Massive 3D Gravity:

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Duff meeting
April '09

- Topological (Deser, Jackiw, Templeton '82)
- New
- General

Bergshoeff, Hohm & PKT '09
arXiv: 0901.1766
+ in progress

$$
\begin{tikzcd}
 & \text{General } (m,\mu) \arrow[dl, "m \to \infty"'] \arrow[dr, "\mu \to \infty"] & \\
\text{Topological } (\mu) & & \text{New } (m)
\end{tikzcd}
$$

[Also: Cosmological GMG (m, μ, λ)]

NMG

$$S = \frac{1}{\kappa^2}\int d^3x \sqrt{g}\left[R + \frac{1}{m^2}K\right]$$

($+--$ signature)

"wrong-sign" EH term!
(like TMG)

$$\boxed{K = R^{\mu\nu}R_{\mu\nu} - \frac{3}{8}R^2}$$

'Magic' property:

Define $$K_{\mu\nu} = \frac{2}{\sqrt{g}}\frac{\delta}{\delta g^{\mu\nu}}\int d^3x \sqrt{g}\,K$$

$$\boxed{g^{\mu\nu}K_{\mu\nu} = K}$$ $\Rightarrow$ no ghosts!

Auxiliary field form

$$S = \frac{1}{\kappa^2}\int d^3x\,\sqrt{g}\left\{ R + f^{\mu\nu}G_{\mu\nu}(g) - \frac{m^2}{4}\left(f^{\mu\nu}f_{\mu\nu} - f^2\right)\right\}$$

auxiliary symmetric tensor

Einstein tensor

$g^{\mu\nu} f_{\mu\nu}$

$$\frac{\delta S}{\delta f^{\mu\nu}} = 0 \quad \Rightarrow \quad \boxed{f_{\mu\nu} = R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R}$$

Back-substitution $\longrightarrow$ NMG

Linearize about Minkowski vacuum

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

$$G_{\mu\nu}(g) = \left[\mathcal{G} h\right]_{\mu\nu} + O(h^2)$$

$$\boxed{\mathcal{G}_{\mu\nu}{}^{\rho\sigma} = \frac{1}{2}\varepsilon_\mu{}^{\eta\rho}\,\varepsilon_\nu{}^{\tau\sigma}\,\partial_\eta\,\partial_\tau}$$

"Einstein Operator"

Linearized NMG $\equiv$ Fierz-Pauli

Define $\tilde{h}_{\mu\nu} = h_{\mu\nu} - f_{\mu\nu}$

$$S_{quad.} = \int d^3x \left\{ \mathcal{L}_2(\tilde{h}) + \mathcal{L}_2(f) \right\}$$

$$\mathcal{L}_2(\tilde{h}) = -\frac{1}{2} \tilde{h}^{\mu\nu} [\mathcal{G} h]_{\mu\nu}$$

← 'wrong-sign' linearized EH term

but propagates no degrees of freedom

$$\mathcal{L}_2(f) = \frac{1}{2} f^{\mu\nu} [\mathcal{G} f]_{\mu\nu} - \frac{1}{4} m^2 \left(f^{\mu\nu} f_{\mu\nu} - f^2 \right)$$

'right-sign' linearized EH term.

Fierz-Pauli mass term.

$\eta^{\mu\nu} f_{\mu\nu}$

$\therefore$ Equivalent to 3D FP theory

Propagates two massive modes of helicities ± 2

Super NMG

Supersymmetrize for $\mathcal{N} = 1, 2$ susy with off-shell tensor calculus (Uematsu Nishino & Rajpoot)

$\mathcal{N} = 1$: get unitary higher-derivative (cubic) eq. for massive spin $3/2$

$\mathcal{N} = 2$: 'Auxiliary' vector propagates with Proca eq. for massive spin 1 ($\therefore$ unitary)

Rep. theory for massive $D=3$ similar to massless $D=4$

$\therefore$ Expect maximally super NMG with $\mathcal{N} = 8$?

Need off-shell-susy action

Perhaps $\mathcal{N} = 4$ is maximal ?

Much to be done here !

3D Factorization : Spin 1 (Pilch, PKT & Van Nieuwenhuizen)

Define $\boxed{\mathcal{O}_\mu{}^\nu(m) = \delta_\mu^\nu + \frac{1}{m}\varepsilon_\mu{}^{\lambda\nu}\partial_\lambda}$

Proca eq $\quad \partial_\mu F^{\mu\nu} - m^2 A^\nu = 0 \quad$ equiv. to

$$[\mathcal{O}(-m)\,\mathcal{O}(m)]_\mu{}^\nu A_\nu = 0 \quad \& \quad \partial\cdot A = 0$$

$\mathcal{O}(m)$: propagates helicity $+1$ with mass m

$\mathcal{O}(-m)$: propagates helicity -1 with mass m

General massive spin 1 :

$$\boxed{\mathcal{O}(-m_-)\,\mathcal{O}(m_+)\,A = 0 \quad \& \quad \partial\cdot A = 0}$$

$m_+ \neq m_- \Rightarrow$ Chern-Simons term in action

$m_- \to \infty \longrightarrow$ "self-dual" spin 1

$\underbrace{}$ equivalent to top. massive spin 1 (Deser & Jackiw)

3D factorization : Spin 2

Equivalent form of linearized NMG eqns :

$$\boxed{\left[\mathcal{O}(-m)\,\mathcal{O}(m)\right]_{\mu}{}^{\rho}\, G^{\text{lin.}}_{\rho\nu} = 0 \quad \& \quad R^{\text{lin}} = 0}$$

propagates helicity −2 with mass m

propagates helicity +2 with mass m

linearized Einstein tensor.

N.B. Symmetry on $\mu\nu$ implied by linearized Bianchi identity & $R^{\text{lin.}} = 0$

Generalize to linearized GMG eqns :

$$\boxed{\mathcal{O}(-m_-)\,\mathcal{O}(m_+)\, G^{\text{lin}} = 0 \quad \& \quad R^{\text{lin}} = 0}$$

$m_- \to \infty$ gives eqs of linearized TMG

GMG

Non-linear field eqs. of GMG are

$$\boxed{G_{\mu\nu} + \frac{1}{\mu} C_{\mu\nu} + \frac{1}{2m^2} K_{\mu\nu} = 0}$$

Cotton tensor

$m = \sqrt{m_+ m_-}$

$\mu = \dfrac{m_+ m_-}{m_- - m_+}$

Look for max. sym. vacua : $G_{\mu\nu} = \Lambda g_{\mu\nu}$

$$\Rightarrow \quad C_{\mu\nu} = 0 \quad \& \quad K_{\mu\nu} = -\frac{1}{2}\Lambda^2 g_{\mu\nu}$$

$$\therefore \quad \underline{\Lambda(4m^2 - \Lambda) = 0}$$

Either $\Lambda = 0$ Mink. vacuum

Or $\Lambda = 4m^2$ dS vacuum

N.B. If we flip sign of EH term then

(i) Mink. vac. has ghosts

(ii) dS $\longrightarrow$ adS $\longleftarrow$ stable!

$\therefore$ Both signs possible

CGMG

Add a cosmological constant to get

$$\boxed{\lambda m^2 g_{\mu\nu} + \sigma G_{\mu\nu} + \frac{1}{\mu} C_{\mu\nu} + \frac{1}{2m^2} K_{\mu\nu} = 0}$$

λ: dimensionless parameter

$$\sigma = \begin{cases} 1 & \text{'right-sign'} \\ -1 & \text{'wrong-sign'} \end{cases}$$

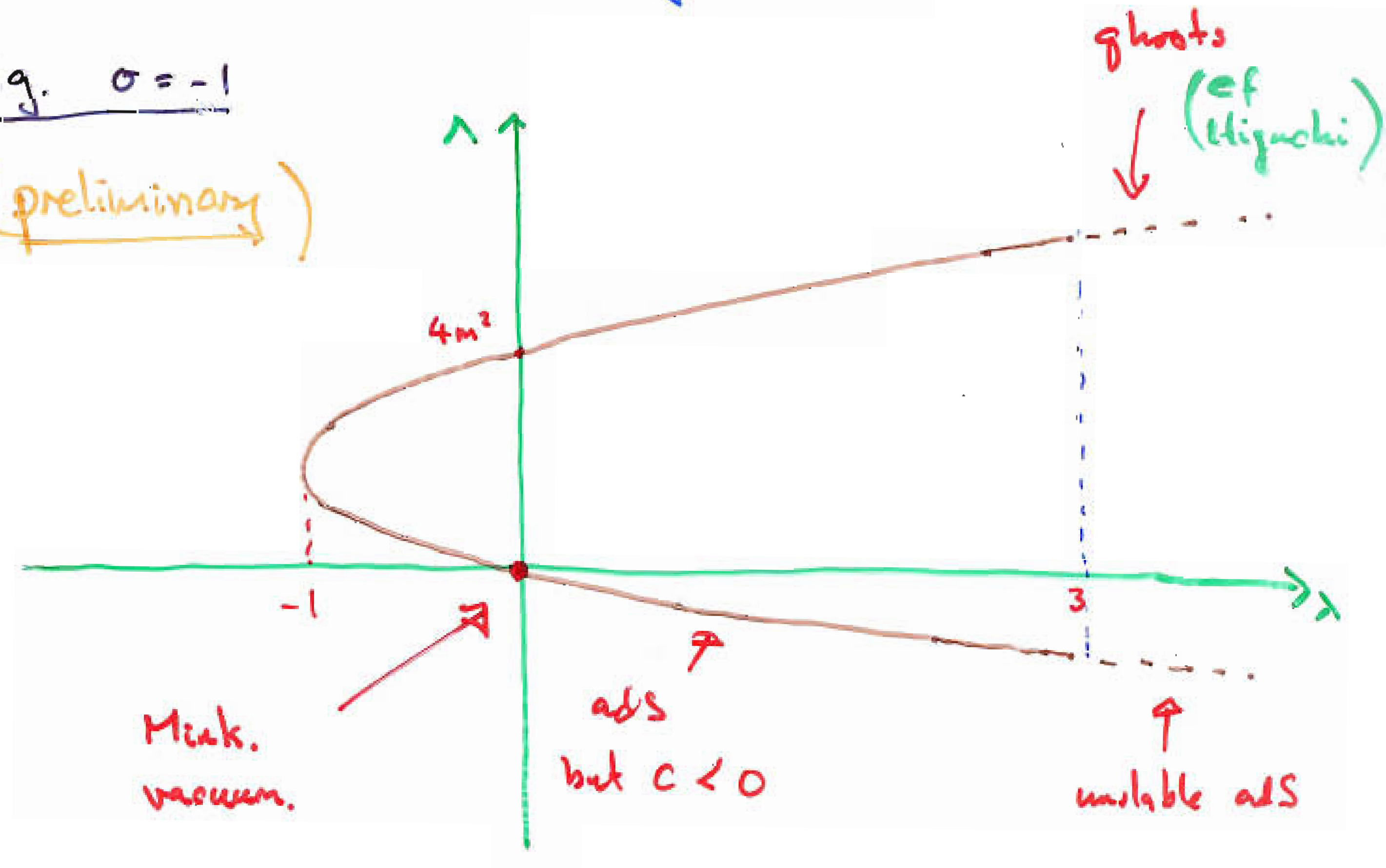
Max. sym. vacua : $G_{\mu\nu} = \Lambda g_{\mu\nu}$

$$\Longrightarrow \quad \boxed{\Lambda = 2m^2\left[\sigma \pm \sqrt{1+\lambda}\,\right]}$$

$\therefore$ $\lambda \geqslant -1$ or no max. sym. vacuum.

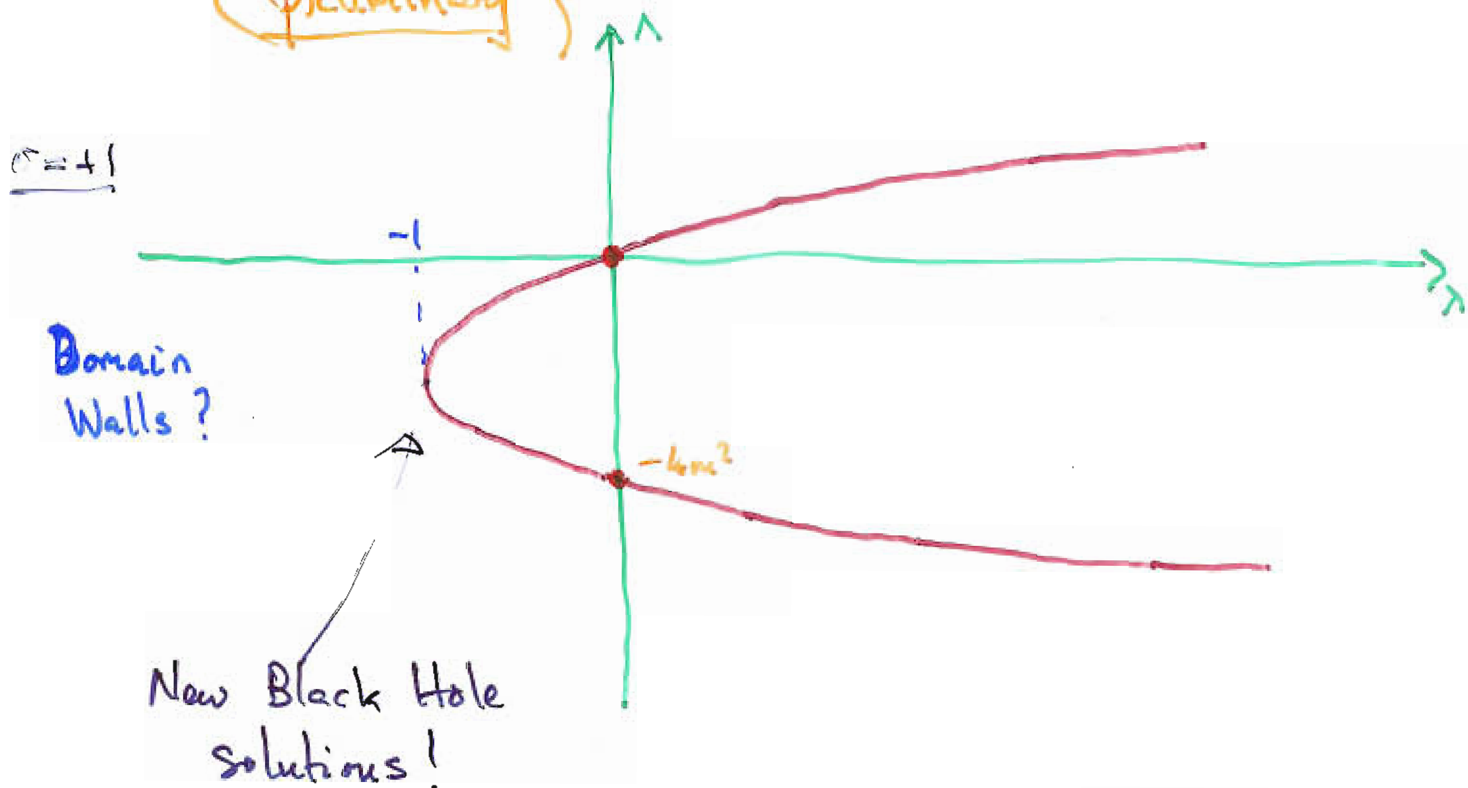
e.g. $\sigma = -1$

(preliminary)



adS/CFT, Black Holes, ...

(preliminary)



$$\boxed{c_{\pm} = \frac{3\ell}{2G_3}\left(1 + \frac{1}{2\ell^2 m^2} \pm \frac{1}{\mu\ell}\right)}$$

(cf. Clement, Liu & Sun)

$\therefore$ Chiral point at $\mu\ell = 1 + \frac{1}{2\ell^2 m^2}$.

- Can CGMG help solve problem of quantum gravity in $D=3$?
- Connection to string/M-theory ?