Newton-Cartan Geometry and Torsion

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Supergravity, Strings and Dualities:

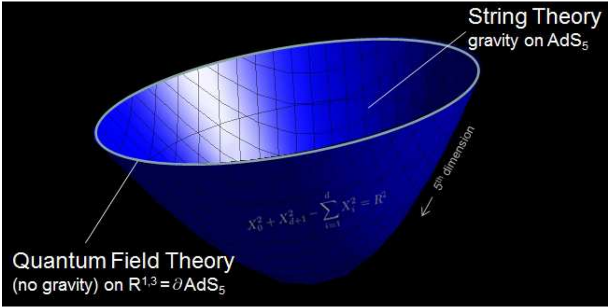
A Meeting in Celebration of Chris Hull's 60th Birthday

London, April 29 2017

rijksuniversiteit groningen

why non-relativistic gravity ?

Holography



Gravity is not only used to describe the gravitational force!

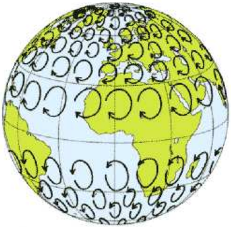
Christensen, Hartong, Kiritsis, Obers and Rollier (2013-2015)

Condensed Matter

Effective Field Theory (EFT) coupled to NC background fields

describe universal features and serve as response functions

compare to



Coriolis force

Luttinger (1964), Greiter, Wilczek, Witten (1989), Son (2005, 2012), Can, Laskin, Wiegmann (2014)

Jensen (2014), Gromov, Abanov (2015), Gromov, Bradlyn (2017)

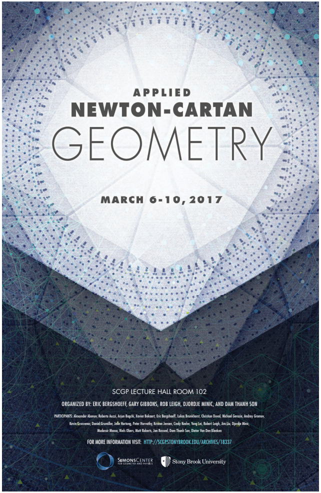
Supersymmetry

supersymmetry allows to apply powerful localization techniques to exactly calculate partition functions of (non-relativistic) supersymmetric field theories

Pestun (2007); Festuccia, Seiberg (2011), Pestun, Zabzine (2016)

This should also apply to the non-relativistic case !

Knodel, Lisbao, Liu (2015)



Simons workshop 'Applied Newton-Cartan Geometry', March 6-10

Outline

A Short Intro to Newton-Cartan Gravity

Outline

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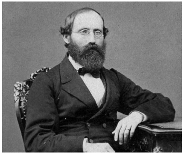
Non-relativistic Gravity

- Free-falling frames: Galilean symmetries

- Constant acceleration: Newtonian gravity/Newton potential $\Phi(x)$

- no frame-independent formulation

  (needs geometry!)



Riemann (1867)

Galilei Symmetries

- time translations: $\delta t = \xi^0$ but not $\delta t = \lambda^i x^i$!
- space translations: $\delta x^i = \xi^i \qquad i = 1, 2, 3$
- spatial rotations: $\delta x^i = \lambda^i{}_j\, x^j$
- Galilean boosts: $\delta x^i = \lambda^i t$

$$[J_{ab}, P_c] = -2\delta_{c[a}P_{b]}\,, \qquad [J_{ab}, G_c] = -2\delta_{c[a}G_{b]}\,,$$

$$[G_a, H] = -P_a\,, \qquad [J_{ab}, J_{cd}] = \delta_{c[a}J_{b]d} - \delta_{a[c}J_{d]b}\,, \qquad a = 1, 2, 3$$

Relativistic versus Non-relativistic Massive Particle

$$S_{\text{relativistic}}(\text{massive}) = -m \int d\tau \sqrt{-\eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} \qquad \mu = 0,1,2,3$$

Lagrangian is invariant under Poincare symmetries

$$S_{\text{non-relativistic}}(\text{massive}) = \frac{m}{2}\int \frac{\dot{x}^i \dot{x}^j \delta_{ij}}{\dot{t}} d\tau \qquad i = 1,2,3$$

Lagrangian is not invariant under Galilean boosts $\delta x^i = \lambda^i t$

The naive non-relativistic limit does not work !

Central Extension

$$\delta L_{\text{non-relativistic}}(\text{massive}) = \frac{d}{d\tau}(m x^i \lambda^j \delta_{ij}) \qquad \Rightarrow$$

modified Noether charge gives rise to central extension:

$$[P_a, G_b] = \delta_{ab} Z$$

Bargmann algebra is centrally-extended Galilei algebra

‘Gauging’ Bargmann

symmetry	generators	gauge field	curvatures
time translations	H	τ_μ	$\mathcal{R}_{\mu\nu}(H)$
space translations	P^a	$e_\mu{}^a$	$\mathcal{R}_{\mu\nu}{}^a(P)$
Galilean boosts	G^a	$\omega_\mu{}^a$	$\mathcal{R}_{\mu\nu}{}^a(G)$
spatial rotations	J^{ab}	$\omega_\mu{}^{ab}$	$\mathcal{R}_{\mu\nu}{}^{ab}(J)$
central charge transf.	Z	m_μ	$\mathcal{R}_{\mu\nu}(Z)$

Imposing Constraints

$\mathcal{R}_{\mu\nu}{}^a(P) = 0\,, \qquad \mathcal{R}_{\mu\nu}(Z) = 0$: solve for spin-connection fields

$\mathcal{R}_{\mu\nu}(H) = \partial_{[\mu}\tau_{\nu]} = 0 \;\rightarrow\; \tau_\mu = \partial_\mu \tau$: absolute time (‘zero torsion’)

The NC Transformation Rules

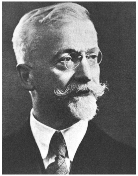
The independent NC fields $\{\tau_\mu, e_\mu{}^a, m_\mu\}$ transform as follows:

$$\delta\tau_\mu = 0\,,$$
$$\delta e_\mu{}^a = \lambda^a{}_b\, e_\mu{}^b + \lambda^a\tau_\mu\,,$$
$$\delta m_\mu = \partial_\mu\sigma + \lambda_a\, e_\mu{}^a$$

The spin-connection fields $\omega_\mu{}^{ab}$ and $\omega_\mu{}^a$ are functions of e, τ and m

The NC Equations of Motion

The NC equations of motion are given by



Élie Cartan 1923

$$\tau^\mu e^\nu{}_a \mathcal{R}_{\mu\nu}{}^a(G) = 0 \qquad \mathbf{1}$$

$$e^\nu{}_a \mathcal{R}_{\mu\nu}{}^{ab}(J) = 0 \qquad \mathbf{a} + (\mathbf{ab})$$

- after gauge-fixing and assuming flat space the first NC e.o.m. becomes $\triangle\Phi = 0$
- there is no known action that gives rise to these equations of motion

Outline

Why Torsion ?

Torsion occurs both in Holography and in Condensed Matter

- Lifshitz Holography

  zero torsion, i.e. $\partial_\mu \tau_\nu - \partial_\nu \tau_\mu = 0$, is not conformal invariant

  Christensen, Hartong, Kiritsis, Obers and Rollier (2013-2015)

- Condensed Matter

  coupling to energy and energy flux requires arbitrary torsion

  Luttinger (1964); Gromov, Abanov (2014); Geracie, Golkar, Roberts (2014); Jensen (2014)

Twistless Torsional

- zero torsion: $\tau_{\mu\nu} \equiv 2\partial_{[\mu}\tau_{\nu]} = 0 \;\rightarrow\; \tau_\mu = \partial_\mu \tau$: absolute time

- twistless torsional or hypersurface orthogonal:

$$\tau_{ab} \equiv e_a^\mu e_b^\nu \tau_{\mu\nu} = 0$$

  is conformal invariant due to identity $e_a{}^\mu \tau_\mu = 0$

- arbitrary torsion : $\tau_\mu(x) = e^{\psi(x)}\,\delta_{\mu,0}$

geometric constraint	Newton-Cartan
$\tau_{0a} = 0\,,\tau_{ab} = 0$	zero torsion
$\tau_{0a} \neq 0\,,\tau_{ab} = 0$	twistless-torsional
$\tau_{0a} \neq 0\,,\tau_{ab} \neq 0$	arbitrary torsion

Null-reduction of General Relativity

Duval, Burdet, Kunzle and Perrin (1985); Julia, Nicolai (1994)

$$x^M = \{x^\mu, v\}\,, \qquad M = (\mu, v)\,, \qquad A = (a, +, -)$$

null Killing vector $\xi = \partial_v$: $\quad \xi^2 = \xi^M \xi^N G_{MN} = 0 \quad \Rightarrow \quad G_{vv} = 0$

Reduction Ansatz (only e.o.m.)

$$E_M{}^A = \begin{array}{c} \\ \mu \\ v \end{array} \begin{array}{c} \begin{array}{ccc} a & - & + \end{array} \\ \begin{pmatrix} e_\mu{}^a & \tau_\mu & -m_\mu \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

$\Lambda_{AB} \quad \Rightarrow \quad \Lambda_{ab}$ (spatial rotations) and Λ_{-a} (Galilean boosts)

Arbitrary Torsion?

off-shell we obtain the transformation rules of NC geometry with arbitrary torsion, i.e. $\tau_{\mu\nu} \equiv \partial_{[\mu}\tau_{\nu]} \neq 0$, leading to modified spin-connections:

$$\Omega_\mu{}^{ab} = \omega_\mu{}^{ab}(\tau, e, m) - m_\mu \tau^{ab}\,, \qquad \Omega_\mu{}^{a} = \omega_\mu{}^{a}(\tau, e, m) + m_\mu \tau_0{}^{a}$$

see also Festuccia, Hansen, Hartong and Obers (2016)

on-shell Julia and Nicolai obtain conditions leading to zero torsion such as

$$\hat{R}_{++} = \tau^{ab}\tau_{ab} = 0 \quad \to \quad \tau_{ab} = 0$$

The reduced E.O.M. are invariant under central charge transformations

Einstein-Hilbert $\Leftrightarrow$ CFT of free real scalar

Poincare gravity = conformal gravity + compensating scalar ϕ

$$\mathcal{L} = \tfrac{1}{2}\phi\partial^\mu\partial_\mu\phi\,, \quad \text{with} \quad \delta\phi = w\Lambda_D\phi\,, \quad \rightarrow$$

$$\mathcal{L} = \tfrac{1}{2}\phi\,(D^a)^c(D_a)^c\phi_{|\phi=1} \quad \rightarrow \quad \mathcal{L} = R$$

$$\mathcal{L} = R \quad \text{and} \quad (e_\mu{}^a)^{\mathrm{P}} = \phi\,(e_\mu{}^a)^{\mathrm{C}}$$

$$\text{plus} \quad (e_\mu{}^a)^{\mathrm{C}} = \delta_\mu{}^a \quad \rightarrow \quad \mathcal{L} = \tfrac{1}{2}\phi\,\partial^\mu\partial_\mu\phi$$

Non-relativistic

- action $\Rightarrow$ E.O.M. (not all!)
- conformal symmetries $\Rightarrow$ $z = 2$ Schrödinger symmetries
- real scalar $\Rightarrow$ complex scalar (dilatations + central charge)

NC Gravity with Zero Torsion

$$\text{foliation constraint}: \quad \partial_\mu \tau_\nu - \partial_\nu \tau_\mu = 0$$

$$\text{NC E.O.M.}: \quad \tau^\mu e^\nu{}_a \mathcal{R}_{\mu\nu}{}^a(G) = 0\,, \qquad e^\nu{}_a \mathcal{R}_{\mu\nu}{}^{ab}(J) = 0$$

Conformal method leads to ($\Psi = \varphi e^{i\chi}$)

$$\text{SFT1}: \quad \partial_0\partial_0\varphi = 0 \quad \text{and} \quad \partial_a\varphi = 0$$

NC Gravity with Arbitrary Torsion

Chatzistavrakidis, Romano, Rosseel + E.B., work in progress

for arbitrary torsion there is no constraint $\partial_a\varphi = 0 \quad \rightarrow$ use the second compensating scalar χ to restore Schrödinger invariance:

$$\delta\chi = M\sigma - \lambda_a x^a \quad \Rightarrow \quad \delta(\partial_a\chi) = -\lambda_a \quad \Rightarrow$$

$$\mathrm{SFT2}: \quad \partial_0\partial_0\varphi - \frac{2}{M}(\partial_0\partial_a\varphi)\partial_a\chi + \frac{1}{M^2}(\partial_a\partial_b\varphi)\partial_a\chi\partial_b\chi = 0$$

Due to the presence of the second compensating scalar χ the central charge transformations are broken.

Option 1: Scherk-Schwarz Null-reduction

Consider GR in $d+1$ dimensions plus a real scalar $\hat{\chi}(\hat{x})$

Perform a Scherk-Schwarz reduction with

$$\hat{\chi}(\hat{x}) = Mv + \chi(x) \quad \rightarrow \quad \partial_v \hat{\chi} = M \quad \Rightarrow$$

$$\hat{R}_{++} = \tau^{ab}\tau_{ab} - M^2 = 0$$

Option 2: Null-reduction of Conformal Gravity

after null-reduction the isotropic scaling $\delta E_M{}^A = \Lambda_{\rm D}\, E_M{}^A$ of conformal gravity becomes an-isotropic:

$$\delta e_\mu{}^a = \lambda_{\rm D}\, e_\mu{}^a\,, \qquad\qquad \delta\tau_\mu = 2\lambda_{\rm D}\,\tau_\mu$$

conformal gravity $\Rightarrow$ $z = 2$ Schrödinger gravity with arbitrary torsion

Note: contraction of conformal algebra gives Galilean Conformal Algebra

Alternative derivation: gauging the $z = 2$ Schrödinger algebra

Outline

Torsion

Torsional NG geometry is needed in holography and condensed matter

Supersymmetric Null-reduction

- 3d NC SUGRA is under control

  R. Andringa, J. Rosseel, E. Sezgin + E.B. (2013)

- SUSY requires Newton potential Φ and dual Newton potential Ψ

- 4d NC SUGRA?

- different realizations Newton potential?, physical effects?

Happy Birthday, Chris !