Weyl Anomalies and D-brane Charges

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ChrisFest

Supergravity, Strings and Dualities
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I feel privileged to be here to celebrate Chris' distinguished career, and to wish him a very

Happy Birthday

Chris is a great person and a brilliant physicist

He is also living proof that crazy ideas are not <u>always</u> wrong !

Chris has (at least) two obsessive themes in physics:

Hunting (everywhere) for Dualities

How many dimensions ??

« Dualities link M-theory to other 11-dimensional theories in signatures **9+2** and **6+5**, and to type II string theories in **all 10-d**imensional **signatures** »

« Branes with various world-volume signatures are possible. For example, the 9+2 dimensional M* theory has membrane-type solutions with world-volumes of signature (3,0) and (1,2), and a solitonic solution with world-volume signature (5,1) »

with Ramzi Khuri '99

Later he let loose the 10/11 taboo:

(Double Field Theory)

18+1 ?

10+10 ?

18+2 ?

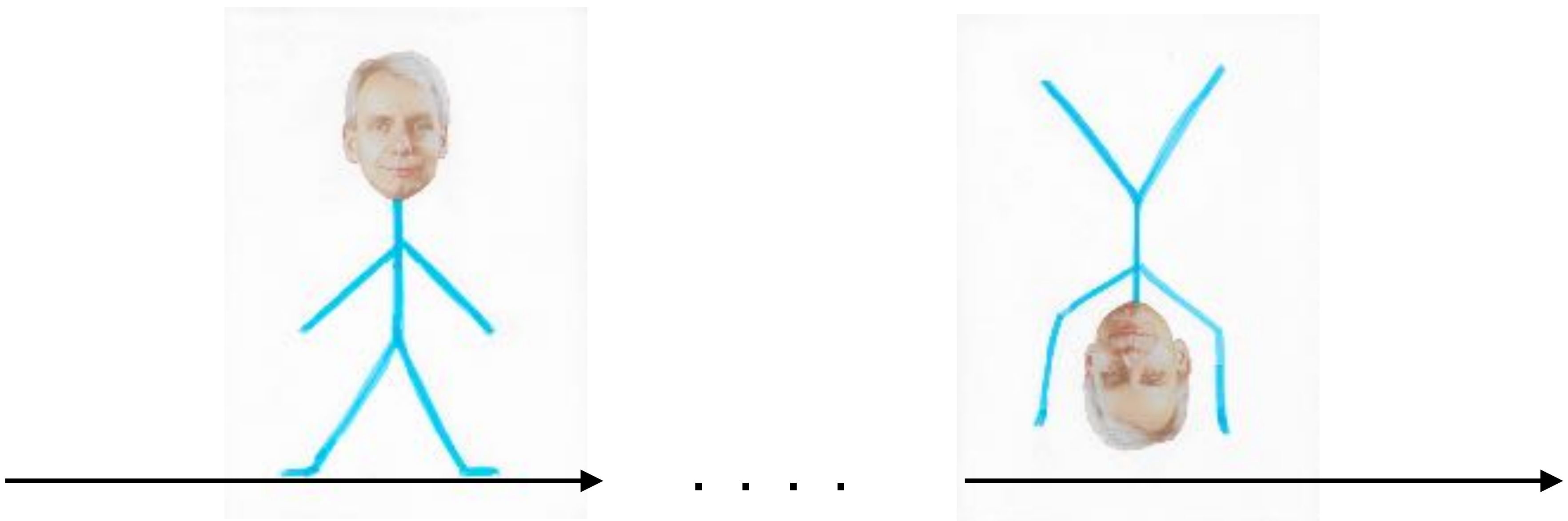
Of course you would not expect Chris to shy away from

T-dualizing time

(Timelike T-duality, de Sitter space ...... C.H. '98)

So it is fitting to offer him for his birthday

A T-fold gift:

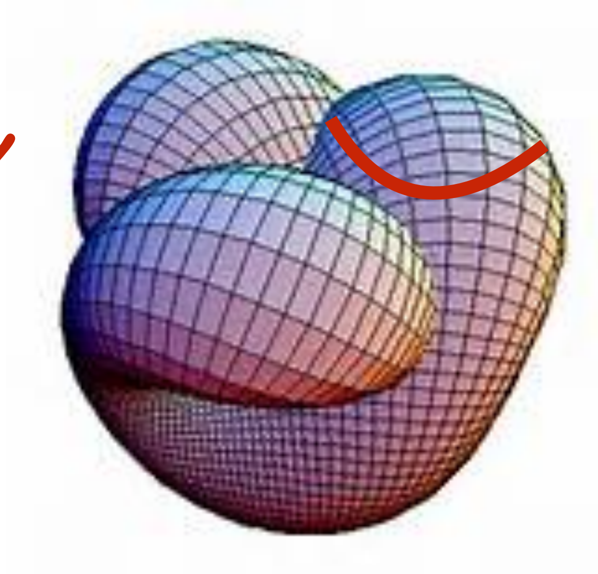


60 1/60

NB: for maximum comfort, do <u>not</u> reverse the arrow of time

1. Introduction

The **moduli** spaces of **Calabi-Yau manifolds,** and the associated **wrapped D-branes** have been intensively studied by string theorists and mathematicians

CY 

$$\times \quad \mathbb{R}^{1,3} \quad \simeq$$ **Our world ?**

Two important quantities: the metric $g_{I\bar{J}}(\lambda)$ and the D-brane masses $M_s(\lambda)$. Computing these is a time-honoured problem.

Gromov-Witten invariants

Recent progress in **susy field theories** has opened a new line of attack for the computation of these quantities

It was conjectured that they are computed by partition functions of N=(2,2) GLSM using **susy localization**

Pestun 2007

. . . .

Benini, Cremonesi 1206.2356

Doroud, Gomis, Le Floch, Lee 1206.2606

$$g_{I\bar{J}} = \partial_I \partial_{\bar{J}} K$$

$$M_s(\lambda)$$

Honda + Okuda 1308.2217

Hori + Romo 1308.2438

Sugishita + Terashima 1308.1973

Well-known facts :

The CY moduli space **factorizes locally**:

but see arXiv:1611.03101
Gomis, Komargodski, Ooguri, Seiberg, Wang

$$\mathcal{M}_c \times \mathcal{M}_{tc}$$

$h^{2,1}$ complex structure
(c,c)

$h^{1,1}$ Kähler moduli
(c,a)

Strong constraints from $\mathcal{N} = 2$ supersymmetry of type-II string theory compactified on CY3:

$$IIA: \quad h^{1,1} \;\; vector \qquad h^{2,1}+1 \;\; hyper$$

$$IIB: \quad h^{2,1} \;\; vector \qquad h^{1,1}+1 \;\; hyper$$

special Kähler (vector) — quaternionic (hyper)

The string coupling is a hyper, and the Kähler moduli include the CY volume

$\Longrightarrow$ metric on complex-structure m.s. is **classical** but
metric on Kähler m.s. has **instanton** corrections ← Gromov-Witten invariants

Assuming mirror symmetry gives the latter from the former when mirror manifold and map is known. But usually it is not.

Jockers, Kumar, Lapan, Morrison, Romo (1208.6244) conjectured that

the **partition function on the 2-sphere** computes the Kähler potential

$$Z(S^2) = \left(\frac{r}{r_0}\right)^{c/3} e^{-K(\lambda,\bar{\lambda})}$$

This was soon after argued for with the help of tt* eqns by
Gomis + Lee (1210.6022)

1,5 years ago **Gomis, Komargodski, Hsin, Schwimmer, Seiberg, Theisen (1509.08511)**

gave an elegant new proof based on a ``new type" of **Weyl anomaly**

Osborn '91

Target-space / worldsheet dictionary:

Target space	**worldsheet**
Calabi-Yau	N=(2,2) SCFT
moduli λ^I	marginal deformations
moduli space	superconformal manifold
metric	Zamolodchikov metric
wrapped brane	boundary conditions Ω
mass	bnry degeneracy g^{Ω}
	Affleck, Ludwig

NEW:

Moduli-dependence	**Integrated anomaly**

With **Daniel Plencner** we generalized the Gomis et al anomaly to worldsheets with boundary

arXiv: 1612.06386

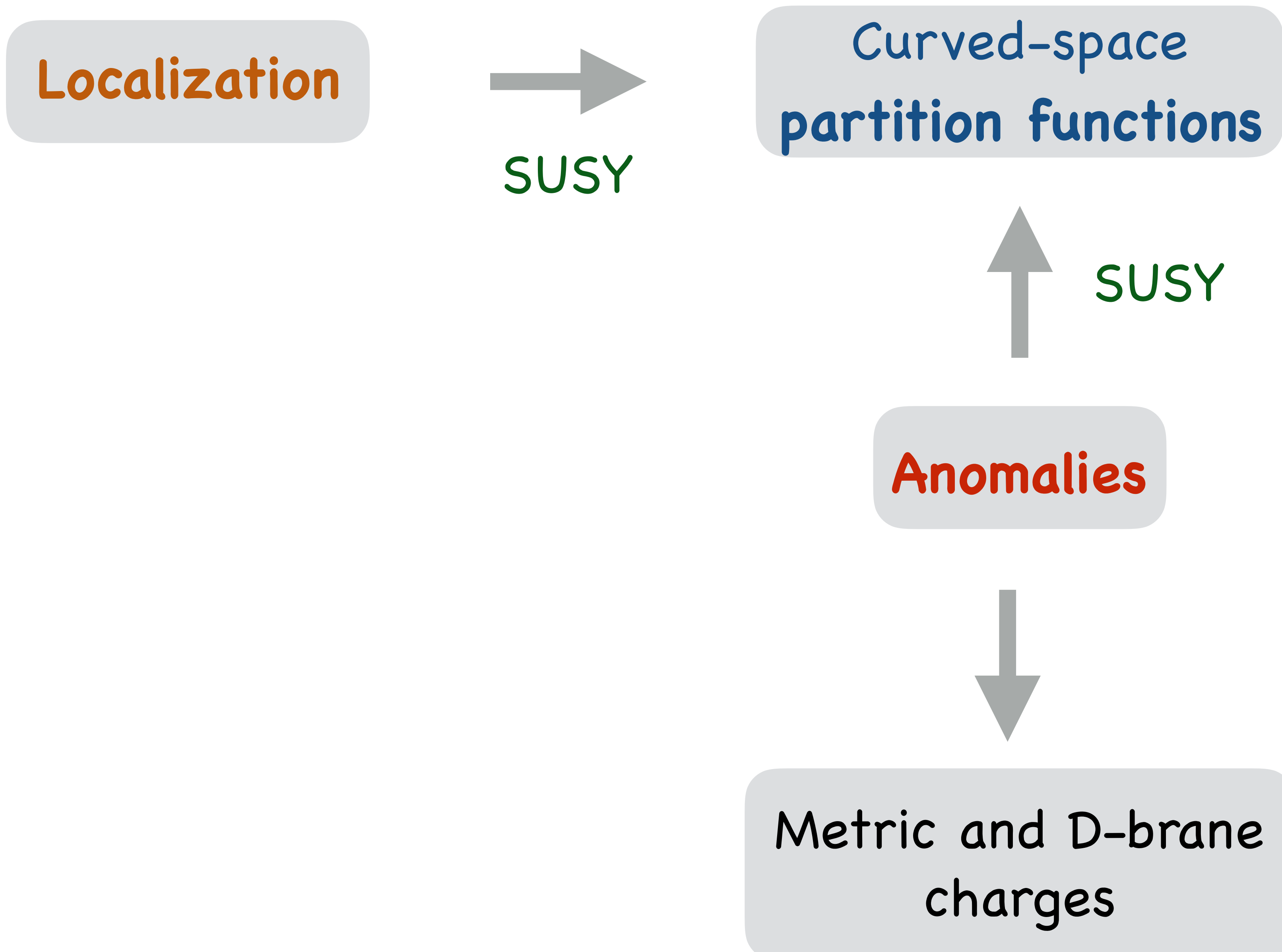
This shows in particular that the hemisphere partition function computes the other important piece of geometric data:

The **central charge** $C^{\Omega}(\lambda)$, and the **mass** of CY D-branes.

confirming conjecture in

Honda + Okuda 1308.2217
Hori + Romo 1308.2438
Sugishita + Terashima 1308.1973

This is a powerful circle of ideas, with non-trivial corollaries and generalizations to higher Dims:



Rest of this talk:

2. Superconformal manifolds & Anomalies

3. Corollaries for $\mathcal{Z}(S^2)$ and CY moduli space

4. Extension to Boundaries and D-brane mass/charge

5. Summary

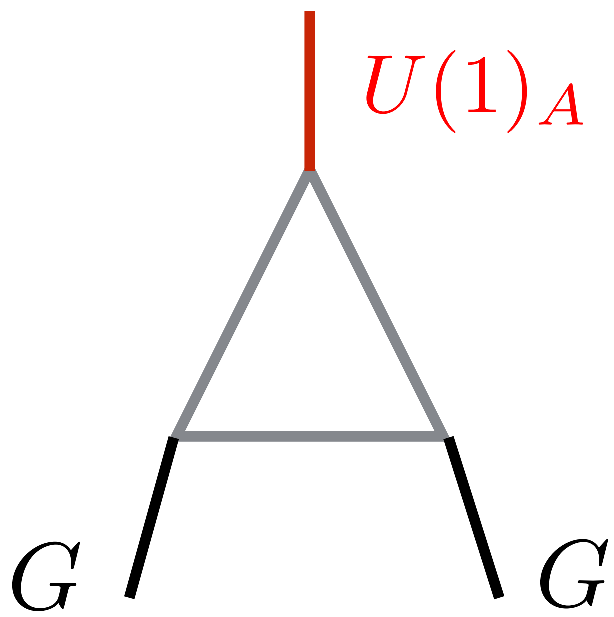
2. "New" super-Weyl anomalies

Anomalies arise when non-conservation in correlation functions:

$$\langle \partial_\mu j^\mu \, \mathcal{O}_1(p_1) \cdots \mathcal{O}_n(p_n) \rangle \neq 0$$

When r.h.s. proportional to **momenta**: non-conservation only in presence of **spacetime-dependent background fields**

e.g.



$$\Longrightarrow \quad \partial_\mu j^\mu_A = F \wedge F$$

axial charge violated by instantons

For chiral anomalies: background is gauge or gravitational

For trace (Weyl) anomaly, can be **exactly-marginal couplings**:

Osborn '91
Osborn, Petkou '93
. . .
Bzowski, McFadden, Skenderis '13 '15

In 2D the 2-point function of marginal operators reads:

Zamolodchikov metric — **anomaly**

$$\langle \mathcal{O}_I(z)\bar{\mathcal{O}}_{\bar{J}}(w)\rangle = g_{I\bar{J}}\,\mathcal{R}\frac{1}{|z-w|^4} = g_{I\bar{J}}\,\frac{1}{2}(\partial\bar{\partial})^2\left[\log(|z-w|^2\mu^2)\right]^2$$

differential regularization of distribution

Turn on space-dependent couplings λ^I :

$$\frac{\partial \mathcal{Z}}{\partial \log \mu} \sim \int_z \int_w \lambda^I(z) \bar{\lambda}^{\bar{J}}(w) \frac{\partial}{\partial \log \mu} \langle \mathcal{O}_I(z) \bar{\mathcal{O}}_{\bar{J}}(w) \rangle \sim \int \partial_\mu \lambda^I \partial^\mu \bar{\lambda}^{\bar{J}} g_{I\bar{J}}$$

Anomaly is **invisible for constant couplings**. But supersymmetry relates it to a term that does not vanish when $\partial_\mu \lambda^I = 0$

Gomis et al (1509.08511)

This term can be removed by non-susy local counterterm;

but **SUSY gives it universal meaning**

Technical details:

To $\mathcal{N}=(2,2)$ SCFTs have $U(1)_V \times U(1)_A$ R-symmetry.

In computing the anomaly we choose to preserve the **vector-like** symmetry, so we must couple it to the $\mathcal{N}=2$ supergravity in which this symmetry is gauged by a field V^μ

Closset + Cremonesi (1404.2636)

In superconformal gauge:

$$g_{\mu\nu} = e^{2\sigma}\eta_{\mu\nu} \ , \quad V^\mu = \epsilon^{\mu\nu}\partial_\nu a$$

Classically σ and a decouple, but in the quantum theory they dont due to the **Weyl** and **axial** anomalies.

Supersymmetry places these fields in a twisted-chiral multiplet

$$\Sigma(y^\mu) = (\sigma + ia) + \theta^+\bar{\chi}_+ + \bar{\theta}^-\chi_- + \theta^+\bar{\theta}^- w$$

with components functions of $y^\pm = x^\pm \mp i\theta^\pm\bar{\theta}^\pm$

The tc field obeys $\overline{D}_+\Sigma = D_-\Sigma = 0$

where $D_\pm = \frac{\partial}{\partial\theta^\pm} - i\bar{\theta}^\pm\partial_\pm \ , \qquad \overline{D}_\pm = -\frac{\partial}{\partial\bar{\theta}^\pm} + i\theta^\pm\partial_\pm \ .$

It is useful to also promote the marginal couplings to vevs of tc fields

$$\Lambda^I = \lambda^I(y^\pm) + \cdots \ , \qquad \bar{\Lambda}^I = \bar{\lambda}^I(\bar{y}^\pm) + \cdots$$

Seiberg

so as to make the susy of the anomaly manifest.

The anomaly $iA(\delta\Sigma) := \delta_\Sigma \log \mathcal{Z}_V(M)$ is the susy invariant

$$A_{\text{closed}} := A^{(1)} + A^{(2)} = \frac{1}{4\pi}\int_M d^2x \int d^4\theta \left[\frac{c}{6}(\delta\Sigma\,\bar{\Sigma} + \delta\bar{\Sigma}\,\Sigma) - (\delta\Sigma + \delta\bar{\Sigma})K(\Lambda,\bar{\Lambda})\right]$$

Gomis et al (1509.08511)

This obeys **Wess-Zumino** consistency $\delta_\Sigma A(\delta\Sigma') - \delta_{\Sigma'} A(\delta\Sigma) = 0$

and can be integrated with the result:

$$\log \mathcal{Z}_V \supset \frac{i}{4\pi}\int_M d^2x \int d^4\theta \left[\frac{c}{6}\Sigma\,\bar{\Sigma} - (\Sigma + \bar{\Sigma})K\right] .$$

super-**Liouville** super-**Osborn**

Expand in components:

$$A^{(1)} = -\frac{c}{12\pi}\int_M d^2x \left[\delta\sigma\,\Box\sigma + \delta a\,\Box a + \frac{1}{2}(\delta w\,\bar{w} + \delta\bar{w}\,w) + \partial^\mu b_\mu^{(1)}\right] + \text{fermions}\ ,$$

$$A^{(2)} = -\frac{1}{2\pi}\int_M d^2x \left[\delta\sigma\,(\partial_\mu\lambda^I\partial^\mu\bar{\lambda}^{\bar{J}})\partial_I\partial_{\bar{J}}K - \frac{1}{2}K\,\Box\delta\sigma - (\partial^\mu\delta a)\mathcal{K}_\mu + \partial^\mu b_\mu^{(2)}\right]$$

where $\mathcal{K}_\mu := \frac{i}{2}(\partial_I K\partial_\mu\lambda^I - \partial_{\bar{I}}K\partial_\mu\bar{\lambda}^{\bar{I}})$ ⟵ Kähler one-form

(Cohomologically) **non-trivial**, real anomalies

Variation of **local invariant counterterm**

$$\sim \int \sqrt{g}R^{(2)}\,K(\lambda,\bar{\lambda})$$

The first term in $A^{(2)}$ is the **scale anomaly in the 2-point function**

as follows from $\delta\sigma = -\delta \log \mu$, $\partial\bar{\partial} \log |z|^2 = \pi \delta^{(2)}(z)$

and $\partial_I \partial_{\bar{J}} K = g_{I\bar{J}}$

contact term

The non-vanishing term for constant couplings is the **red** one

It could be removed by **change of scheme** in bosonic theory,

but supersymmetry relates it to the non-trivial **blue** terms !

Similar remarks for 4D Casimir energy

Assel, Cassani, Di Pietro, Komargodski, Lorenzen, Martelli 1503.05537

3. Corollaries

Integrating the anomaly for constant couplings gives

$$\int_{S^2} K\,\Box\sigma = -4\pi K \quad\Longrightarrow\quad Z_V^E(S^2) = \left(\frac{r}{r_0}\right)^{c/3} e^{-K(\lambda,\bar\lambda)}$$

so the 2-sphere free energy computes the Kähler potential

on the SCFT2 moduli space (both chiral and twisted chiral)

A puzzle

$Z_V^E(S^2)$ not invariant under **Kähler-Weyl** transformations

$$K'(\lambda,\bar\lambda) = K(\lambda,\bar\lambda) + H(\lambda) + \bar H(\bar\lambda)$$

Resolution

The difference amounts to change of **renormalization scheme**:

$$\Delta_{\rm KW} A^{(2)} = -\frac{1}{4\pi}\int_M d^2x \int d^4\theta\, (\delta\Sigma + \delta\bar{\Sigma})H + c.c.$$

$$= \; -\frac{1}{4\pi}\int_M d^2x \int d\theta^+ d\bar{\theta}^- \,(\bar{D}_+ D_- \delta\bar{\Sigma})H + \int_M d^2x\; (\partial^\mu Y_\mu) + c.c.$$

twisted F-term

$$\mathcal{R} = \bar{D}_+ D_- \bar{\Sigma} = \; -\bar{w} \; + \; \theta^+\bar{\theta}^- \partial_+\partial_-(\sigma - ia) + \cdots$$

curvature superfield

So **local, susy and diffeo-invariant counterterm** compensates the Kähler-Weyl (gauge) transformation !

An interesting conjecture

Gomis et al (1509.08511)

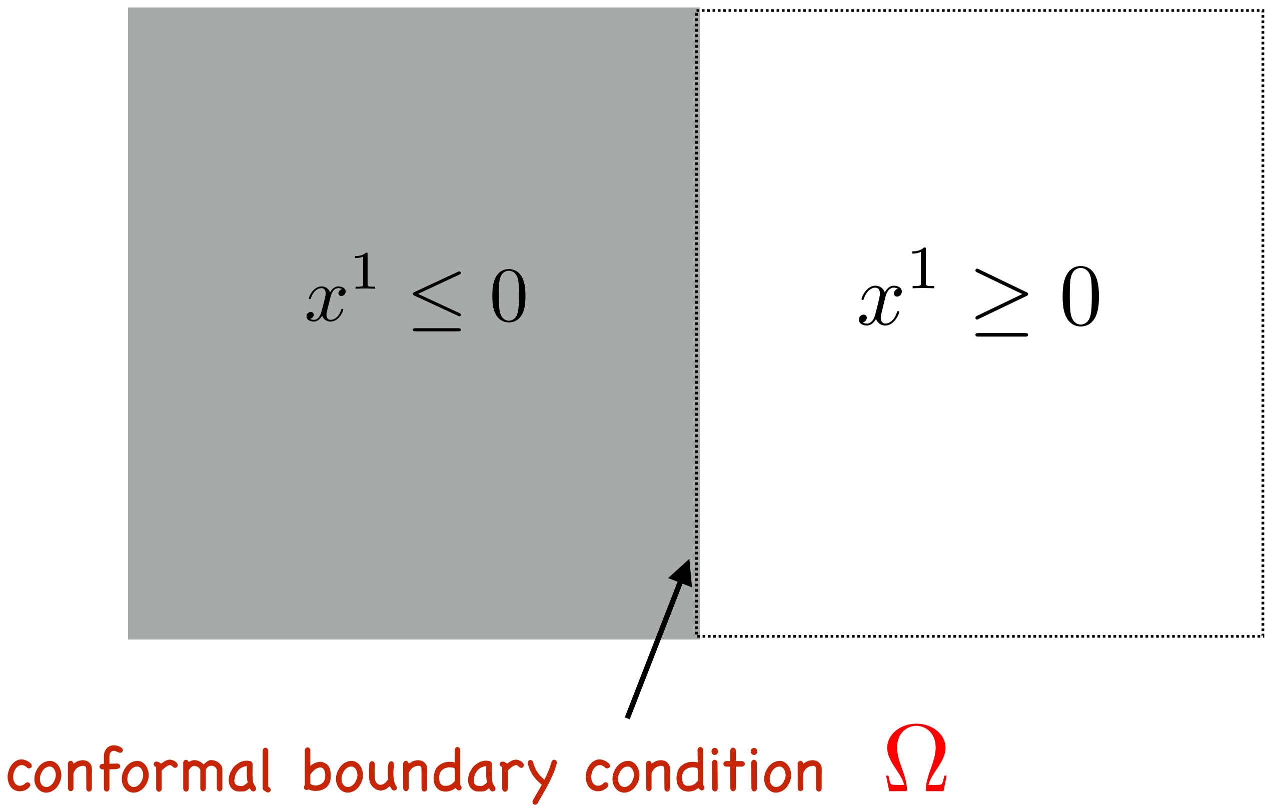
If the moduli space had non-vanishing Kähler class one could pick $\lambda^I(x)$ such that $S^2 \to \mathcal{M}$ is non-trivial 2-cycle

Then there would be no global renormalization scheme !

Way out: **Moduli space has Kähler class = 0**

4. Boundary anomaly

Consider half space:



One-point functions of marginal operators:

$$\langle \mathcal{O}_I(x) \rangle_\Omega = d_I^\Omega \mathcal{R} \frac{1}{|x_1|^2} = d_I^\Omega \, \partial_1^2 \left[\Theta(-x^1) \log |x^1 \mu| \right]_\Omega$$

Focus on **B-type** brane in region of **Kähler** moduli space with no walls of marginal stability.

The 1pt-function coefficients are related to a **holomorphic boundary charge** $c^{\Omega}(\lambda)$

$$4d_I = \frac{c^{\Omega}_I}{c^{\Omega}} = \partial_I (K + \log c^{\Omega})$$

Ooguri, Oz, Yin '96

Argument: vacuum projection of boundary state

$$\Pi_{\text{vac}} \, |\Omega\rangle\rangle := c^{\Omega} \, |0\rangle_{\text{RR}} + \sum_I c^{\Omega}_I \, |I\rangle_{\text{RR}}$$

is flat section of the improved connection $\nabla - C$ on moduli space

structure constants of chiral ring

Our result: prove these relations from Weyl-Osborn anomaly, and show that hemisphere p.f. computes bnry charge

$$\mathcal{Z}_+(D^2) = \left(\frac{r}{r_0}\right)^{c/6} c^\Omega(\lambda) \ , \qquad \mathcal{Z}_-(D^2) = \left(\frac{r}{r_0}\right)^{c/6} c^\Omega(\bar\lambda) \ .$$

Under Kähler Weyl transformations $c^\Omega \ \rightarrow \ c^\Omega \, e^F$

The **boundary entropy** is the scheme-independent combination

$$g^\Omega = \frac{|c^\Omega|}{e^{-K/2}} \ = \sqrt{\frac{\mathcal{Z}_+(D^2)\mathcal{Z}_-(D^2)}{\mathcal{Z}(S^2)}}$$

D-brane mass

In string-theory compactifications, g^Ω and c^Ω are the **mass** and **RR charge** of the 1/2 BPS D-brane states

↑

dyons in field-theory limits

These are related to worldsheet anomalies !

Technical details:

3 steps in calculation:

- Take into account the divergence terms in $A_{\rm closed}$

$$b_\mu^{(1)} = \frac{1}{4}(\partial_\mu \delta\sigma)\sigma - \frac{3}{4}\delta\sigma\,\partial_\mu\sigma + \frac{1}{4}(\partial_\mu \delta a)a - \frac{3}{4}\delta a\,\partial_\mu a$$

$$b_\mu^{(2)} = \frac{1}{4}(\partial_\mu \delta\sigma)K - \frac{1}{4}\delta\sigma\,\partial_\mu K\ .$$

- Add `minimal' boundary term needed for susy

- Extra boundary-superinvariant additions using formalism of **boundary superspace**

Reference boundary completion

top component

Consider the D-term :

$$\int_M d^2x \int d^4\theta\, \mathcal{S} = \int_M d^2x\, [\mathcal{S}]_{\rm top}$$

The **type-B susy** generator is $\mathcal{D}_{\rm susy} = \epsilon\,(Q_+ + Q_-) - \bar{\epsilon}\,(\bar{Q}_+ + \bar{Q}_-)$

where $\mathcal{Q}_\pm = \frac{\partial}{\partial\theta^\pm} + i\bar{\theta}^\pm\partial_\pm\ ,\qquad \overline{\mathcal{Q}}_\pm = -\frac{\partial}{\partial\bar{\theta}^\pm} - i\theta^\pm\partial_\pm$

The transformation of the D-term is a total derivative

$$\Delta_{\rm susy}[\mathcal{S}]_{\rm top} = \int d^4\theta\ \mathcal{D}_{\rm susy}\mathcal{S} = i\epsilon \int d^4\theta\ (\bar{\theta}^+\partial_+\mathcal{S} + \bar{\theta}^-\partial_-\mathcal{S})\ +\ c.c.$$

We want to write as the susy transformation of a boundary term.

Standard manipulations give:

$$\Delta_{\rm susy}[\mathcal{S}]_{\rm top} = -\Delta_{\rm susy}(\partial_1[S]_{\rm bnry}) + \partial_0 Y$$

with $$[\mathcal{S}]_{\rm bnry} = -\frac{i}{2}\left([\mathcal{S}]_{\theta^+\bar\theta^-} - [\mathcal{S}]_{\theta^-\bar\theta^+}\right) - \frac{1}{4}\partial_1[\mathcal{S}]_{\emptyset}$$

so that $$\mathrm{I}_D(\mathcal{S}) := \int d^2x\, [\mathcal{S}]_{\rm top} + \int dx^0\, [S]_{\rm bnry}$$

is our susy-invariant standard completion.

For the case of interest, the integrated superfield is $\delta\mathcal{S}$

with $$\mathcal{S} = \frac{1}{4\pi}\left[\frac{c}{6}\Sigma\bar\Sigma - (\Sigma + \bar\Sigma)K\right]$$

Boundary superspace

Hori (hep-th/0012179)

$$x^+ = x^- \,, \qquad \theta \equiv e^{-i\beta}\,\theta^+ = e^{i\beta}\,\theta^- \,, \qquad \bar\theta \equiv e^{i\beta}\,\bar\theta^+ = e^{-i\beta}\,\bar\theta^-$$

Restrictions of bulk superfields, e.g.

$$\Sigma|_{\partial M} = \sigma + ia + \theta\bar\chi_+ + \bar\theta\chi_- + \theta\bar\theta[w - i\partial_1(\sigma + ia)]$$

Usual D-term and F-term integrals of bnry superfields are invariant

Brunner + Hori (hep-th/0303135)

WZ-consistency, locality and parity covariance leads to ansatz for

boundary-superinvariant contribution to anomaly:

$$\int dx^0\,[\mathcal{B}]_{\theta\bar\theta} \qquad \text{where} \qquad \mathcal{B} = \frac{i}{8\pi}\left[\#\frac{c}{12}(\Sigma^2 - \bar\Sigma^2) + \bar\Sigma\,G^\Omega(\Lambda,\bar\Lambda) - \Sigma\,G^\Omega(\bar\Lambda,\Lambda)\right]\Bigg|_{\partial M}$$

and reality condition $\quad G^\Omega(\bar\Lambda,\Lambda) = [G^\Omega(\Lambda,\bar\Lambda)]^\star$

Collecting everything:

$$A_{\text{open}} = \int_M d^2x \, [\delta \mathcal{S}]_{\text{top}} + \int_{\partial M} dx^0 \left([\delta \mathcal{S}]_{\text{bnry}} + [\delta \mathcal{B}]_{\theta\bar{\theta}} \right)$$

where

$$\mathcal{S} = \frac{1}{4\pi} \left[\frac{c}{6} \Sigma \bar{\Sigma} - (\Sigma + \bar{\Sigma}) K \right]$$

$$[\mathcal{S}]_{\text{bnry}} = -\frac{i}{2} \left([\mathcal{S}]_{\theta^+ \bar{\theta}^-} - [\mathcal{S}]_{\theta^- \bar{\theta}^+} \right) - \frac{1}{4} \partial_1 [\mathcal{S}]_{\emptyset}$$

$$\mathcal{B} = \frac{i}{8\pi} \left[\# \frac{c}{12} (\Sigma^2 - \bar{\Sigma}^2) + \bar{\Sigma} \, G^{\Omega}(\Lambda, \bar{\Lambda}) - \Sigma \, G^{\Omega}(\bar{\Lambda}, \Lambda) \right] \Big|_{\partial M}$$

central-charge anomaly (red) — Weyl-Osborn anomaly (green)

cf Polchinski; Solodukhin for higher D

Susy Ward identity: $\langle \int \delta \mathcal{L}_{\rm sugra} \int \delta \mathcal{L}_{\rm SCFT} \rangle = 0$ if $\delta \bar{\Sigma} = \bar{\Lambda}^I = 0$

$\Longrightarrow$ no terms propto $\delta \Sigma \, \Lambda^I$

$$\Longrightarrow \quad G^\Omega(\lambda, \bar{\lambda}) = K(\lambda, \bar{\lambda}) + 2 \log c^\Omega(\lambda)$$

Kähler-Weyl covariance (up to local counterterms) requires

$c^\Omega := e^{h^\Omega}$ section of holomorphic line bundle

$$K \to K + H + \bar{H} \qquad h^\Omega \to h^\Omega - H$$

final ingredient: susy hemisphere

. . . . Seiberg, Festuccia 1105.0689

$$A_{\rm open} \supset \delta \Big\{ -\frac{1}{4\pi}\int d^2x \left[\Box(\sigma - ia)h^\Omega + \Box(\sigma + ia)\bar{h}^\Omega\right] + \frac{i}{4\pi}\int dx^0 \left[\bar{w}\, h^\Omega - w\, \bar{h}^\Omega\right]\Big\}$$

integrated anomaly subtracted so as to vanish for infinitesimal disk depends only the holomorphic boundary charge, plus the auxiliary field of the metric.

Killing-spinor equations imply

$$w = \quad 2i\frac{\zeta^-}{\zeta^+}\partial_z(\sigma + ia + \log\zeta^-) = \quad 2i\frac{\bar\zeta^+}{\bar\zeta^-}\partial_{\bar z}(\sigma + ia + \log\bar\zeta^+)\,,$$
$$\bar w = -\,2i\frac{\zeta^+}{\zeta^-}\partial_{\bar z}(\sigma - ia + \log\zeta^+) = -2i\frac{\bar\zeta^-}{\bar\zeta^+}\partial_z(\sigma - ia + \log\bar\zeta^-)\,.$$

where the unbroken superconformal symmetries are

$$\epsilon_+ = \epsilon\,\zeta^-(z)\ ,\quad \epsilon_- = -\epsilon\,\zeta^+(\bar z)\ ,\quad \bar\epsilon_+ = \bar\epsilon\,\bar\zeta^-(z)\ ,\quad \bar\epsilon_- = -\bar\epsilon\,\bar\zeta^+(z)$$

Two solutions for hemisphere with B-type bnry condition:

$$(+):\quad \zeta^- = 1,\quad \zeta^+ = \bar z,\qquad \bar\zeta^- = z,\quad \bar\zeta^+ = 1,$$
$$(-):\quad \zeta^- = z,\quad \zeta^+ = -1,\quad \bar\zeta^- = 1,\quad \bar\zeta^+ = -\bar z$$

Supersymmetric hemispheres with B-type bnry condition:

$$\sigma = -\log(1+z\bar{z}) + \text{constant} \;, \qquad a = 0$$

$$(+):\quad w = \bar{w} = -\frac{2i}{1+z\bar{z}} \;, \qquad (-):\quad w = \bar{w} = \frac{2i}{1+z\bar{z}}$$

which implies

$$Z_+(D^2,\Omega) = \mathcal{Z}_0\, c^{\Omega}(\lambda)\,, \qquad Z_-(D^2,\Omega) = \mathcal{Z}_0\, c^{\Omega}(\bar{\lambda})\,.$$

qed

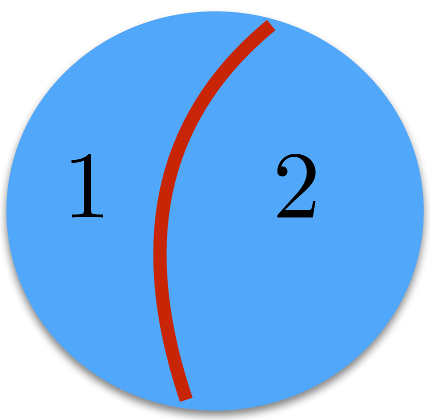
5. Summary + outlook

- Computed the super-Weyl anomaly for $\mathcal{N} = (2,2)$ models on a surface with boundary generalizing the result of

  Gomis, Komargodski, Hsin, Schwimmer, Seiberg, Theisen (1509.08511)

- Not only the **Kähler potential** but also the brane **charge & mass** are given by an (`Osborn-type') anomaly. They can be computed by localization of the hemisphere partition function

- Argument easily extended to sphere partition function with moduli-changing interface



$$C^{\mathcal{I}} = e^{-K(\lambda_1,\bar{\lambda}_2)}$$

$$2\log g^{\mathcal{I}} = K(\lambda_1,\bar{\lambda}_1) + K(\lambda_2,\bar{\lambda}_2) - K(\lambda_1,\bar{\lambda}_2) - K(\lambda_2,\bar{\lambda}_1)$$

Calabi's diastasis function

CB, Brunner, Douglas, Rastelli (1311.2202)

- Extension to higher dimensions and other co-dimension defects

[in progress with Daniel]

Many thanks to the organizers

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and all the best to Chris !