Relating the RNS and Pure Spinor Formalisms of the Superstring

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Outline

1) non-minimal RNS variables and non-minimal pure spinor variables

2) RNS BRST operator = pure spinor BRST operator

3) Relation of RNS and pure spinor picture-changing operators

4) RNS vertex operators = pure spinor vertex operators (integer picture for bosonic states and half-integer picture for fermionic states)

5) RNS amplitude prescription = pure spinor amplitude prescription (up to subtleties involving regulator)

Non-minimal variables

$$\text{minimal } RNS : (x^m, \psi^m) \quad (c, b) \quad (\gamma, \beta)$$

$$\text{non} - \text{minimal } RNS : (\widehat{\theta}^\alpha, \widehat{p}_\alpha) \quad (\lambda^\alpha, w_\alpha) \quad (r_\alpha, s^\alpha) \quad (\bar{\lambda}_\alpha, \bar{w}^\alpha)$$

$$\lambda\gamma^m\widehat{\theta}^\alpha = \lambda\gamma^m\lambda = 0, \quad \bar{\lambda}\gamma^m r = \bar{\lambda}\gamma^m\bar{\lambda} = 0$$

$$\text{minimal } PS : (x^m, \theta^\alpha, p_\alpha) \quad (\lambda^\alpha, w_\alpha)$$

$$\text{non} - \text{minimal } PS : (r_\alpha, s^\alpha) \quad (\bar{\lambda}_\alpha, \bar{w}^\alpha) \quad (c, b) \quad (\widehat{\gamma}, \widehat{\beta})$$

$$\bar{\lambda}\gamma^m r = \bar{\lambda}\gamma^m\bar{\lambda} = 0$$

56 bosonic + 56 fermionic variables

$$Q_{RNS} = \int dz(cT_{RNS} + \gamma\psi^m \partial x_m + \gamma^2 b) + \int dz(\lambda^\alpha \widehat{p}_\alpha + \bar{w}^\alpha r_\alpha)$$

$$S = \int d^2 z[\frac{1}{2}\partial x^m \bar{\partial} x_m + \frac{1}{2}\psi^m \bar{\partial}\psi_m + b\bar{\partial}c + \beta\bar{\partial}\gamma$$

$$+\widehat{p}_\alpha \bar{\partial}\widehat{\theta}^\alpha + w_\alpha \bar{\partial}\lambda^\alpha + s^\alpha \bar{\partial} r_\alpha + \bar{w}^\alpha \bar{\partial}\bar{\lambda}_\alpha]$$

$$Q_{PS} = \int dz(\lambda^\alpha d_\alpha) + \int dz(\bar{w}^\alpha r_\alpha + b\widehat{\gamma})$$

$$d_\alpha = p_\alpha + \partial x^m (\gamma_m \theta)_\alpha + \frac{1}{8}(\gamma^m \theta)_\alpha (\theta\gamma_m \partial\theta)$$

$$S = \int d^2 z[\frac{1}{2}\partial x^m \bar{\partial} x_m + p_\alpha \bar{\partial}\theta^\alpha + w_\alpha \bar{\partial}\lambda^\alpha$$

$$+s^\alpha \bar{\partial} r_\alpha + \bar{w}^\alpha \bar{\partial}\bar{\lambda}_\alpha + b\bar{\partial}c + \widehat{\beta}\bar{\partial}\widehat{\gamma}]$$

Relation between RNS and PS variables

$$\theta^\alpha = \widehat{\theta}^\alpha + \frac{\gamma}{2(\lambda\bar{\lambda})}\psi^m(\gamma_m\bar{\lambda})^\alpha$$

$$p_\alpha = \widehat{p}_\alpha + \frac{1}{\gamma}\psi^m(\gamma_m\lambda)_\alpha$$

$$\widehat{\gamma} = \gamma^2, \quad \widehat{\beta} = \frac{\beta}{\gamma}$$

$$\gamma = \eta e^\phi, \quad \frac{1}{\gamma} = \xi e^{-\phi}, \quad \beta = \partial\xi e^{-\phi}$$

Can invert relation to obtain $\psi^m = \frac{1}{\gamma}(\lambda\gamma^m\theta) + \frac{\gamma}{2(\lambda\bar{\lambda})}(\bar{\lambda}\gamma^m p)$

With these relations, $S_{RNS} = S_{PS}$

$$T = T_{RNS} + w_\alpha\partial\lambda^\alpha + \widehat{p}_\alpha\partial\widehat{\theta}^\alpha + \bar{w}^\alpha\partial\bar{\lambda}_\alpha + s^\alpha\partial\bar{r}_\alpha$$

$$= T_{PS} + \bar{w}^\alpha\partial\bar{\lambda}_\alpha + s^\alpha\partial\bar{r}_\alpha + b\partial c + \widehat{\beta}\partial\widehat{\gamma} + \partial(bc + \widehat{\beta}\widehat{\gamma})$$

Relation of BRST operators

$$e^{-R}Q_{RNS}e^{R} = \int dz e^{-R}(cT_{RNS} + \gamma\psi^m \partial x_m + \gamma^2 b + \lambda^\alpha \widehat{p}_\alpha + \bar{w}^\alpha r_\alpha)e^R$$

$$= \int dz[cT + \gamma\psi^m \partial x_m + \lambda^\alpha p_\alpha + \bar{w}^\alpha r_\alpha + \gamma^2(b + w_\alpha \partial\theta^\alpha + s^\alpha \partial\bar{\lambda}_\alpha + ...)]$$

$$= \int dz[cT + \lambda^\alpha(p_\alpha + \partial x_m(\gamma^m\theta)_\alpha) + \bar{w}^\alpha r_\alpha + \widehat{\gamma}(b + \frac{1}{2(\lambda\bar{\lambda})}(\bar{\lambda}\gamma^m p)\partial x_m + w_\alpha \partial\theta^\alpha + s^\alpha \partial\bar{\lambda}_\alpha + ...)]$$

$$= \int dz[cT + \lambda^\alpha d_\alpha + \bar{w}^\alpha r_\alpha + \widehat{\gamma}(b + B)]$$

$$= \int dz e^{-R'}(\lambda^\alpha d_\alpha + \bar{w}^\alpha r_\alpha + \widehat{\gamma} b)e^{R'} = \int dz e^{-R'} Q_{PS} e^{R'}$$

$$R = \int dz\ c(w_\alpha \partial\theta^\alpha + s^\alpha \partial\bar{\lambda}^\alpha + ...), \quad R' = \int dz\ c(\widehat{\beta}\partial c + B)$$

$B = \frac{1}{2(\lambda\bar{\lambda})}(\bar{\lambda}\gamma^m d)\partial x_m + ...$ is composite operator satisfying $\{Q_{PS}, B\} = T_{PS}$

$$e^{-R} Q_{RNS} e^{R} = e^{-R'} Q_{PS} e^{R'} \text{ implies } V_{RNS} = e^{R} e^{-R'} V_{PS} e^{R'} e^{-R}$$

Non-zero picture in pure spinor formalism?

$$RNS : \gamma = \eta e^{\phi}, \quad \beta = \partial \xi e^{-\phi}$$

$$Z = \{Q, \xi\} = e^{\phi} \partial x^m \psi_m + e^{2\phi} b \partial \eta + \ldots, \quad Y = c \partial \xi e^{-2\phi}$$

$$PS : \widehat{\gamma} = \widehat{\eta} e^{\widehat{\phi}}, \quad \widehat{\beta} = \partial \widehat{\xi} e^{-\widehat{\phi}}$$

$$\widehat{Z} = \{Q, \widehat{\xi}\} = e^{\widehat{\phi}} (b + B), \quad \widehat{Y} = c e^{-\widehat{\phi}}$$

$\widehat{\gamma} = \gamma^2 = \eta \partial \eta e^{2\phi}$ and regular OPEs with $\theta^\alpha = \widehat{\theta}^\alpha + \frac{\gamma}{2(\lambda\bar{\lambda})} \psi^m (\gamma_m \bar{\lambda})^\alpha$ implies

$$\widehat{\eta} = e^{-\frac{\phi}{2}} \lambda^\alpha \Sigma_\alpha, \quad \widehat{\xi} = e^{\frac{\phi}{2}} \frac{\bar{\lambda}_\alpha}{(\lambda\bar{\lambda})} \Sigma_\alpha, \quad e^{\widehat{\phi}} = \eta \partial \eta e^{\frac{5\phi}{2}} \frac{\bar{\lambda}_\alpha}{(\lambda\bar{\lambda})} \Sigma_\alpha$$

So $\widehat{\xi}$ and $\widehat{Z}$ carry picture $+\frac{1}{2}$ and $\widehat{Y}$ carries picture $-\frac{1}{2}$

Vertex operators

$$V_{PS} = \lambda^\alpha A_\alpha(x,\theta)$$

$$= a_m e^{ikx}[\lambda\gamma^m\theta + k_n(\lambda\gamma_p\theta)(\theta\gamma^{mnp}\theta) + ...] + \chi^\alpha e^{ikx}[(\lambda\gamma^m\theta)(\gamma_m\theta)_\alpha + ...]$$

$$e^R e^{-R'} V_{PS} e^{R'} e^{-R} = a_m e^{ikx}[\lambda\gamma^m\theta + c(\partial x^m + k_n(\theta\gamma^{mn}p) + ...)]$$

$$+\chi^\alpha e^{ikx}[(\lambda\gamma^m\theta)(\gamma_m\theta)_\alpha + c(p_\alpha + ...)]$$

$$= a_m e^{ikx}[\frac{\lambda\gamma^m\gamma^n\bar\lambda}{2(\lambda\bar\lambda)}\gamma\psi^m + c(\partial x^m + k_n\psi^m\psi^n) + ...]$$

$$+\chi^\alpha e^{ikx}[\gamma^2\psi^m\psi^n\frac{(\gamma_m\gamma_n\bar\lambda)_\alpha}{2(\lambda\bar\lambda)} + c(\widehat{p}_\alpha + \frac{1}{\gamma}\psi^m(\gamma_m\lambda)_\alpha) + ...]$$

$$= a_m e^{ikx}[\gamma\psi^m + c(\partial x^m + k_n\psi^m\psi^n) + ...]$$

where ... involves RNS non-minimal variables

$$V_{PS}^{-\frac{1}{2}} = \widehat{Y}\lambda^\alpha A_\alpha(x,\theta) = ce^{-\widehat{\phi}}\lambda^\alpha A_\alpha(x,\theta)$$

$$= a_m e^{ikx} ce^{-\widehat{\phi}}[\lambda\gamma^m\theta + k_n(\lambda\gamma_p\theta)(\theta\gamma^{mnp}\theta) + ...] + \chi^\alpha e^{ikx} ce^{-\widehat{\phi}}[(\lambda\gamma^m\theta)(\gamma_m\theta)_\alpha + ...]$$

$$e^R e^{-R'} V_{PS}^{-\frac{1}{2}} e^{R'} e^{-R} = a_m e^{ikx} ce^{-\widehat{\phi}}[(\lambda\gamma^m\theta) + ...]$$

$$+\chi^\alpha e^{ikx} ce^{-\widehat{\phi}}[(\lambda\gamma^m\theta)(\gamma_m\theta)_\alpha + ...]$$

$$= a_m e^{ikx} ce^{-\frac{5\phi}{2}}\xi\partial\xi\lambda^\alpha\Sigma_\alpha[(\gamma\psi^m) + ...]$$

$$+\chi^\alpha e^{ikx} ce^{-\frac{5\phi}{2}}\xi\partial\xi\lambda^\alpha\Sigma_\alpha[\gamma^2\psi^m\psi^n\frac{(\gamma_m\gamma_n\bar{\lambda})_\alpha}{2(\lambda\bar{\lambda})} + ...]$$

$$= \chi^\alpha e^{ikx}[ce^{-\frac{\phi}{2}}\Sigma_\alpha + ...]$$

where ... involves RNS non-minimal variables

Massless vertex operators in other pictures obtained in similar manner, and massive vertex operators obtained by taking OPEs of massless vertex operators.

Amplitude prescriptions

$$\mathcal{A}^{tree}_{RNS} = \langle \xi \mathcal{N}\ U_1 U_2 U_3 \int dz_4 V_4 ... \int dz_N V_N \rangle$$

where sum of pictures of vertex operators is $P = -2$

$\mathcal{N} = e^{-Q(\widehat{\theta}\bar{\lambda})} = e^{-(\lambda\bar{\lambda} + r_\alpha \widehat{\theta}^\alpha)}$ absorbs non-minimal zero modes

$$\mathcal{A}^{tree}_{RNS} = \langle (\xi Y) \mathcal{N}\ U_1 U_2 U_3 \int dz_4 V_4 ... \int dz_N V_N \rangle$$

where sum of pictures of vertex operators is $P = -1$

$$\xi Y = c \xi \partial \xi e^{-2\phi} = \widehat{\widehat{\xi Y}}$$

$$\mathcal{A}^{tree}_{PS} = \langle \widehat{\xi} (c e^{-\widehat{\phi}})^3 \mathcal{N} U_1 U_2 U_3 \int dz_4 V_4 ... \int dz_N V_N \rangle$$

where sum of pictures of vertex operators is $P = 0$

$$\mathcal{A}^{tree}_{PS} = \langle (\widehat{\widehat{\xi Y}}) \mathcal{N}\ U_1 U_2 U_3 \int dz_4 V_4 ... \int dz_N V_N \rangle$$

where sum of pictures of vertex operators is $P = -1$

$$\mathcal{A}^{g-loop}_{RNS} = \int d^{3g-3}\tau \langle \xi (\oint \eta)^g \mathcal{N} \, (\int \mu b)^{3g-3} \int dz_1 V_1 ... \int dz_N V_N \rangle$$

where sum of pictures of vertex operators is $P = 2g - 2$

$$\mathcal{A}^{g-loop}_{RNS} = \int d^{3g-3}\tau \langle (\xi Y)(\oint j_{BRST})^g \mathcal{N} \, (\int \mu b)^{3g-3} \int dz_1 V_1 ... \int dz_N V_N \rangle$$

where sum of pictures of vertex operators is $P = g - 1$

$$\mathcal{A}^{g-loop}_{PS} = \int d^{3g-3}\tau \langle \widehat{\xi} (\oint \widehat{\eta})^g (\widehat{Z})^{3g-3} \mathcal{N} \, (\int \mu B)^{3g-3} \int dz_1 V_1 ... \int dz_N V_N \rangle$$

$$= \int d^{3g-3}\tau \langle \widehat{\xi} (\oint \widehat{\eta})^g \mathcal{N} \, (\int \mu B e^{\widehat{\phi}} (b + B))^{3g-3} \int dz_1 V_1 ... \int dz_N V_N \rangle$$

$$= \int d^{3g-3}\tau \langle (\widehat{\xi Y})(\oint j_{BRST})^g \widehat{Z}^{2g-2} \mathcal{N} \, (\int \mu b)^{3g-3} \int dz_1 V_1 ... \int dz_N V_N \rangle$$

$$= \int d^{3g-3}\tau \langle (\widehat{\xi Y})(\oint j_{BRST})^g \mathcal{N} \, (\int \mu b)^{3g-3} \int dz_1 V_1 ... \int dz_N V_N \rangle$$

where sum of pictures of vertex operators is $P = g - 1$

Subtleties in definition of $\mathcal{N}$ for both RNS and pure spinor g-loop amplitudes