Supergravity Amplitudes in the Horava-Witten Background

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Chris Hull at 60

Imperial College, April 27 2017

Chris – thanks for the memorable times over many years

HAPPY BIRTHDAY, CHRIS

Introductory comments

- Connections between superstring scattering amplitudes and perturbative supergravity.

No UV divergences in string theory.

Technical fantasy: How do field theory UV divergences arise in low energy field theory limit?

- Supergravity duality: Scalar fields parametrize coset space $G(\mathbb{R})/K(\mathbb{R})$ where $G(\mathbb{R})$ is the duality symmetry group of classical supergravity.

  Maximal compact subgroup

  Preserved in perturbation theory in D=4 dimensions for N>4 supergravity.

- Superstring duality group is $G(\mathbb{Z})$: No continuous duality group – even at tree level.

  Moduli space $G(\mathbb{Z})\backslash G(\mathbb{R})/K(\mathbb{R})$

- Rich dependence of string amplitudes on moduli – strong coupling duality.

  Fascinating connections with mathematics of automorphic forms, Jacobi forms, …….Different Talk

Amplitudes with Maximal supersymmetry

Low energy expansion involves low order higher-derivative BPS interactions

$C_1(\mu_r)\, R^4$ $\quad C_2(\mu_r)\, D^4 R^4$ $\quad C_3(\mu_r)\, D^6 R^4$ $\quad C_4(\mu_r)\, D^8 R^4$??

1/2-BPS 1/4-BPS 1/8-BPS

Where μ_r are the moduli (coupling constants) for ten-dimensional type II theories compactified on tori.

- The exact expressions for $C_1,\ C_2,\ C_3$ are known in all dimensions.

- They are automorphic functions under the action of arithmetic duality groups $E_{d+1}(\mathbb{Z})$.

D dimensions	10B	9	8	7	6	5	4	3
	$SL(2,\mathbb{Z})$	$SL(2,\mathbb{Z})$	$SL(3,\mathbb{Z})\times SL(2,\mathbb{Z})$	$SL(5,\mathbb{Z})$	$SO(5,5,\mathbb{Z})$	$E_{6(6)}(\mathbb{Z})$	$E_{7(7)}(\mathbb{Z})$	$E_{8(8)}(\mathbb{Z})$

- Rich content:
  - These only contribute up to 3 loops in string perturbation theory;
  - Correct spectrum of BPS instantons

AMPLITUDES WITH ½-MAXIMAL SUPERSYMMETRY

We would like to use the structure of string/M-theory to constrain properties of low energy expansion of HETEROTIC/TYPE I scattering amplitudes.

This will make use of the Horava-Witten description of this background.

MBG, Arnab RUDRA

c,f, low order higher-derivative BPS interactions in maximal SUGRA :

$C_1(\mu_r)\, R^4$ $C_2(\mu_r)\, D^4 R^4$ $C_3(\mu_r)\, D^6 R^4$ $C_4(\mu_r)\, D^8 R^4$ **??**

1/2-BPS   1/4-BPS   1/8-BPS

Where μ_r are the moduli for ten-dimensional type II theories compactified on tori.

Fascinating connections with mathematics of automorphic forms, Jacobi forms, …….DIFFERENT TALK

Issues to address:

- The realization of S-duality between Heterotic and Type I theories.

- Absence of a three-loop divergence in four-graviton scattering in N=4 SUGRA in 4 dimensions. Is this implied by supersymmetry?

- Extensions of the strong coupling duality results of maximal (type II) superstring? There has been little discussion of this (but see Bossard, Cosnier-Horeau, Pioline for discussion of ½- and ¼-BPS gauge interactions in D=3 heterotic string with duality group $O(24, 8, \mathbb{Z})$).

M-THEORY AND N=1 SUPERSTRING DUALITIES

Horava and Witten (1995):

ELEVEN-DIMENSIONAL SUPERGRAVITY IN AN INTERVAL OF LENGTH $L = \pi R_{11} \ell_{11}$

Introduce E_8 gauge fields on each boundary – necessary for anomaly cancellation

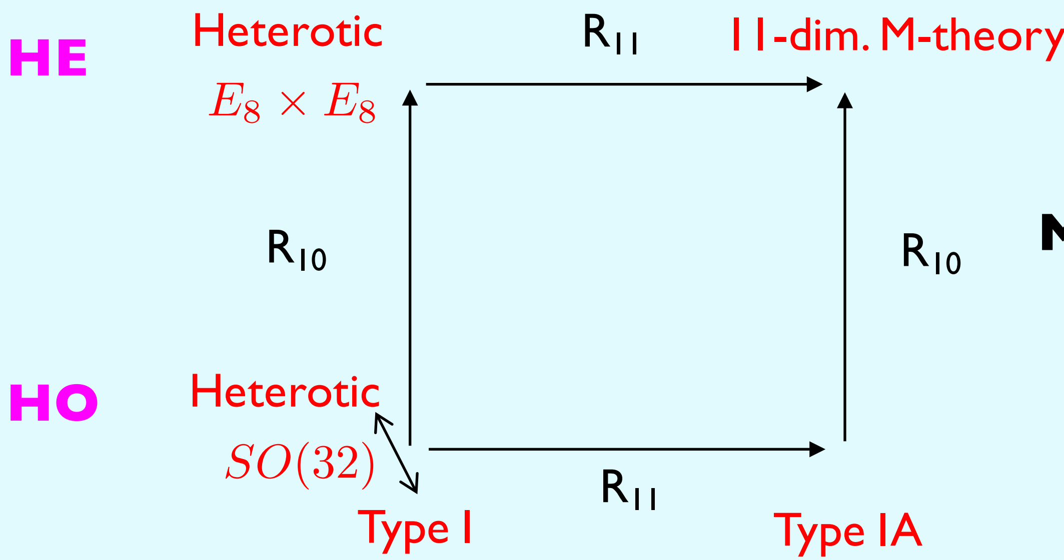
Compactify on a further circle of radius $R_{10} \ell_{11}$ to nine dimensions –

Break symmetry to $SO(16)$ with Wilson lines on circle.

T-duality between heterotic theories

$$SO(16) \times SO(16) \subset E_8 \times E_8$$
$$SO(16) \times SO(16) \subset SO(32)$$

DUALITIES BETWEEN M-THEORY AND N=1 SUPERSTRING THEORIES



M-THEORY ON ORBIFOLD OF A TORUS

M-theory on $\mathbb{M}^9 \times \mathbb{S}^1 \times \mathbb{S}^1/\mathbb{Z}_2$

Based on a number of CLASSICAL SUPERGRAVITY/GAUGE THEORY arguments

I will here consider quantum contributions in PERTURBATIVE SUPERGRAVITY/YANG-MILLS in order to address some long-standing questions (see e.g. Tseytlin, …..) MBG, Arnab RUDRA

M-THEORY/STRING THEORY DICTIONARY

String lengths:

$$\ell_{het} = \frac{\ell_{11}}{\sqrt{R_{11}}} \qquad \ell_I = \frac{\ell_{11}}{\sqrt{R_{10}}}$$

Relations between couplings:

$$g_{he} = R_{11}^{3/2} \qquad g_{IA} = R_{10}^{3/2}$$

HETEROTIC/TYPE I DUALITY $\quad g_{ho} = \dfrac{R_{11}}{R_{10}} = \dfrac{1}{g_I}$

Relations between radii

T-DUALITY $\quad r_{he} = R_{10}\sqrt{R_{11}} = \dfrac{1}{r_{ho}} \qquad r_{IA} = R_{11}\sqrt{R_{10}} = \dfrac{1}{r_I}$

HE $\leftrightarrow$ **HO** $\qquad$ **IA** $\leftrightarrow$ **I**

A question ?

Heterotic SO(32) and Type I theories should have the same amplitudes (or effective actions) in the Einstein frame when

$$g_{ho} \leftrightarrow g_I^{-1}$$

not naively BPS

e.g. In the STRING FRAME perturbative terms of order R^4 arise from:

Type I world-sheet	Sphere	Disk	Torus	....
Type I string frame	g_I^{-2}	g_I^{-1}	$(g_I)^0$	
Einstein frame H0/type I duality	$(g_I)^{-\frac{3}{2}} = g_{ho}^{\frac{3}{2}}$	$g_I^{-\frac{1}{2}} = g_{ho}^{\frac{1}{2}}$	$(g_I)^{\frac{1}{2}} = g_{ho}^{-\frac{1}{2}}$	
HO string frame	g_{ho}	$(g_{ho})^0$	g_{ho}^{-1}	
	nonsense	1-loop	nonsense	

But HO perturbation theory is an expansion in powers of g_{ho}^2.
So naïve term-by-term HO/type I duality gives nonsense – apart from Type I disk/ HO torus.

But the R^4 interaction must transform sensibly under HO/type I duality.

How is this realised ?

It might be that HO/type I duality of these terms is extremely complicated.

But note the very simple modular ($SL(2,\mathbb{Z})$- invariant) function of the coupling that arises in type IIB theory (where this is a BPS interaction).

Feynman rules in Horava-Witten background

Interaction vertices originate from:

- Bulk eleven-dimensional supergravity. $G_{MN}\,,\quad C_{MNP}$ $\quad M,N,P=1,\ldots,11$
- Ten-dimensional gauge theory in each boundary. A_μ^a $\quad \mu=1,\ldots,10$
- Boundary interactions of bulk supergravity fields

  Gauge and Lorentz Chern-Simons interactions induced by boundary – couple to bulk graviton and to $C_{11\mu\nu}\equiv B_{\mu\nu}$

Feynman propagator

(MBG, Gaberdiel)

On circle:
$$G(p,x^{11}-y^{11})=\frac{1}{2L}\sum_{m\in\mathbb{Z}}e^{i\frac{m(x^{11}-y^{11})}{R_{11}\ell_{11}}}\frac{1}{\mathbf{p}^2+\frac{m^2}{R_{11}^2\ell_{11}^2}}$$
$$\mathbf{p}=\left(p_1,\ldots,p_9,\frac{m_1}{R_{10}\ell_{11}}\right)$$

On orbifold: Identify x_{11} with $-x_{11}$ i.e. m with $-m$

$$\begin{aligned}\mathcal{G}(\mathbf{p},x^{11},y^{11})&=G(\mathbf{p},x^{11}-y^{11})+G(\mathbf{p},x^{11}+y^{11})\\&=\frac{1}{L}\sum_{m\in\mathbb{Z}}\frac{1}{\mathbf{p}^2+p_{11}^2}\cos(p_{11}x^{11})\cos(p_{11}y^{11})\end{aligned}$$
$$p_{11}=\frac{m}{R_{11}\ell_{11}}$$

We want to determine the coupling constant dependence of low-lying terms in the low energy expansion of string scattering amplitudes.

Superinvariants

Tseytlin, ...

R^4 superinvariants of maximal supergravity/YM

Type IIA and IIB invariant $\qquad J_0 = t_8 t_8 R^4 - \frac{1}{8}\epsilon_{10}\epsilon_{10} R^4$

Type IIA invariant $\qquad \mathcal{I}_2 = -\frac{1}{8}\,\epsilon_{10}\epsilon_{10} R^4 + 6\,\epsilon_{10} B\, Y_8^{vw}(R)$

$$Y^{vw}(R) = B \wedge \left(\mathrm{tr}\,(R\wedge R\wedge R\wedge R) - \frac{1}{4}\mathrm{tr}\,(R\wedge R)\,\mathrm{tr}\,(R\wedge R)\right)$$

½-maximal supergravity: - three combinations of R^4 :

$$t_8 t_8\, R^4 - 24\, t_8\, \mathrm{tr} R^4 + 6\, t_8\, \mathrm{tr}(R^2)^2 = 0$$

Superinvariants of ½-maximal supergravity/YM:

$$X_1 = t_8 \mathrm{tr} R^4 - \tfrac{1}{4}\epsilon_{10} B\, \mathrm{tr} R^4 \qquad\qquad X_2 = t_8 (\mathrm{tr} R^2)^2 - \tfrac{1}{4}\,\epsilon_{10} B\, (\mathrm{tr} R^2)^2$$

Parity-conserving + parity-violating.

Anomaly cancelling (parity-violating) invariants:

$$\epsilon_{10} B\, Y^{gs}\,(R,F) \qquad Y^{gs}(R,F) = \left(8\,\mathrm{tr} F^4 + \mathrm{tr} R^4 + \frac{1}{4}\,(\mathrm{tr} R^2)^2 - \mathrm{tr} F^2\, \mathrm{tr} R^2\right)$$

8-form

In particular, note that $\qquad Y^{gs}(R,0) = Y^{vw}(R) + \frac{1}{2}(\mathrm{tr} R^2)^2$

Parity violating part of $\qquad X_1 + \frac{1}{4} X_2 = (t_8 - \frac{1}{4}\epsilon_{10} B)\, Y^{gs}(R,0)$

We aim to determine the coupling constant dependence of low-lying terms in the low energy expansion of string scattering amplitudes.

- PARITY VIOLATING HIGHER DERIVATIVE INTERACTIONS DETERMINED BY ANOMALY CANCELLATION

One-loop five-point function

Boundary $\frac{1}{2}\epsilon_{10}\, B\,(\mathrm{tr}R^2)^2 + \boxed{\epsilon_{11}\, C\, Y_8^{(vw)}(R)}$ Bulk (Vafa-Witten)

$$Y_8(R)^{(vw)} = \left(\mathrm{tr}\, R^4 - \frac{1}{4}(\mathrm{tr}\, R^2)^2\right)$$

Anomaly cancelling $= \epsilon_{10}\, B\, Y_8^{(gs)}(R)$ $\quad Y_8(R)^{(gs)} = \left(\mathrm{tr}\, R^4 + \frac{1}{4}(\mathrm{tr}\, R^2)^2\right)$

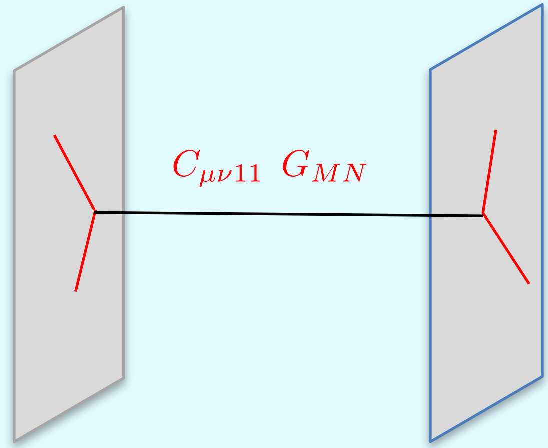
(+ analogous gauge terms)

- These higher derivative terms have PARITY CONSERVING PARTNERS that fill out ten-dimensional $\mathcal{N} = 1$ supermultiplets.

Different kinds of **TREE DIAGRAMS**:

(MBG, Gaberdiel)

e.g. gauge bosons on separate boundaries



$$\frac{R_{10}}{R_{11}\,\ell_{11}^3}\,\mathrm{tr}_1(T_1^{(1)}\,T_1^{(2)})\,\mathrm{tr}_2(T_2^{(3)}\,T_2^{(4)})\,\sum_{m\in Z}\frac{(-1)^m}{-s+\frac{m^2}{R_{11}^2\ell_{11}^2}}$$

LOW ENERGY EXPANSION: expansion in powers of $(-s)\,R_{11}^2\,\ell_{11}^2$

HE powers of $(-s)\,g_{he}^2\ell_H^2$

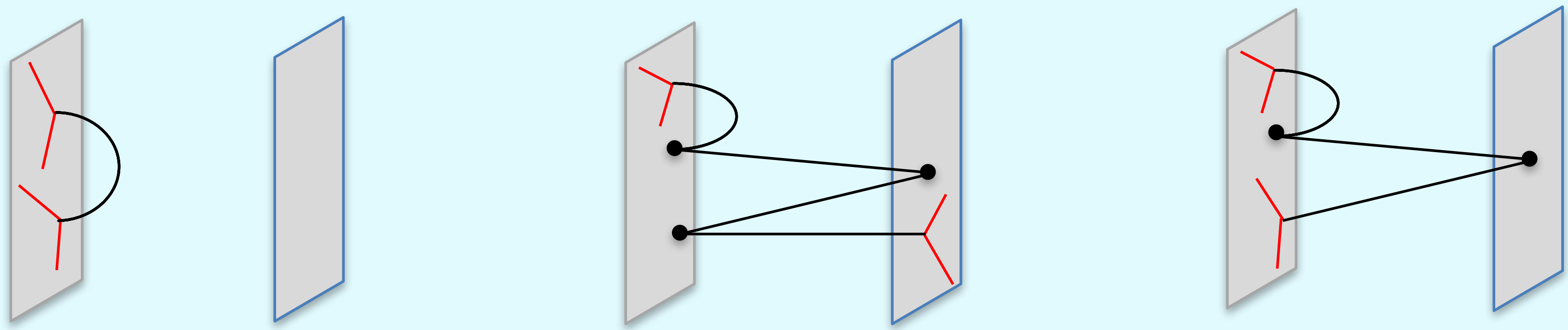
tree-level pole + higher loop corrections

$m=0$ $\qquad$ $m\neq 0$

Type I powers of $(-s)\,\ell_I^2\,r_{IA}^2$

all terms arise from one loop (annulus) in string perturbtion theory. $\quad (g_{IA})^0$

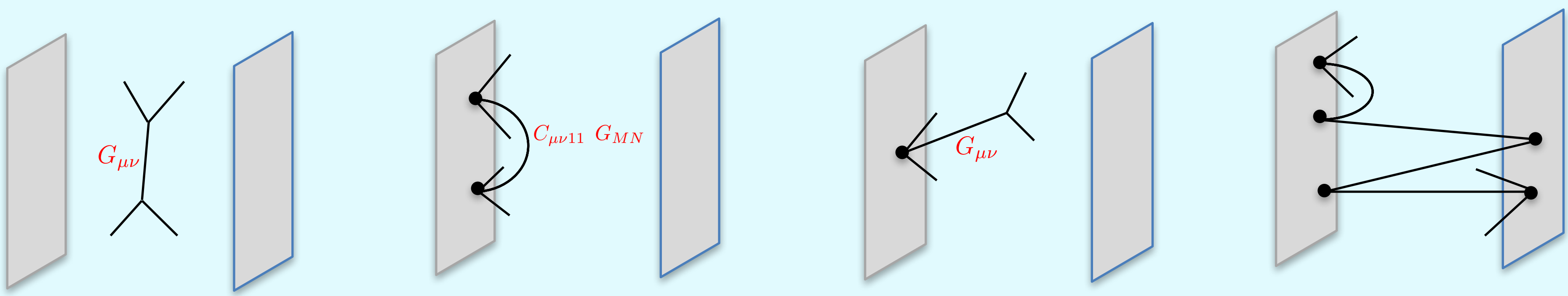
Reproduces HE theory up to terms of order $s(\mathrm{tr}F^2)(\mathrm{tr}F^2)$ - i.e. up to genus two.
These are expected to be BPS “protected” terms.



MULTIPLE PROPAGATOR CONTRIBUTIONS TO 4-GAUGE PARTICLE SCATTERING

REPRODUCES HIGHER DERIVATIVE TERMS OF FORM $s^n\, t_8\, (\mathrm{tr} F^2)^2$ $\quad n = -1,\, 0,\, 1$

in ten-dimensional HE - up to two loops – as expected from BPS conditions



MULTIPLE PROPAGATOR CONTRIBUTIONS TO 4-GRAVITON SCATTERING

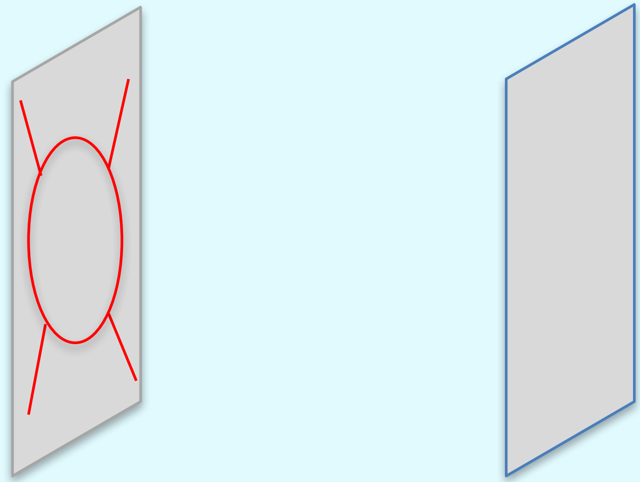
This generates various expected combinations of $t_8\, \mathrm{tr} R^4\,, \quad t_8\, (\mathrm{tr} R^2)^2$

4 GAUGE-PARTICLE AMPLITUDE

GAUGE LOOP

in $\mathbb{M}^9 \times \mathbb{S}^1$

10-dimensional SYM



E_8 gauge loop on x^{10} circle of radius $r_{he} = 1/r_{ho}$ with appropriate Wilson lines

- Massless $SO(16)$ gauge particles + massive spinor states + KK recurrences
- Zero winding number sector is UV divergent – killed by counterterm $\sim r_{he}/g_{he}^{2/3}\ell_{HE}$

  Coefficient must vanish
- Non-zero winding terms give finite expression $\sim 1/r_{he}$

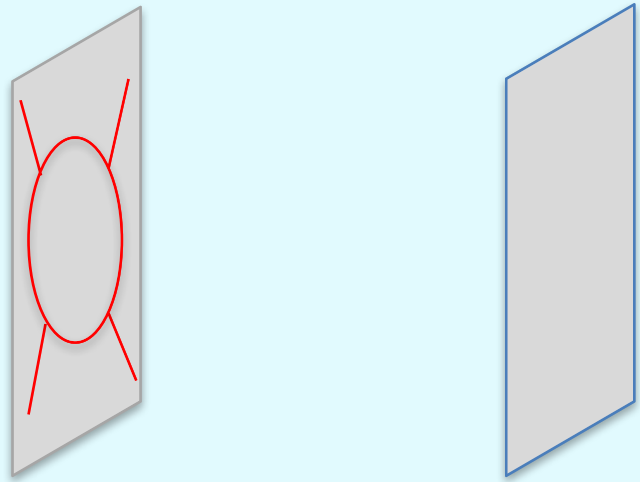
Trace in the FUNDAMENTAL rep. of $SO(16)$

$$\frac{r_{ho}}{\ell_{het}}\zeta(2)\,\mathrm{tr}_1 F^4 = \frac{r_I}{g_I \ell_I}\zeta(2)\,\mathrm{tr}_1 F^4$$

1-LOOP IN HETEROTIC $SO(32) \to SO(16) \times SO(16)$ $\longrightarrow$ DISK AMP. IN TYPE I

4 gauge-particle amplitude

GAUGE LOOP



E_8 gauge loop on x^{10} circle with appropriate Wilson lines

- Massless $SO(16)$ gauge particles + massive spinor states + KK recurrences
- POISSON RESUMMATION expresses this as a sum over windings of loop
- UV divergence in 0-winding sector $\sim C\,\dfrac{r_{he}}{g_{he}^{2/3}\,\ell_{het}}\,(\mathrm{tr}_1 F^2)\,(\mathrm{tr}_1 F^2) \to 0$

Renormalised

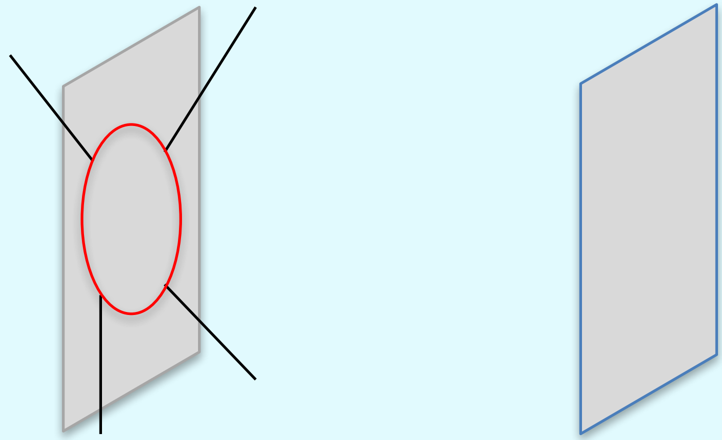
$C = 0$

- Sum over non-zero windings $\dfrac{r_{ho}}{\ell_{het}}\,\zeta(2)\,\mathrm{tr}_1 F^4 = \dfrac{r_I}{g_I \ell_I}\,\zeta(2)\,\mathrm{tr}_1 F^4$

1-LOOP IN HETEROTIC $SO(32) \to SO(16) \times SO(16) \longrightarrow$ DISK AMP. IN TYPE I

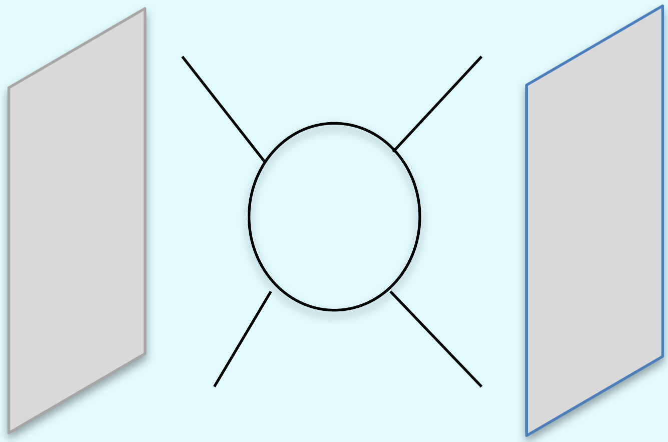
4-GRAVITON LOOP AMPLITUDES

GAUGE LOOP



BULK GRAVITON LOOP

4-GRAVITON amplitude



Sum over supergravity states in the eleven-dimensional loop.

Bulk graviton loop

Orbifold conditions on states: $\Omega: \ x^{11} \to -x^{11}$ $\quad \Omega\,|m\rangle \to |-m\rangle$ $\quad p_{11} = \frac{m}{R_{11}\ell_{11}}$

Decompose $SO(10,1)$ spinor into chiral $SO(9,1)$ spinors $\mathcal{S} = (S_+^A,\, S_-^{\bar{A}})$

Projected states

$$\frac{1}{2}(1+\Omega)\,|A_+;m\rangle = \frac{1}{2}\left(|A_+;m\rangle + |A_+;-m\rangle\right)$$

$$\frac{1}{2}(1+\Omega)\,|\bar{A}_-;m\rangle = \frac{1}{2}\left(|\bar{A}_-;m\rangle - |\bar{A}_-;-m\rangle\right)$$

(i) $m=0$ $\quad |\bar{A}_-;0\rangle = 0$ $\quad \mathcal{N}=1$ ten-dimensional superspace

(ii) $m \neq 0$ $\quad \mathcal{N}=2$ eleven-dimensional superspace

(apart from $m=0$ sector)

Evaluate loop using a world-line superspace vertex operator formalism

(i) Total $m=0$ sector (adding gauge loop to $m=0$ bulk loop) gives HO effective action

$$\frac{1}{\ell_H}\zeta(2)\int d^{10}x\,\sqrt{-G}\,t_8\left(\mathrm{tr}R^4 + \frac{1}{4}(\mathrm{tr}R^2)^2\right) = \frac{1}{\ell_H}\zeta(2)\int d^{10}x\,\sqrt{-G}\,t_8\,Y^{gs}$$

- Forms $\mathcal{N}=1$ supermultiplet with parity violating anomaly cancelling terms in the HO theory. (not renormalised beyond 1 loop).
- HO/type I duality with disk contribution in the type I theory.

(ii) $m \neq 0$ sector – after some work :

$$\left(g_{ho}^{-\frac{1}{2}}\, E_{\frac{3}{2}}(i\, g_{ho}^{-1}) - 2\zeta(2)\right) \int d^{10}x \sqrt{-G}\, t_8 t_8\, R^4$$

Real analytic Eisenstein Series

c.f. Type IIB coefficient $E_{\frac{3}{2}}\left(C^{(0)} + i\, g_{IIB}^{-1}\right)$ with $C^{(0)} = 0$

where $$E_{\frac{3}{2}}(i\, g_{ho}^{-1}) = \sum_{(m_1,m_2)\neq(0,0)} \frac{g_{ho}^{-\frac{3}{2}}}{(m_1^2\, g_{ho}^{-2} + m_2^2)^{\frac{3}{2}}}$$

$$= 2\zeta(3)\, g_{ho}^{-\frac{3}{2}} + 2\zeta(2)\, g_{ho}^{\frac{1}{2}} + \sum_{n\in\mathbb{Z}^+} 8\pi\, \sigma_{-1}(|n|)\, e^{-\frac{2\pi|n|}{g_{ho}}}\, (1 + O(g_{ho}))$$

Tree-level | One loop (**CANCELS** with $-2\zeta(2)$) | "D-instantons"

- Invariant under HO/type I duality $E_{\frac{3}{2}}(i\, g_I^{-1}) = E_{\frac{3}{2}}(i\, g_I) = E_{\frac{3}{2}}(i\, g_{ho}^{-1})$

BUT **NOT** INVARIANT TERM BY TERM IN PERT. EXPANSION

- SUGGESTS NON-RENORMALISATION OF $t_8 t_8\, R^4$ BEYOND TREE LEVEL IN HO THEORY.

  No further M-theory SUGRA diagrams that can contribute to order R^4

  The one-loop action is $(t_8 + \epsilon_{10} B)\left(\mathrm{tr}R^4 + \frac{1}{4}(\mathrm{tr}R^2)^2\right)$

  Agrees with explicit perturbative heterotic string calculations

- Explicit SUGRA calculations show absence of expected R^4 three-loop counterterm.

  Bern, Davies, Dennen, Huang

WHY?

- Is there some symmetry protecting this (apparently non-BPS) interaction?

- The presence of TYPE I D-INSTANTON contributions is expected and results from KK modes in the x^{10} direction with world-lines wound around the x^{11}direction. These are Z_2 instantons that break O(32) to SO(32).

- The HO D-INSTANTON correspond to KK modes in the x^{11} direction - these are UNSTABLE since p_{11} is not conserved) with world-lines wound around x^{10}.

Comments

- More generally, moduli space is a symmetric space $G(R)/K(R)$ with discrete identifications

$$G(\mathbb{Z})\backslash G(R)/K(R)$$

where $G(\mathbb{Z})$ is a a discrete arithmetic subgroup of the supergravity duality group; Multi-graviton/gauge amplitudes transform as automorphic functions under $G(\mathbb{Z})$.

- Non-perturbative information involving D-instantons.

- Makes contact with BPS black hole counting formulas - D-instantons in D dimensions are wrapped euclidean world-lines of D=4 black holes.

- Well-studied in the low energy expansion of type II (maximal SUSY) but not in the heterotic/type I case.

- However suggested formulas for ½-BPS F^4 and ¼-BPS d^2F^4 interactions in the D=3 dimensional SUGRA/YM theory Bossard, Cosnier-Horeau, Pioline

$$G(\mathbb{Z}) = O(24, 8, \mathbb{Z})$$

Instantons correspond to the Dikgraaf, Verlinde, Verlinde genus-two modular form that counts ¼-BPS black hole states in D=4 dimensions.

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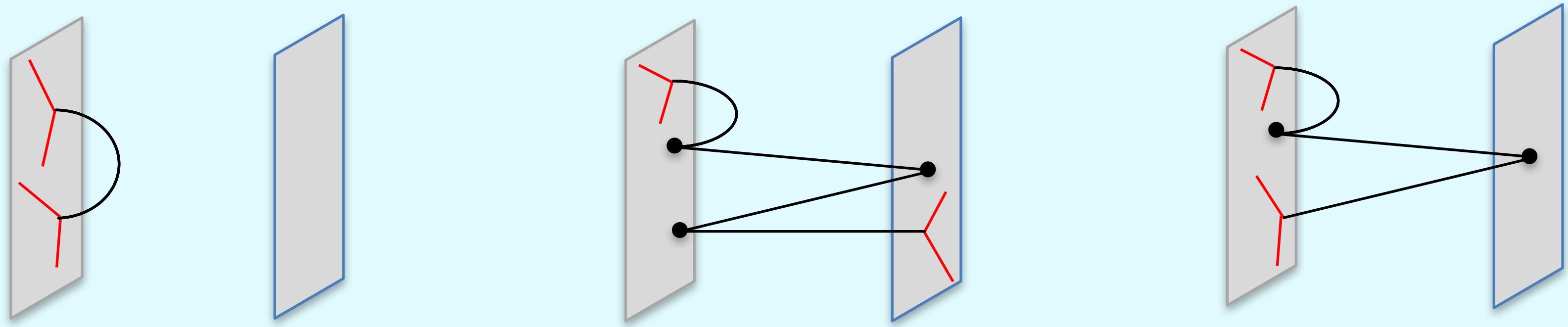
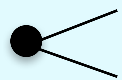


MULTIPLE PROPAGATOR CONTRIBUTIONS TO 4-GAUGE PARTICLE SCATTERING

Power series in  $\ell_{11}^2\, s^2/R_{11} = \ell_{het}^2\, s = g_I\, \ell_I^2\, s$

Tree level HE   Multiloop type I

REPRODUCES HIGHER DERIVATIVE TERMS OF FORM $s^n\, t_8\, (\mathrm{tr} F^2)^2$ $n = -1,\, 0,\, 1$

in ten-dimensional HE - up to two loops – as expected from BPS conditions

$$s^n\, t_8\, (\mathrm{tr}_1 F^2)^2 \qquad s^n\, t_8\, (\mathrm{tr}_1 F^2)\, (\mathrm{tr}_2 F^2) \qquad s^n\, t_8\, (\mathrm{tr}_2 F^2)^2$$