Happy 60'th Birthday, Michael Duff

Martin Roček

April 1, 2009

Historical comment: When I first heard of Mike, he was famous for doing hard calculations and getting them right; for example, in collaboration with Steve Christensen, he was the first to correctly calculate the chiral anomaly of the spin $\frac{3}{2}$ field.

So I was very happy as a young postdoc to get the chance to write a paper with Mike:

S. M. Christensen, M. J. Duff, G. W. Gibbons and M. Rocek,

“Vanishing One Loop Beta Function In Gauged $N > 4$ Supergravity,”

Phys. Rev. Lett. **45** (1980) 161.

Vanishing One-Loop β Function in Gauged $N > 4$ Supergravity

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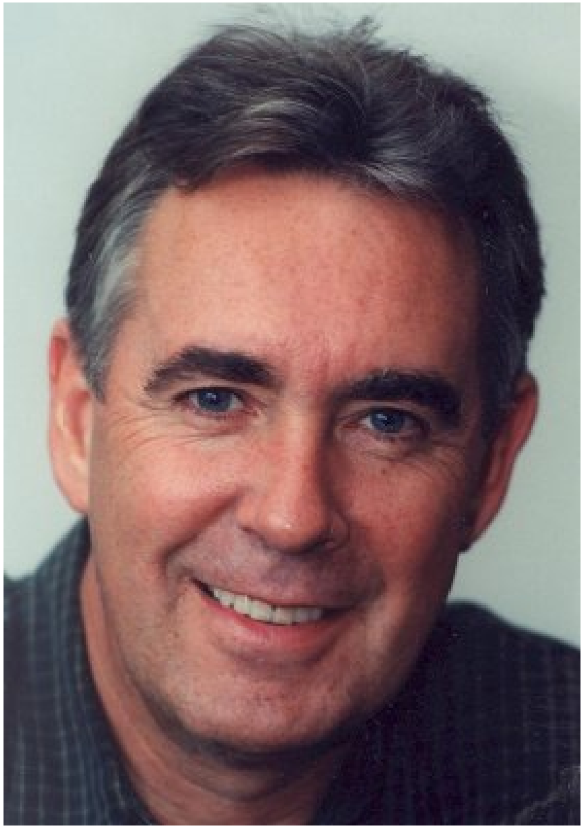
In O(N) extended supergravity theories, the usual arguments for one-loop finiteness (modulo topological terms) cease to apply when the internal symmetry is gauged because
of the appearance of a cosmological constant related to the gauge coupling e. For $N \leq 4$, 3
we find that infinite renormalizations are required. Remarkably, the particle content of theories with $N > 4$ results in a cancellation of these infinities, implying, in particular a vanishing one-loop $\beta(e)$ function.

PACS numbers: 11.10 Np, 04.60+n, 11.10 Gh, 11.30.Pb

While I'm reminiscing, let me just comment that we were all younger, but few of us have aged as gracefully as Mike:



By the way, the other guy is only 47.

The basic observation in the paper is simple: as is well known, pure one-loop gravity is on-shell $\boldsymbol{finite}$ because $\boldsymbol{R = R_{\mu\nu} = 0}$ reduces divergences to a total derivative (Gauss-Bonnet). Finiteness holds for supergravity as well.

But in the presence of a cosmological term, $\boldsymbol{R \propto \Lambda}$, so pure gravity with $\boldsymbol{\Lambda}$ is one-loop $\boldsymbol{renormalizable}$.

The analogous statement is true for gauged supergravity; however, $\boldsymbol{\Lambda}$ is proportional to $\boldsymbol{g^2}$, the gauge coupling, so its renormalization gives a running gauge coupling constant.

We calculated this for $\boldsymbol{N \geq 1}$ supergravity, and found nontrivial divergences and hence running of $\boldsymbol{\Lambda}$ for $\boldsymbol{N \leq 4}$, whereas for $\boldsymbol{N > 4}$, $\boldsymbol{\Lambda}$ and hence $\boldsymbol{g^2}$ is $\boldsymbol{not}$ renormalized.

It would be interesting to reconsider this calculation in a well defined perturbative theory of quantum supergravity: String Theory compactified on a suitable background preserving different amounts of supersymmetry, such as $\boldsymbol{AdS(4) \times CP(3)}$.


COMPLETELY
DIFFERENT

New Gauge Multiplets in D=2

with

Ulf Lindström, Itai Ryb,

Rikard von Unge, and Maxim Zabzine

arxiv.org/0705.3201,arxiv.org/0808.1535,

more to come!

Recall $D = 4$ SYM: Chiral (antichiral) super-fields transform with chiral (antichiral) gauge parameters, and the gauge superfield $\boldsymbol{V}$

transforms with both:

$$\boldsymbol{\Phi' = e^{i\Lambda}\Phi \ , \quad \bar{\Phi}' = \bar{\Phi} e^{-i\bar{\Lambda}} \ ,}$$

$$\boldsymbol{e^{V'} = e^{i\bar{\Lambda}} e^{V} e^{-i\Lambda} \ .}$$

The super-lagrangian

$$\boldsymbol{\bar{\Phi} e^{V} \Phi}$$

is gauge invariant, and the gauge covariant derivatives are constructed out of $\boldsymbol{V}$.

In $D = 2$, there are more kinds of multiplets:

$$\boldsymbol{Chiral: \ \bar{D}_{\pm}\Phi = 0 \ ,}$$

$$\boldsymbol{Twisted\ Chiral: \ \bar{D}_{+}\chi = D_{-}\chi = 0 \ ,}$$

$$\boldsymbol{Semichiral: \ \bar{D}_{+}X_{L} = \bar{D}_{-}X_{R} = 0 \ ,}$$

and their respective complex conjugates.

They naturally transform with gauge parameters

$$\mathbf{\Lambda}, \tilde{\mathbf{\Lambda}}, \mathbf{\Lambda}_{\boldsymbol{L}}, \mathbf{\Lambda}_{\boldsymbol{R}}$$

that obey the same chirality constraints, respectively. It is natural to introduce gauge superfields analogous to $\boldsymbol{V}$ that transform with pairs of these gauge parameters. For example, a symmetry that acts only on chiral superfields $\mathbf{\Phi}$ and their complex conjugates can be gauged by the single Hermitian potential $\boldsymbol{V}$ described above (for clarity, in $D = 2$, we refer to such a potential as $\boldsymbol{V}^{\phi}$).

Similarly, a symmetry that acts only on twisted chiral superfields χ can be gauged by a single Hermitian potential V^χ which transforms as

$$e^{V^{\chi\prime}} = e^{i\bar{\tilde{\Lambda}}} e^{V^\chi} e^{-i\tilde{\Lambda}} \ .$$

Things get more interesting when we consider symmetries that act on **both** Φ and χ; then naively we would need to introduce six potentials, but we can impose several gauge invariant restrictions, leading to only three independent potentials:

$$e^{V^{\phi}} = e^{i\bar{\Lambda}} e^{V^{\phi}} e^{-i\Lambda} \;, \quad e^{V^{\chi'}} = e^{i\bar{\tilde{\Lambda}}} e^{V^{\chi}} e^{-i\tilde{\Lambda}} \;,$$

are real, whereas

$$e^{i\bar{V}'_R} = e^{i\bar{\Lambda}} e^{i\bar{V}_R} e^{-i\tilde{\Lambda}} \;, \quad e^{-iV'_L} = e^{i\tilde{\Lambda}} e^{-iV_L} e^{-i\Lambda}$$

are complex.

These six potentials obey four gauge-covariant constraints:

$$e^{V^\chi} e^{-iV_L} = e^{-iV_R} \ , \quad e^{i\bar{V}_L} e^{V^\chi} = e^{i\bar{V}_R} \ ,$$

$$e^{V^\phi} e^{iV_L} = e^{i\bar{V}_R} \ , \quad e^{-i\bar{V}_L} e^{V^\phi} = e^{-iV_R} \ ,$$

which however are not independent–the constraints obey one reality condition, leaving real three independent potentials. This multiplet contains many component fields, and is called the **Large Vector Multiplet**.

Precisely the same structure arises for isometries that act on the complex semichiral superfield $\boldsymbol{X_L, X_R, \bar{X}_L, \bar{X}_R}$; here the gauge parameters are less restricted, and there are fewer component fields; because of the nature of the gauge parameters, the multiplet is called the **Semichiral Vector Multiplet**. (This multiplet was also discussed by Gates, Merrell, and Vaman for the Abelian case).

In our conventions, we have:

$$e^{i\mathbb{V}'} = e^{i\Lambda_L} e^{i\mathbb{V}} e^{-i\Lambda_R} \ , \quad e^{i\tilde{\mathbb{V}}'} = e^{i\Lambda_L} e^{i\tilde{\mathbb{V}}} e^{-i\bar{\Lambda}_R}$$

$$e^{\mathbb{V}^{L\prime}} = e^{i\bar{\Lambda}_L} e^{\mathbb{V}^L} e^{-i\Lambda_L} \ , \quad e^{\mathbb{V}^{R\prime}} = e^{i\bar{\Lambda}_R} e^{\mathbb{V}^R} e^{-i\Lambda_R} \ .$$

where $\mathbb{V}^{L,R}$ are Hermitian and $\mathbb{V}, \tilde{\mathbb{V}}$ are complex; the constraints these obey are

$$e^{i\tilde{\mathbb{V}}} e^{\mathbb{V}^R} = e^{i\mathbb{V}} \ , \quad e^{i\bar{\mathbb{V}}} e^{\mathbb{V}^R} = e^{i\bar{\tilde{\mathbb{V}}}} \ ,$$

$$e^{\mathbb{V}^L} e^{i\tilde{\mathbb{V}}} = e^{i\bar{\mathbb{V}}} \ , \quad e^{\mathbb{V}^L} e^{i\mathbb{V}} = e^{i\bar{\tilde{\mathbb{V}}}} \ ,$$

Because the gauge parameters are less constrained for Semichiral Vector Multiplet, it has fewer physical components and a simpler geometry. Just as in $D = 4$, there are different representations of the gauge group, such as chiral and antichiral rep, here we have four represenations: Left semichiral, right semichiral, left antisemichiral, right antisemichiral .

Let's consider the derivatives in left-chiral rep. Then the parameter $\mathbf{\Lambda}_L$ obeys $\bar{D}_+\mathbf{\Lambda}_L = 0$, and the spinor covariant derivatives are uniquely determined to be:

$$\bar{\boldsymbol{\nabla}}_+ = \bar{\boldsymbol{D}}_+$$

$$\bar{\boldsymbol{\nabla}}_- = e^{i\mathbb{V}}\bar{\boldsymbol{D}}_- e^{-i\mathbb{V}}$$

$$\boldsymbol{\nabla}_+ = e^{-\mathbb{V}^L}\boldsymbol{D}_+ e^{\mathbb{V}^L}$$

$$\boldsymbol{\nabla}_- = e^{i\tilde{\mathbb{V}}}\boldsymbol{D}_- e^{-i\tilde{\mathbb{V}}}$$

Other representations are reached by similarity transformations by the appropriate potential, and vector derivatives and field-strengths can be computed in the usual manner; one finds a chiral and a twisted chiral field-strength:

$$\boldsymbol{F} = i\{\bar{\boldsymbol{\nabla}}_+ , \bar{\boldsymbol{\nabla}}_-\}$$

$$\tilde{\boldsymbol{F}} = i\{\bar{\boldsymbol{\nabla}}_+ , \boldsymbol{\nabla}_-\} \ .$$

The Large Vector Multiplet is more surprising and complicated. Once more we can construct derivatives appropriate to different representations; however, because the gauge parameters are more strongly constrained, there are *two* sets of covariant derivatives, e.g., in chiral rep,

$$\bar{\nabla}_+ = \bar{D}_+ \quad \textbf{and} \quad \hat{\bar{\nabla}}_+ = e^{iV_L} \bar{D}_+ e^{-iV_L}$$

are *both* covariant (recall $e^{iV'_L} = e^{i\Lambda} e^{iV_L} e^{-i\tilde{\Lambda}}$)

This means that there are new dimension 1/2 spinor field strengths in the theory, *e.g.*,

$$\boldsymbol{G_+ = i(\hat{\bar{\bar{\nabla}}} - \bar{\nabla}_+)} \ .$$

We are just beginning to understand the applications of these new multiplets; in particular, they help us understand aspects of generalized Kähler geometry, another subject that I find fascinating. Thank you for giving me opportunity to tell you about them, and thank you Mike for providing this happy occasion for all of us to meet.

HAPPY BIRTHDAY!