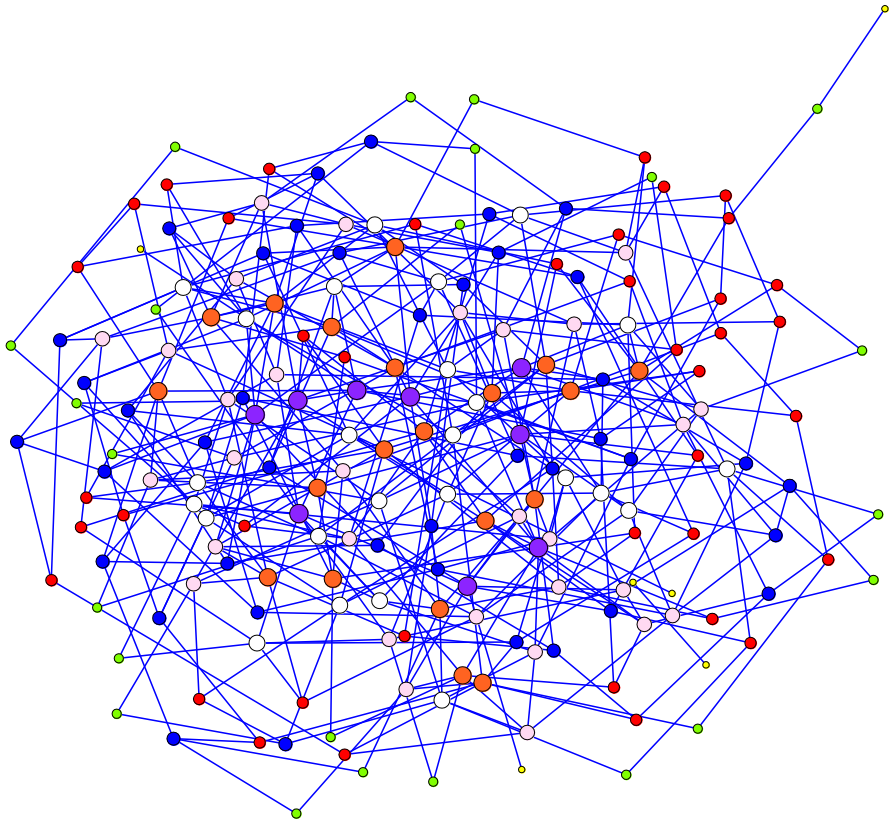
Does “Network Science” really exist?

- No coherent definition
- Too broad to be a single area
- New name for old work = ***Hype***
- Too early to say
- No need to define a new field

[“Network Science”, nap.edu, 2005]

Are networks providing new insights?

- Just another approach to statistical analysis and data mining
- Sometimes this is a better way to analyse
  - Gives new questions e.g. Small world definition
  - Gives new answers e.g. Small world models
- Brings the tools of Complexity
  - Scaling



THANKS

See http://www.imperial.ac.uk/people/T.Evans
or
search for *Tim Evans Networks*

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