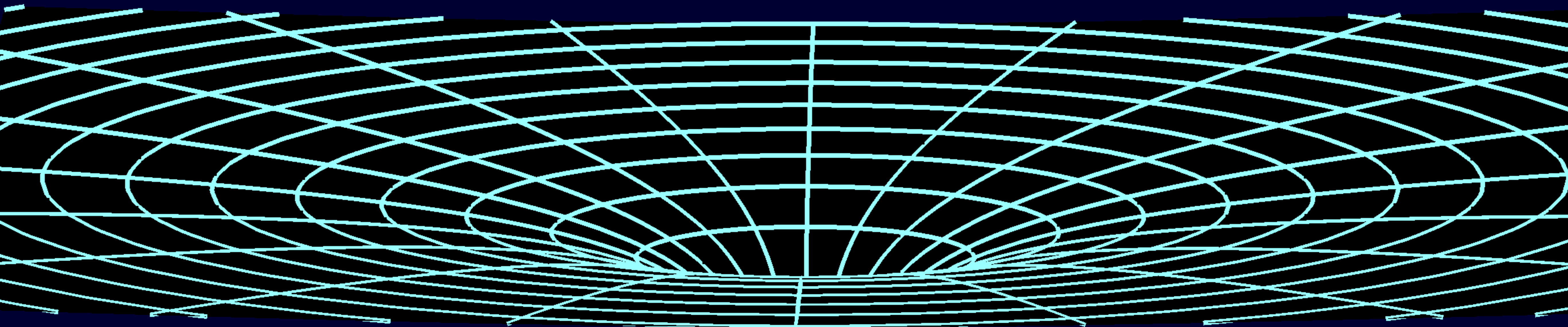
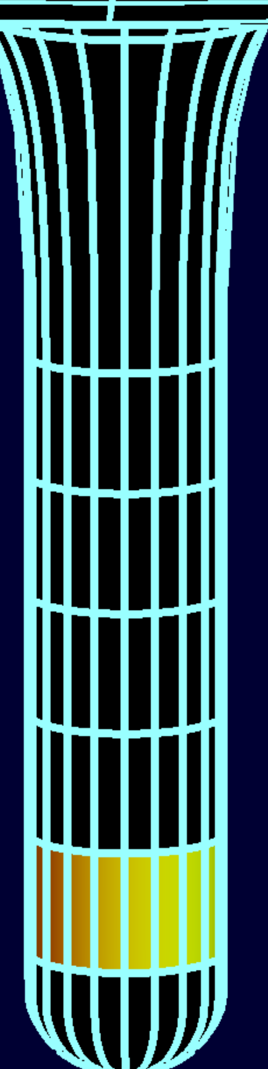
Microstate Geometries





Deep Inside the
Black-Hole Regime

Nick Warner, HullFest, April 28, 2017

Chris Hull

Chris Hull

Chris Hull


CHRIS HULL
WEEKDAYS
4PM-7PM
the breeze


MEET
CHRIS HULL
"EVENTS COORDINATOR"
An Oscar Mike Production
OSCARMIKE


Happy Birthday, Chris!

Outline

Microstate Geometries and Holography

- Some families of states in the D1-D5-CFT

- Holographic duals

- The MSW string

- Holographic duals of some MSW states

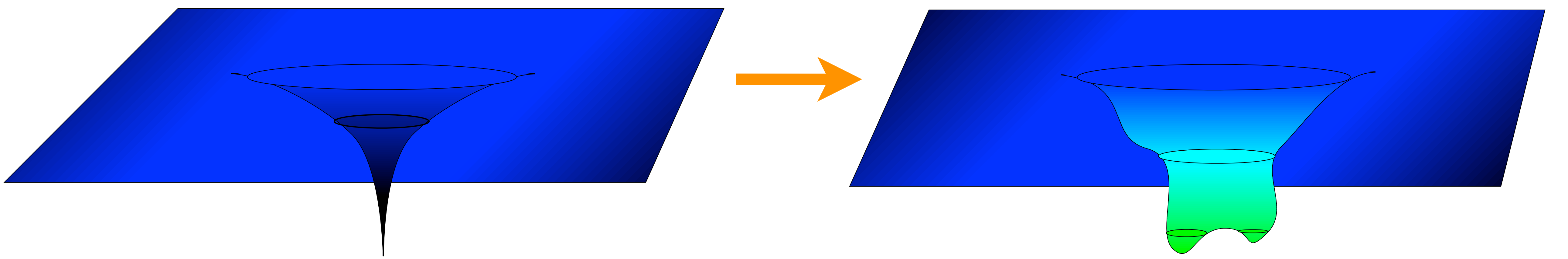
Based on Collaborations with:

I. Bena, S. Giusto, E. Martinec, R. Russo, M. Shigemori, D. Turton.

arXiv:1607.03908, arXiv:1703.10171, arXiv:17summer.XXXXX

Microstate Geometry Program

Microstate Geometry ≡ Smooth, horizonless solutions to the bosonic sector of ***supergravity*** with the same asymptotic structure as a given black hole/ring



Supergravity because we seek stringy resolutions on horizon scale

- *Very long-range effects* ⇒ Massless limit of strings …

What is the form of generic, BPS, time-independent horizonless, smooth solutions in supergravity?

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Singularity resolved; Horizon removed

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- ***A Mechanism to support structure at the horizon scale***

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Microstate Geometries

- ***A Mechanism to support structure at the horizon scale***
- ***How much of the microstate structure can supergravity encode?***

Black-Hole Microstates and CFT's

Black-Hole Microstates and CFT's

- **D1-D5 CFT:** A (4,4) supersymmetric CFT with $c = 6\, N_1\, N_5$

¼ BPS states = (R,R)-ground states

⅛ BPS states = $(\underbrace{\textit{any left-moving state}}_{N_P}$, R ground state)

Strominger-Vafa state counting for BPS black hole in five dimensions:

$$S \;=\; 2\,\pi\,\sqrt{N_1 N_5 N_P}$$

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- **MSW String:** A (0,4) supersymmetric CFT *(Maldacena-Strominger-Witten)*

  M5 brane wrapping a divisor in a $\mathbf{CY_3}$. Dual class, $\boldsymbol{P} \in H^2(CY_3, \mathbb{Z})$

  MSW string CFT lives on remaining (1+1) dimensions of M5 brane

  Central charge $\mathbf{c = 6\, D}$, $\quad D = \frac{1}{6}\int_{CY_3} P^3$

  State counting for BPS black hole in four dimensions: $\quad S = 2\pi\sqrt{D\, N_P}$

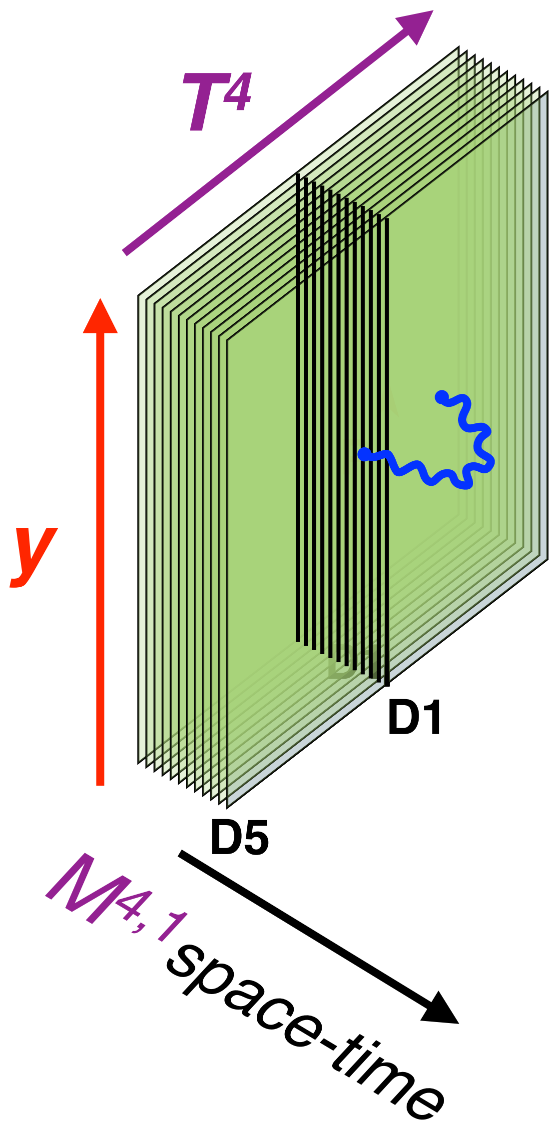
One Focus of the Microstate Geometry Program

Describe the strongly coupled gravity duals of these CFT states.

To what extent can these CFT states be captured in supergravity?

The D1-D5 CFT

Open D1-D5 superstrings moving in T^4
with $N \equiv N_1 N_5$ Chan-Paton labels: $(T^4)^N/S_N$



⇒ CFT on common **D1-D5** direction, $(t,y) \Leftrightarrow (u,v)$
(4,4) supersymmetric CFT with $c = 6\, N_1 N_5$

$y \equiv y+2\pi R$

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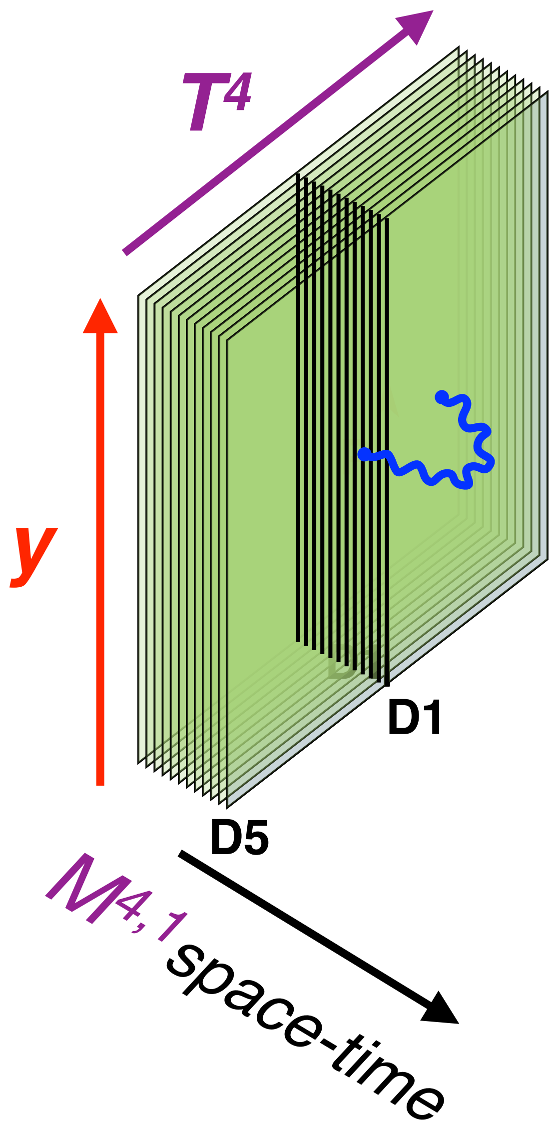

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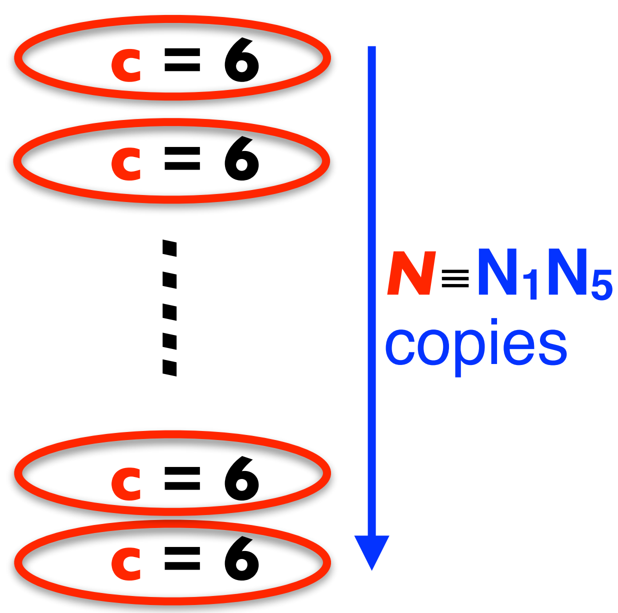
Maximally spinning RR-ground state:



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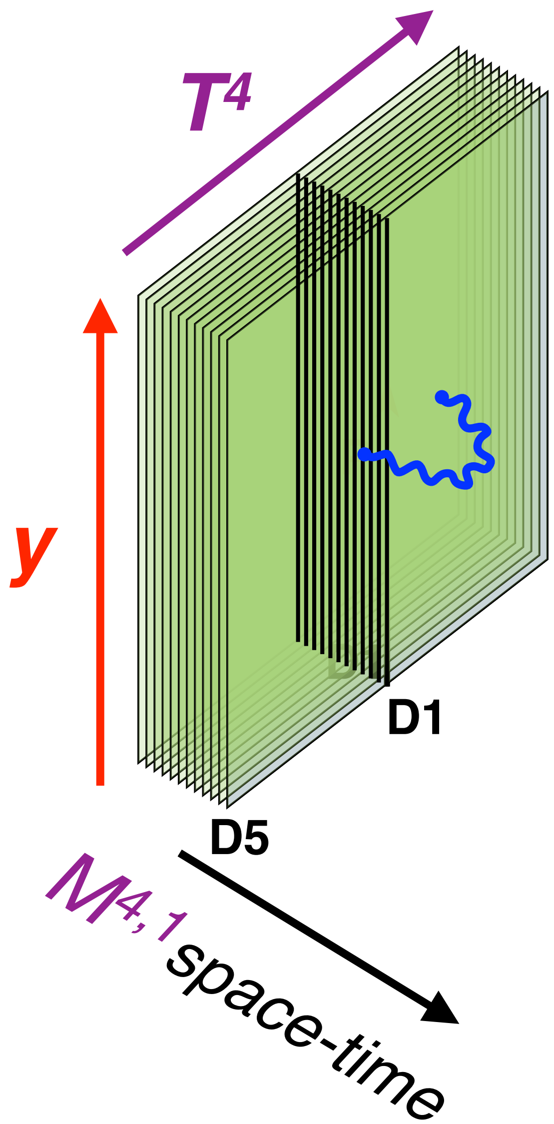

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Maximally spinning RR-ground state:

$c = 6$		$(+,+)$
$c = 6$		$(+,+)$
$\vdots$	$N \equiv N_1 N_5$ copies	$\vdots$
$c = 6$		$(+,+)$
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space-time angular momenta

SU(2) × SU(2) R-symmetry

$$(j_L, j_R) = \tfrac{1}{2}(N, N)$$

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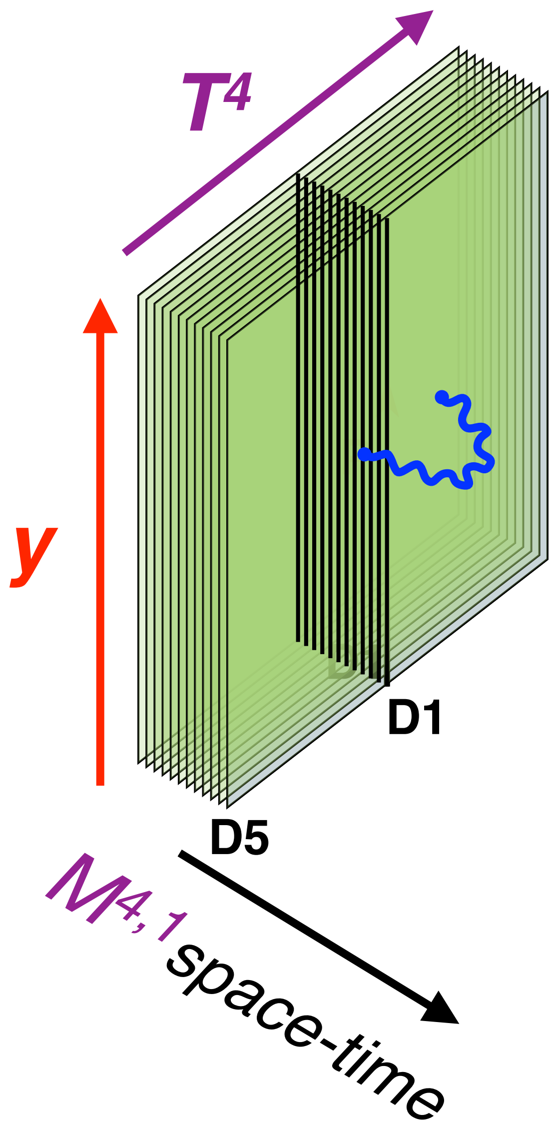

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(4,4) supersymmetric CFT with c = 6 $N_1 N_5$

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Maximally spinning RR-ground state:

c = 6		(+,+)	space-time angular momenta
c = 6		(+,+)	SU(2) × SU(2) R-symmetry
⋮	$N \equiv N_1 N_5$ copies	⋮	$(j_L, j_R) = \tfrac{1}{2}(N, N)$
c = 6		(+,+)	
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Holographic dual: Maximally spinning supertube in $R^{4,1}$

Supertube profile spins out into $M^{4,1}$ space-time

$$(g_1(v), g_2(v), g_3(v), g_4(v)) \in \mathbb{R}^4$$

$$g_1(v) + i g_2(v) = a\, e^{2\pi i v/R}$$

$$g_3(v) = g_4(v) = 0$$

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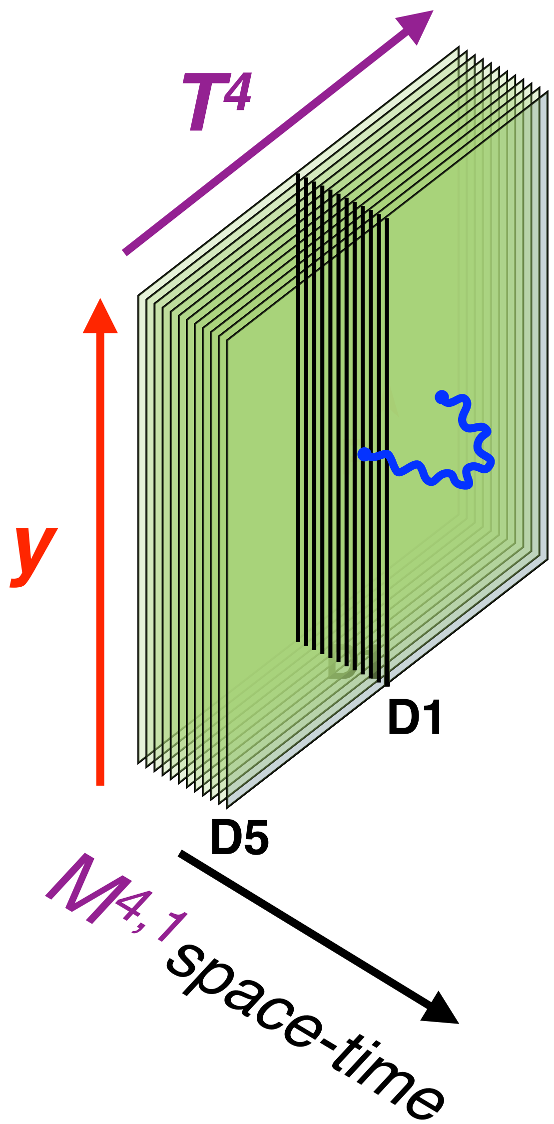

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space-time angular momenta

SU(2) × SU(2) R-symmetry

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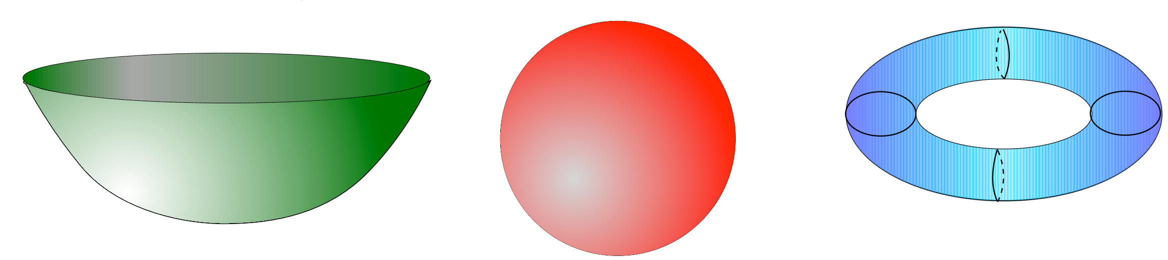
Holographic dual: Maximally spinning supertube in $R^{4,1}$

Supertube profile spins out into $M^{4,1}$ space-time

$$Q_1\, Q_5 = R^2\, \mathbf{a}^2$$

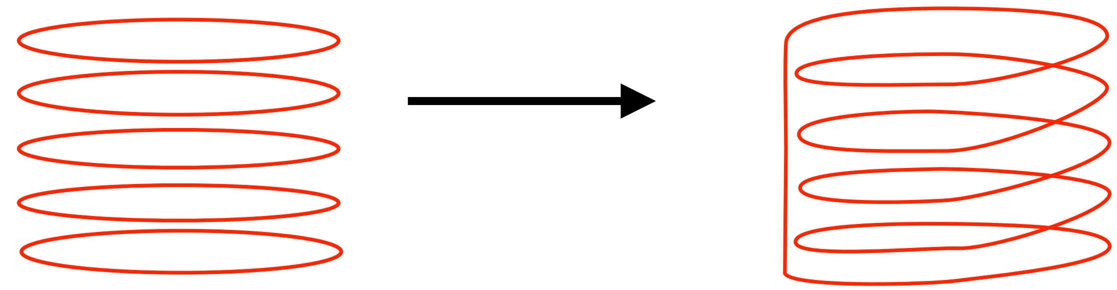
$$\left.\begin{aligned} &(g_1(v), g_2(v), g_3(v), g_4(v)) \in \mathbb{R}^4 \\ &g_1(v) + i g_2(v) = a\, e^{2\pi i v/R} \\ &g_3(v) = g_4(v) = 0 \end{aligned}\right\} \xrightarrow{\textit{back-react}}$$

AdS$_3$ × S^3 × T^4



More general ¼ BPS profiles

Orbifold CFT: ***k*** twisted sector



k* loops** → ***Length k loop

$|+\tfrac{1}{2},+\tfrac{1}{2}\rangle^{k}$ → $|+\tfrac{1}{2},+\tfrac{1}{2}\rangle_{k}$

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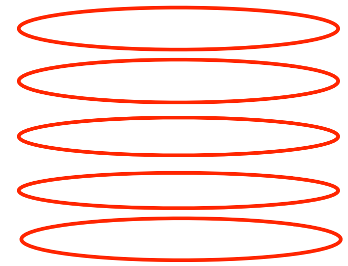

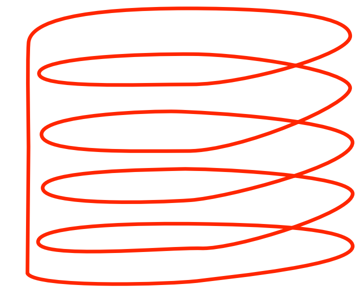

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Act with fermion zero modes

$|0,0\rangle^{k}$ → $|0,0\rangle_{k}$

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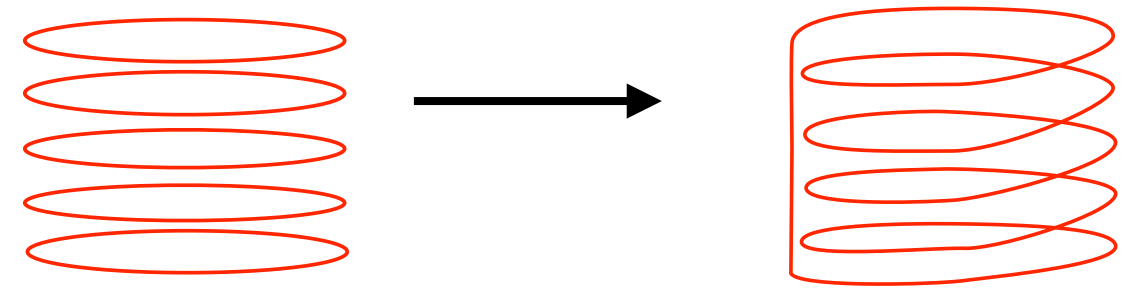

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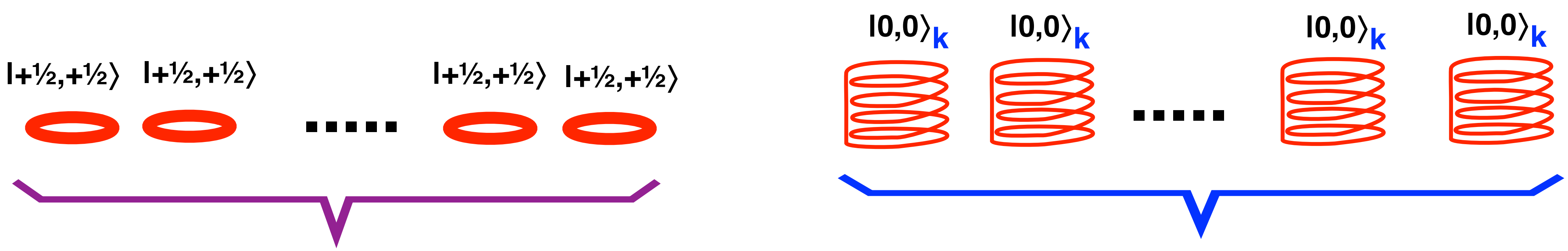
More general class of D1-D5 ground state



~ $\mathbf{a}^2$ copies ~ $\mathbf{b}^2$ copies

Holographic dual supertube profile

$$g_1(v) + i g_2(v) = \mathbf{a}\, e^{2\pi i v/R} \qquad \text{“}g_5(v)\text{”} = \mathbf{b}\, \sin(2\pi\, \mathbf{k}\, v/R)$$

Partitioning of charges: $Q_1\, Q_5 \;=\; R^2\, (\mathbf{a^2} + \mathbf{b^2})$

Families ⅛ BPS states in the D1-D5-P system

Generic ⅛ BPS state: Add general left-moving excitations

Momentum charge, Q_P $= L_{0,left}$ $$S = 2\pi\sqrt{Q_1 Q_5 Q_P}$$ *(Strominger-Vafa)*

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Very special families of momentum excitations: "Supergraviton gas"

$$\left(|+\tfrac{1}{2}\,,+\tfrac{1}{2}\rangle_1\right)^{N_0} \otimes \left(\frac{1}{m!\,n!}\,(J^{+}_{-1})^m\,(L_{-1} - J^{3}_{-1})^n\,|00\rangle_k\right)^{N_{k,m,n}}$$

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Quantum numbers

Define $\mathcal{N} \;=\; \dfrac{N_1 N_5}{a^2+b^2}$

$$j_L \;=\; \tfrac{1}{2}\,\mathcal{N}\left(\mathbf{a^2}+\frac{m}{k}\,\mathbf{b^2}\right)\,, \qquad \tilde{j}_R = \tfrac{1}{2}\,\mathcal{N}\,\mathbf{a^2}\,, \qquad N_P = \tfrac{1}{2}\,\mathcal{N}\,\frac{m+n}{k}\,\mathbf{b^2}$$

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D1-D5 |+½,+½⟩ residue

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Special forms:

Adding pure momentum: m = 0.

Vanishing angular momentum: m = 0, $\mathbf{a} \to \mathbf{0}$.

The "Supergraviton gas"

Linearity of BPS equations ⇒ We know the supergravity duals of arbitrary superpositions of states of the form:

$$\left(|+\tfrac{1}{2}\,,+\tfrac{1}{2}\rangle_1\right)^{N_0} \otimes \left[\ \bigotimes_{k_i,m_i,n_i} \left(\frac{1}{m_i!\,n_i!}\,(J^+_{-1})^{m_i}\,(L_{-1}-J^3_{-1})^{n_i}\,|00\rangle_{k_i}\right)^{N_{k_i,m_i,n_i}}\right]$$

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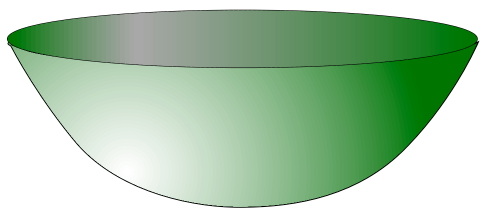
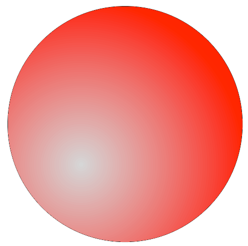
Holographic duals

Add momentum and angular momentum excitations to D1-D5 profiles:



AdS3 (u,**v**,r)



S^3 (θ,**ψ**,**ϕ**)

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to give: $$j_L=\tfrac{1}{2}\mathcal{N}\left(\mathbf{a^2}+\frac{m}{k}\,\mathbf{b^2}\right), \qquad \tilde{j}_R=\tfrac{1}{2}\mathcal{N}\,\mathbf{a^2}, \qquad N_P=\tfrac{1}{2}\mathcal{N}\,\frac{m+n}{k}\,\mathbf{b^2}$$

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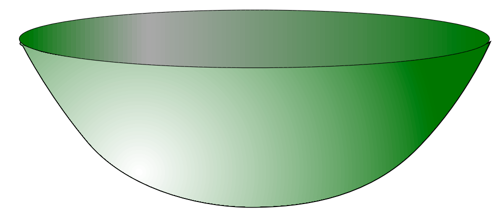
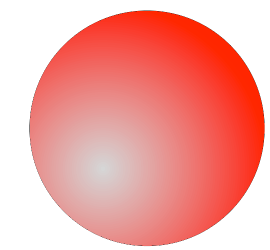


AdS_3 (u,**v**,r)



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to give: $$j_L = \tfrac{1}{2}\mathcal{N}\left(\mathbf{a^2}+\frac{m}{k}\,\mathbf{b^2}\right), \qquad \tilde{j}_R = \tfrac{1}{2}\mathcal{N}\,\mathbf{a^2}, \qquad N_P = \tfrac{1}{2}\mathcal{N}\,\frac{m+n}{k}\,\mathbf{b^2}$$

Three mode numbers, (k,m,n) $\Rightarrow$ Supergravity duals depend on:

$$\chi_{k,m,n} \equiv R^{-1}(m+n)\,v + \tfrac{1}{2}(k-2m)\,\psi - \tfrac{1}{2}\,k\,\phi$$

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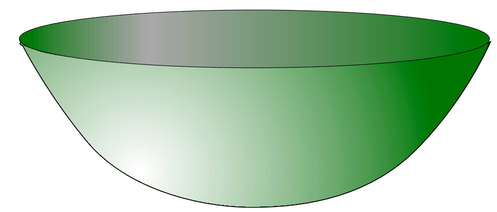
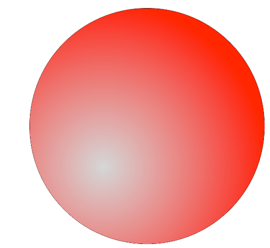
Holographic duals





Add momentum and angular momentum excitations to D1-D5 profiles:

$AdS_3\,(u,\mathbf{v},r)$ $\qquad$ $S^3\,(\theta,\boldsymbol{\psi},\boldsymbol{\phi})$

$$g_1(v)+ig_2(v)=\mathbf{a}\,e^{2\pi i v/R} \qquad \text{“}g_5(v)\text{”}=\mathbf{b}\,\sin(2\pi\,\mathbf{k}\,v/R)$$

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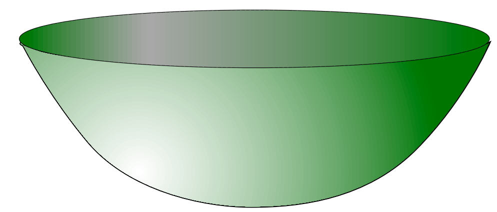
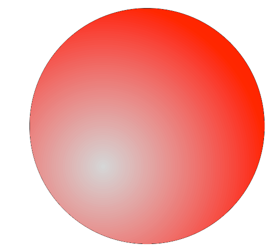


AdS3 (u,**v**,r)



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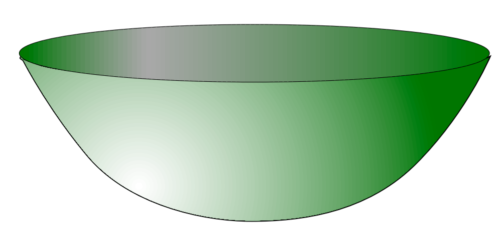

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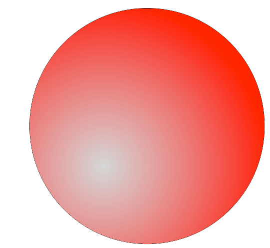

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n-mode (v) $\leftrightarrow$ N_P responsible for $N_P = \tfrac{1}{2}\mathcal{N}\,\frac{\mathbf{n}}{\mathbf{k}}\,\mathbf{b^2}$

Building the Fluctuating BPS Microstate Geometries

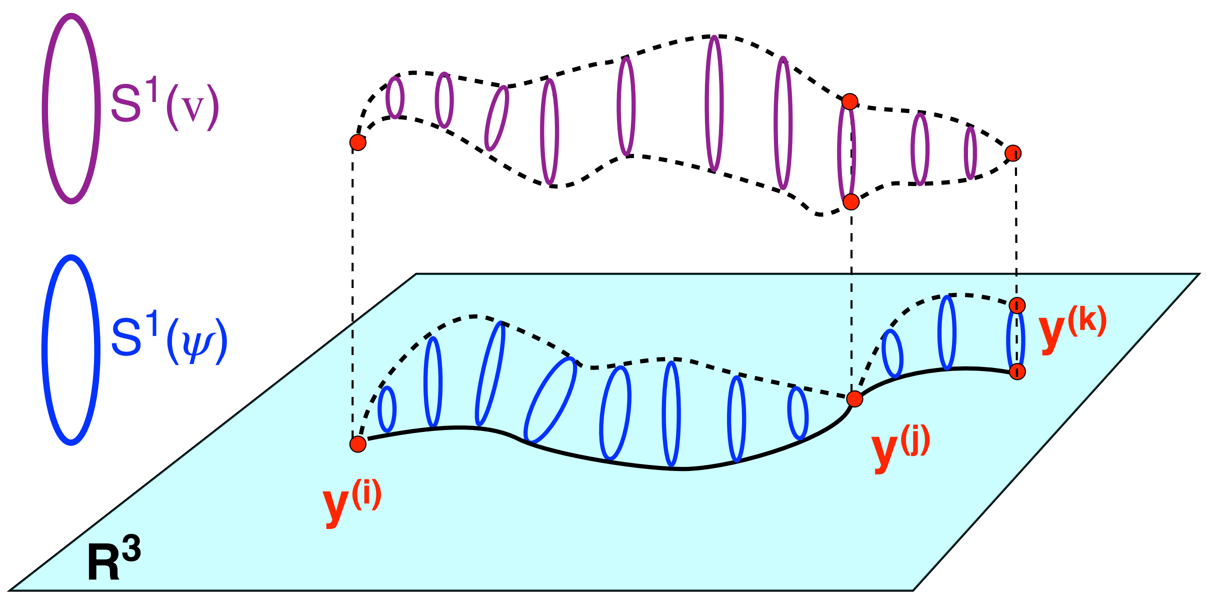
IIB Supergravity on T^4: Supergravity + two (anti-self-dual) tensor multiplets in six-dimensions

Six-dimensional metric ansatz: *(Gutowski, Martelli and Reall)*

$$ds_6^2 = -\frac{2}{\sqrt{\mathcal{P}}}(dv+\beta)\big(du+\omega-\tfrac{1}{2}Z_3(dv+\beta)\big) + \sqrt{\mathcal{P}}\,V^{-1}(d\psi+A)^2 + \sqrt{\mathcal{P}}\,V\,d\vec{y}\cdot d\vec{y}$$

u = null time; (v, ψ) define a double S^1 fibration over a flat R^3 base with coordinates, y.

The scale of everything is set by the "warp factors:" ***V, P*** and Z_3



The non-trivial homology cycles are defined through the pinching off of the $S^1 \times S^1$ fibration at special points in the R^3 base.

Maxwell Fields

Electric Potentials

Magnetic Fluxes

$$G^{(a)} = d\Big[-\frac{1}{2}\,\frac{\eta^{ab}Z_b}{\mathcal{P}}(du+\omega)\wedge(dv+\beta)\Big] + \tfrac{1}{2}\eta^{ab} *_4 DZ_b + \tfrac{1}{2}(dv+\beta)\wedge\Theta^{(a)}$$

$$\mathcal{P} \equiv \tfrac{1}{2}\eta^{ab}Z_aZ_b \equiv Z_1Z_2 - \tfrac{1}{2}{Z_4}^2$$

The BPS Equations

The BPS Equations

Layer 1: Conditions on Maxwell Fields

*A homogeneous **linear** system*

$$\Theta^{(a)} = *_4 \Theta^{(a)} \,, \qquad *_4 D(\partial_v Z_a) = \eta_{ab} D\Theta^{(b)} \,, \qquad D *_4 D Z_a = -\eta_{ab} \Theta^{(b)} \wedge d\beta \,.$$

where $D\Phi \equiv d_{(4)}\Phi - \beta \wedge \partial_v \Phi$

$\left(Z_a \,, \Theta^{(a)}\right)$ depend upon (r, θ) and

$$\chi_{k,m,n} \equiv R^{-1}\,(m+n)\,v + \tfrac{1}{2}\,(k-2m)\,\psi - \tfrac{1}{2}\,k\,\phi$$

General solution known!

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General solution known!

Layer 2: Conditions on Metric pieces

*An inhomogeneous **linear** system*

$$ds_6^2 \;=\; -\frac{2}{\sqrt{\mathcal{P}}}\,(dv+\beta)\big(du+\omega-\tfrac{1}{2}\,Z_3\,(dv+\beta)\big) \;+\; \sqrt{\mathcal{P}}\,V^{-1}\,(d\psi+A)^2 \;+\; \sqrt{\mathcal{P}}\,V\,d\vec{y}\cdot d\vec{y}$$

$$D\omega + *_4 D\omega - Z_3\,d\beta \;=\; Z_a \Theta^{(a)}$$

$$*_4 D *_4 \Big(\partial_v \omega + \tfrac{1}{2}\,D Z_3\Big) \;=\; \partial_v^2 \mathcal{P} - \big((\partial_v Z_1)(\partial_v Z_2) - \tfrac{1}{2}(\partial_v Z_4)^2\big) - \tfrac{1}{4}\eta_{ab} *_4 \Theta^{(a)} \wedge \Theta^{(b)}$$

$(Z_3 \,, \omega)$ depend upon (r, θ) and (quadratic) products of harmonics that depend upon

$$\chi_{k_i,m_i,n_i} \;=\; R^{-1}\,(m_i+n_i)\,v \;+\; \tfrac{1}{2}\,(k_i-2m_i)\,\psi \;-\; \tfrac{1}{2}\,k_i\,\phi$$

Interesting families of particular solutions known. General solution not known.

Linear system: Arbitrary superpositions easily constructed

A Microstate Geometry deep in the Black-Hole Regime

Add pure momentum states

$$\left(|+\tfrac{1}{2},+\tfrac{1}{2}\rangle_1\right)^{N_0}\otimes\left(\frac{1}{n!}\,(L_{-1}-J^3_{-1})^n|00\rangle_1\right)^{N_{1,0,n}}$$

$$Q_1\,Q_5 \;=\; R^2\,(\mathbf{a^2}+\mathbf{b^2})$$

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All angular momentum

Angular momentum ≡ 0

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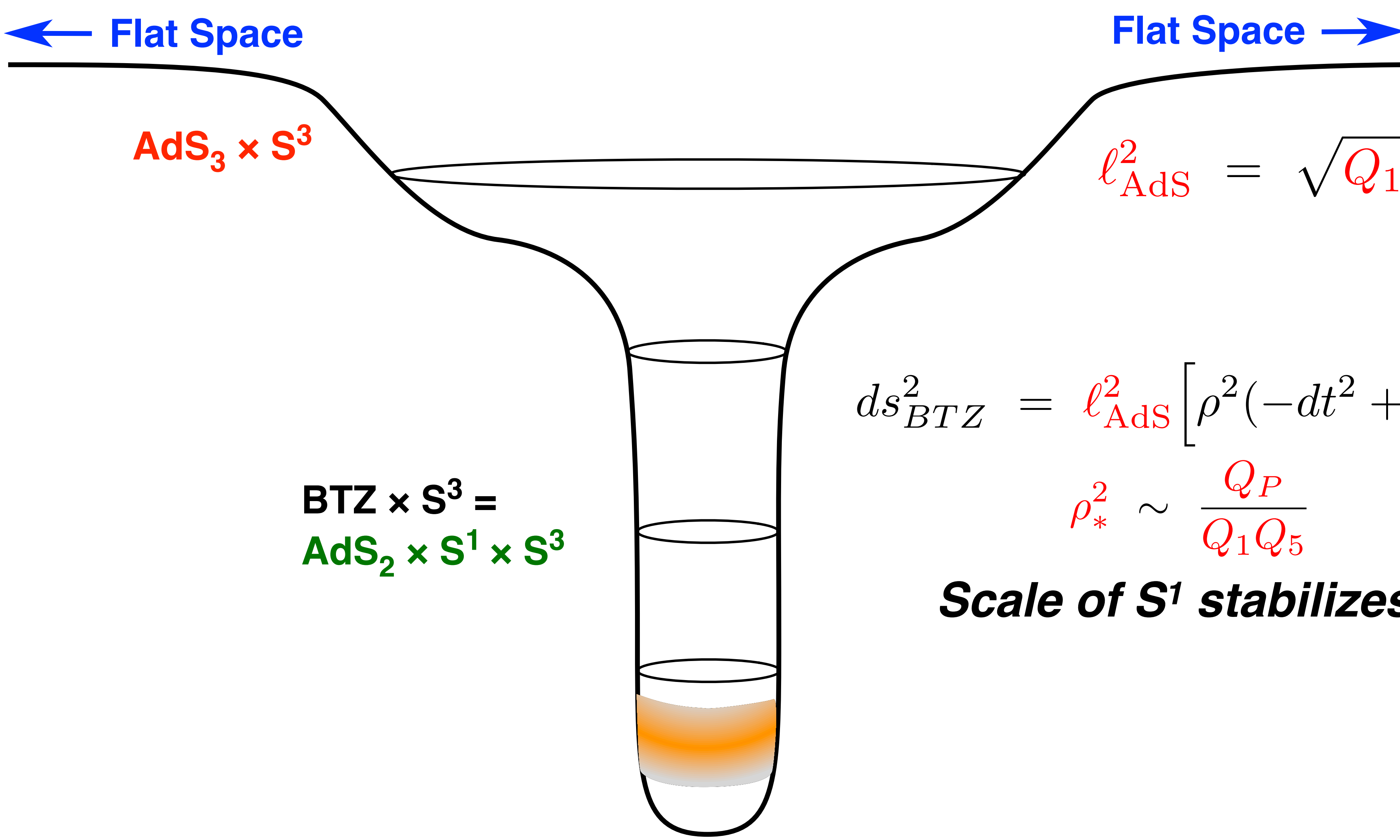
Geometry:



$$\ell^2_{\rm AdS} \;=\; \sqrt{Q_1 Q_5}$$

$$ds^2_{BTZ} \;=\; \ell^2_{\rm AdS}\left[\rho^2(-dt^2 + dy^2) + \frac{d\rho^2}{\rho^2} + \rho_*^2\,(dt+dy)^2\right]$$

$$\rho_*^2 \;\sim\; \frac{Q_P}{Q_1 Q_5}$$

Scale of S^1 stabilizes at $\rho_*\,\ell_{AdS}\,R$

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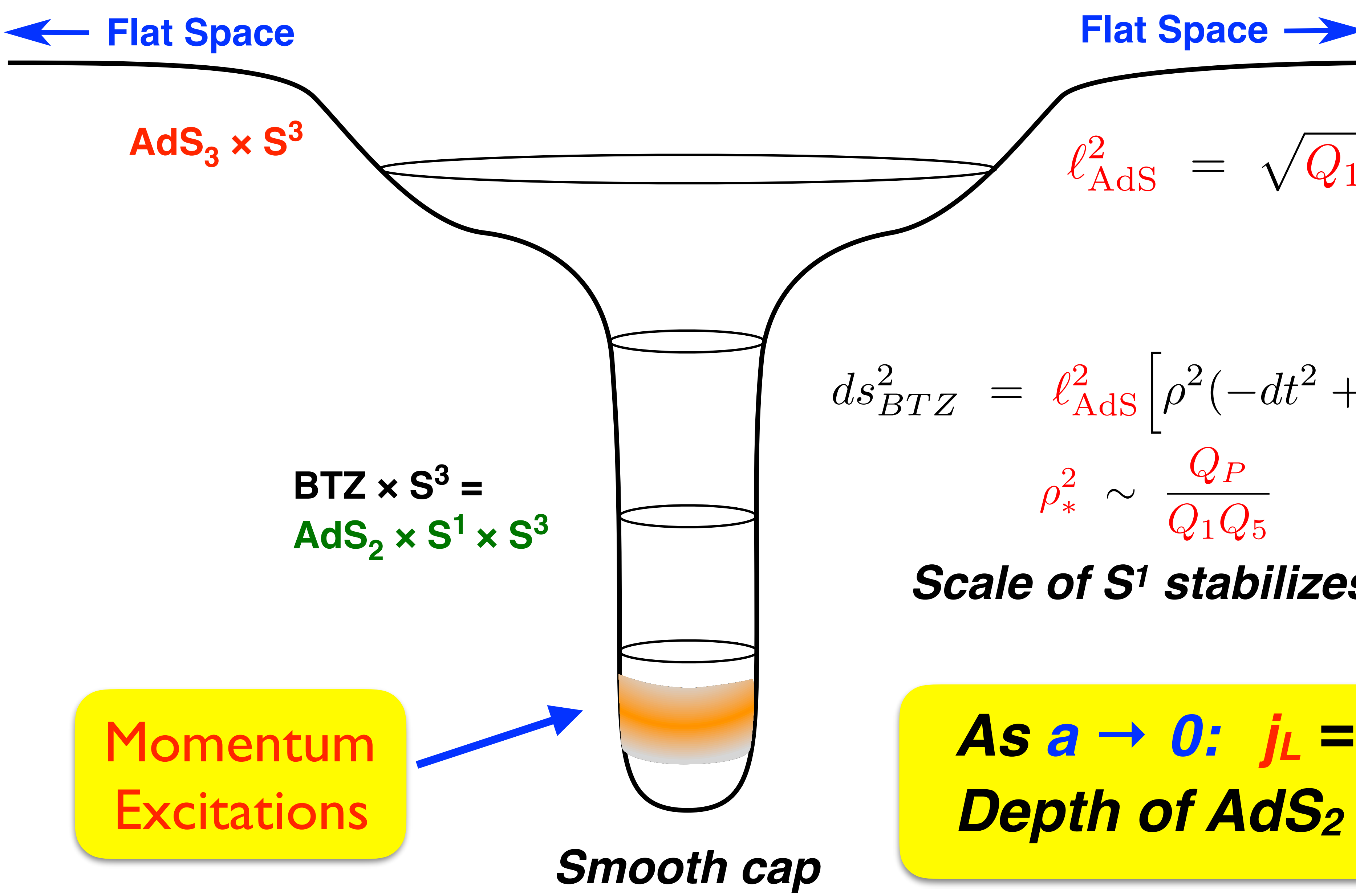

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Scale of S^1 stabilizes at $\rho_*\, \ell_{AdS}\, R$

As $a \rightarrow 0$: $j_L = j_R \rightarrow 0$

Depth of AdS_2 throat → ∞

Several significant results

- *First deep, scaling microstate geometry in Black-Hole regime with $j_L = j_R \to 0$*
- *Deep, scaling microstate geometry that goes to BTZ*
- *Deep, scaling ⇒ Arbitrarily large red-shifts*
  Microstate Geometry ⇒ Smooth cap-off
- *Momentum excitations localize at the bottom of the BTZ throat*
- *Holographic dictionary in AdS_3 for deep AdS_2/BTZ throat*
- *Geometry dual to states counted by Strominger-Vafa*

Microstate Geometries for MSW Black Holes

The hint of interesting new families of solutions comes from:

$$\chi_{k,m,n} \equiv R^{-1}\,(m+n)\,v \;+\; \tfrac{1}{2}\,(k-2m)\,\psi \;-\; \tfrac{1}{2}\,k\,\phi$$

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For *k = 2m* the solutions are independent of $\boldsymbol{\psi}$, the Hopf fiber of the ***S*³**

➔ Reduction to five-dimensional microstate geometries: *capped BTZ* × ***S*²**

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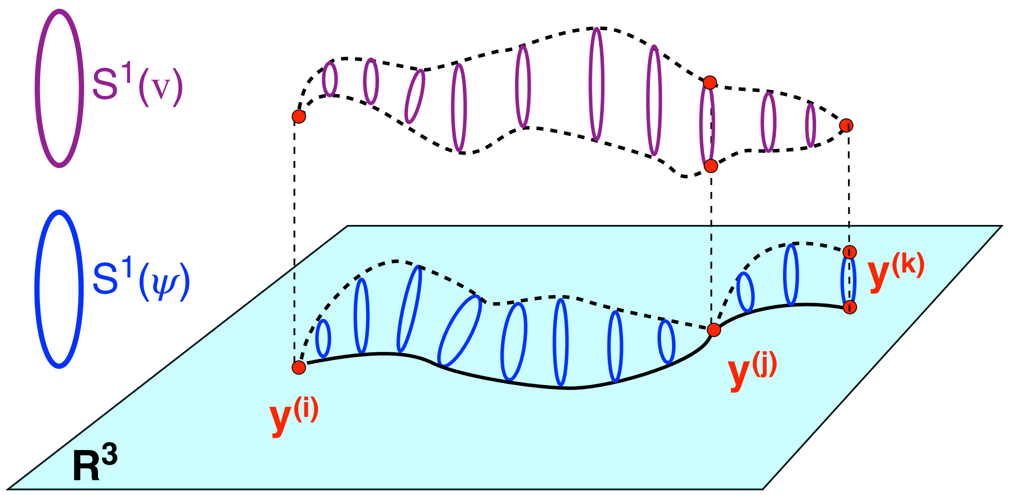
Enriching the family of solutions

Solutions are a T^2 fibration over R^3



Spectral transformations;

Fractional spectral flows …

$$\begin{pmatrix} \hat{v} \\ \hat{\psi} \end{pmatrix} = \mathbf{S} \begin{pmatrix} v \\ \psi \end{pmatrix}, \qquad \mathbf{S} \in SL(2,\mathbb{Q})$$

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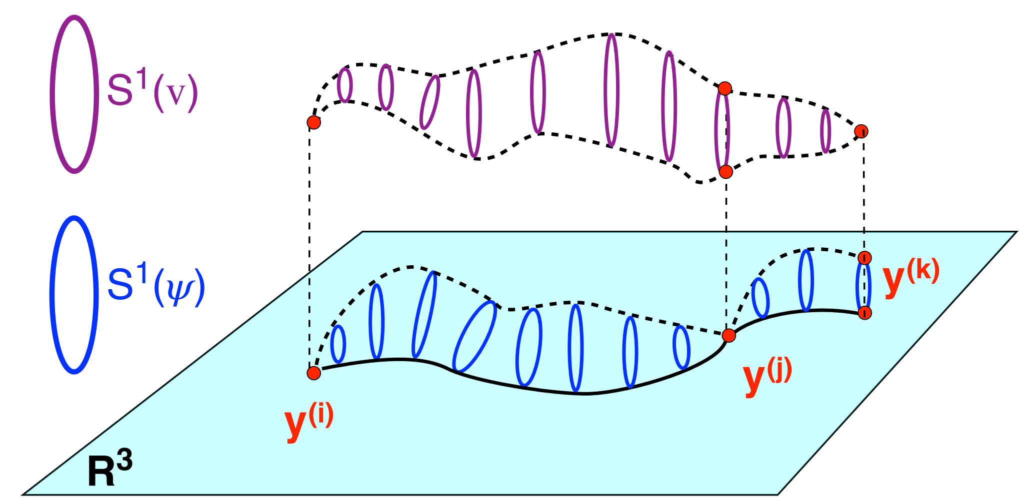

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Rational transformations:
Generate new solutions but may need to “re-declare the lattice of $(\hat{v},\hat{\psi})$

Standard supertube: D1-D5 charges + KKM dipole charge, $\boldsymbol{\kappa}$

➔ Supertube with D1-D5-KKM charges: $(\boldsymbol{Q_1, Q_5, \kappa})$

… presumably dual to D1-D5-KKM CFT. What is this exactly?

Some T-dualities

IIB	0	1	2	3	4	5	6	7	8	9
D1	⬆	✶	✶	✶	✶	⬆	↔	↔	↔	↔
D5	⬆	✶	✶	✶	✶	⬆	⬆	⬆	⬆	⬆
KKM	⬆	✶	✶	✶	⬆	⬆	⬆	⬆	⬆	⬆

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KKM	↑	✶	✶	✶	↑	↑	↑	↑	↑	↑

T-dualize 3 times to IIA:

IIA	0	1	2	3	4	5	6	7	8	9
D4	↑	✶	✶	✶	↑	↑	↔	↔	↑	↑
D4	↑	✶	✶	✶	↑	↑	↑	↑	↔	↔
NS5	↑	✶	✶	✶	↔	↑	↑	↑	↑	↑

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KKM	⬆	✶	✶	✶	⬆	⬆	⬆	⬆	⬆	⬆

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D4	⬆	✶	✶	✶	⬆	⬆	⬆	⬆	↔	↔
NS5	⬆	✶	✶	✶	↔	⬆	⬆	⬆	⬆	⬆

Uplift to M theory

M	0	1	2	3	5	4	10	6	7	8	9
M5	⬆	✶	✶	✶	⬆	⬆	⬆	↔	↔	⬆	⬆
M5	⬆	✶	✶	✶	⬆	⬆	⬆	⬆	⬆	↔	↔
M5	⬆	✶	✶	✶	⬆	↔	↔	⬆	⬆	⬆	⬆

M-theory background

D1-D5-KKM solution ➔ M5-M5-M5 charges: ($\boldsymbol{Q_1}$, $\boldsymbol{Q_5}$, $\boldsymbol{\kappa}$)
+ dipolar/dissolved M2-M2-M2 charges

Compactification/new (v, ψ) lattice:
D1-D5-KKM (4,4) supersymmetry ➔ M5-M5-M5 (0,4) supersymmetry

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Add momentum along common circle (5) … untouched in duality

IIB	0	1	2	3	4	5	6	7	8	9
D1	↑	✶	✶	✶	✶	↑	↔	↔	↔	↔
D5	↑	✶	✶	✶	✶	↑	↑	↑	↑	↑
KKM	↑	✶	✶	✶	↑	↑	↑	↑	↑	↑
P	↑					↑				

M	0	1	2	3	5	4	10	6	7	8	9
M5	↑	✶	✶	✶	↑	↑	↑	↔	↔	↑	↑
M5	↑	✶	✶	✶	↑	↑	↑	↑	↑	↔	↔
M5	↑	✶	✶	✶	↑	↔	↔	↑	↑	↑	↑
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KKM	↑	✶	✶	✶	↑	↑	↑	↑	↑	↑
P	↑					↑				

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M5	↑	✶	✶	✶	↑	↑	↑	↑	↑	↔	↔
M5	↑	✶	✶	✶	↑	↔	↔	↑	↑	↑	↑
P	↑				↑						

➔ *Momentum excitations of MSW string wrapping (5) direction ..*

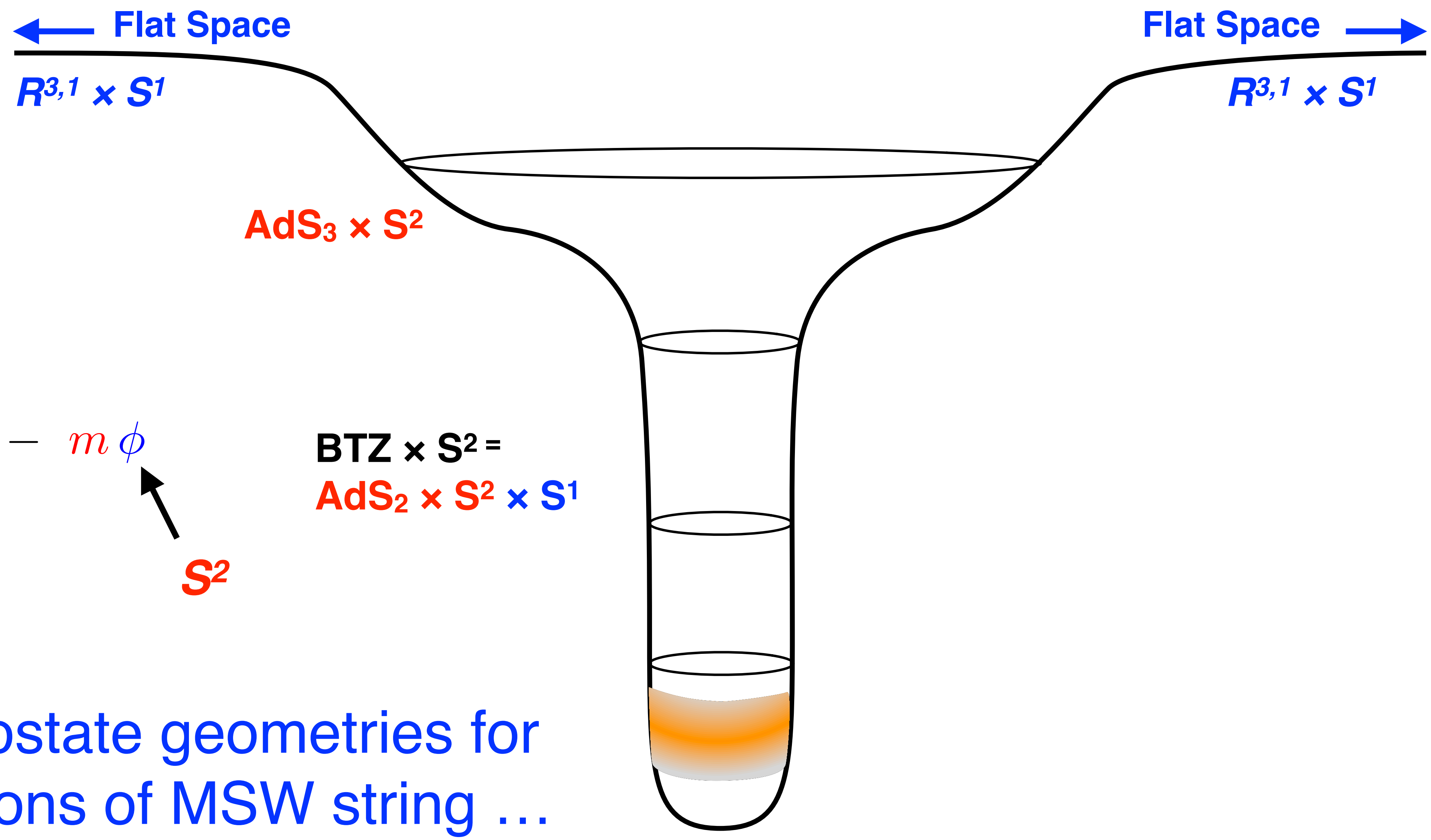
Fluctuating Microstate Geometries for MSW Strings

Previous picture compactified on Hopf fiber of S^3.

Fluctuations:

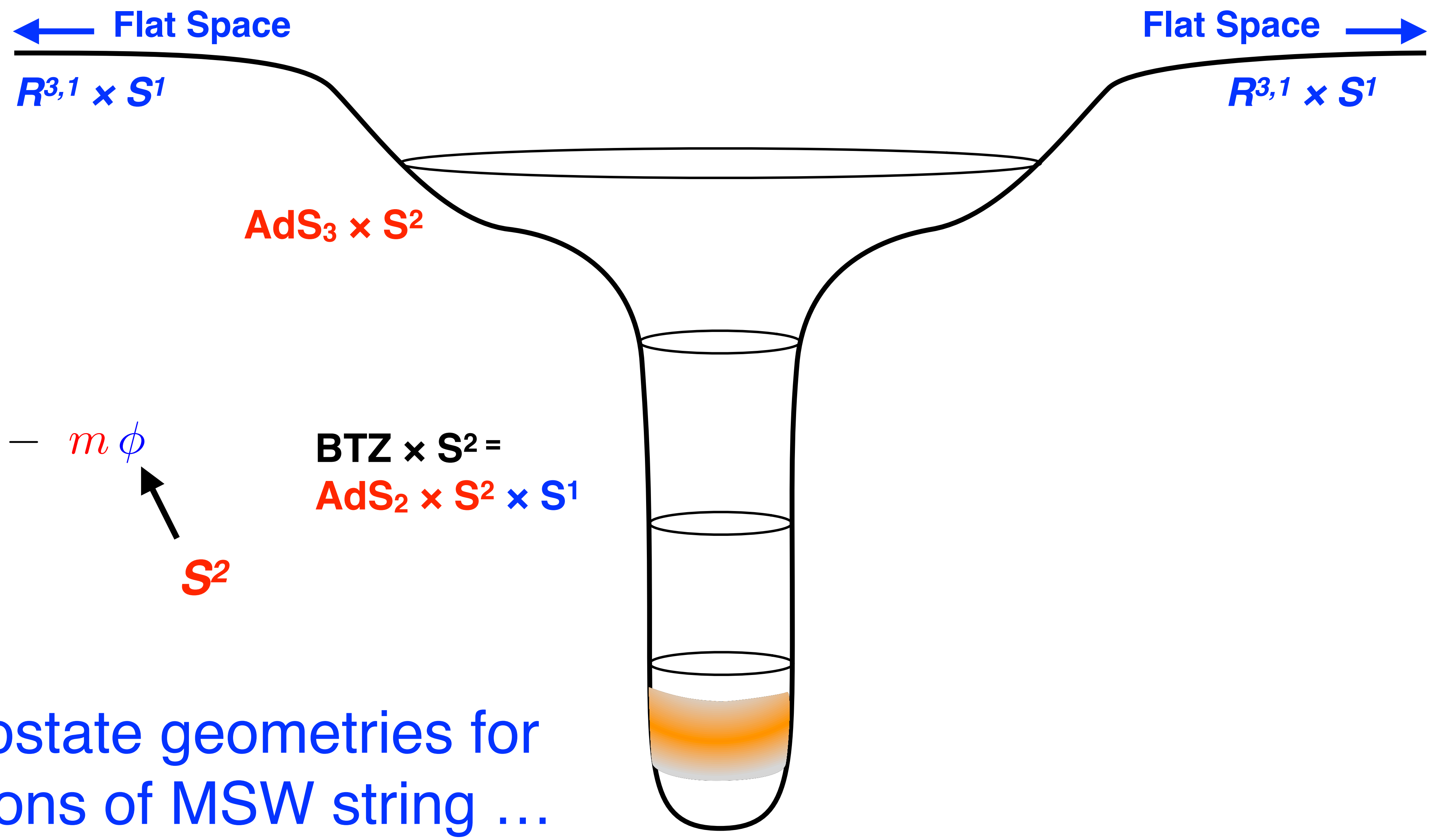
$$\chi_{m,n} \equiv R^{-1}(m+n)\,v - m\,\phi$$

AdS$_3$ ***or*** *S*^{1} *S*^{2}



Deep scaling, microstate geometries for momentum excitations of MSW string …

Fluctuating Microstate Geometries for MSW Strings



Previous picture compactified on Hopf fiber of S^3.

Fluctuations:

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AdS_3* or *S^1   ***S^2***

Deep scaling, microstate geometries for momentum excitations of MSW string …

Deconstruction: *Attempts to realize black-hole microstate structure with perturbative/singular D0 branes or perturbative momenta on "Deconstructed" MSW string*

Here: Precise, fully back-reacted, capped-off BTZ × S^2 realization of the deconstructed configurations …

….. related to D1-D5-P microstate structure

Conclusions

- Microstate geometries that are holographic duals to very particular D1-D5-P CFT states
- First deep, scaling microstate geometry in *Black-Hole Regime* with $j_L = j_R \to \mathbf{0}$
- Deep, scaling geometry going to BTZ × S^3 or BTZ × S^2
- Momentum excitations localize at bottom of throat and create smooth cap
- Holographic dictionary in AdS_3 for deep AdS_2/BTZ throat
- Microstate geometries for MSW … and that fully realize deconstruction

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Open issues

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  Some very limited families known
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Happy Birthday, Chris!