Some properties of superconformal M2 branes

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Duff fest

Imperial College, London

April 1-2, 2009

Talk based on:

- *“Three-dimensional N=8 superconformal gravity and its coupling to BLG M2 branes”*
  Ulf Gran and Bengt E.W. Nilsson,
  `arXiv:0809.4478 [hep-th] (v.3), in JHEP`
- *"Three dimensional topologically gauged N=6 superconformal ABJM type theories"*
  Xiaoyong Chu and Bengt E.W. Nilsson, in prep

More references

see also

- *"On relating M2 and D2 branes*,
  Ulf Gran, Bengt E.W. Nilsson and Christoffer Petersson
  `arXiv:0804.1784[hep-th]`, in JHEP
- *“Superconformal M2-branes and generalized Jordan triple systems”*
  Bengt E.W. Nilsson and Jakob Palmkvist,
  `arXiv:0807.5134 [hep-th], in CQG`
- *"Light-cone analysis of ungauged and topologically gauged BLG*
  Bengt E.W. Nilsson,
  `arXiv:0811.3388 [hep-th]`

Introduction: Motivation

Motivation:

3-dim superconformal field theories are of interest in

- M-theory: M2-branes, AdS_4/CFT_3, 3d mirror symmetry, microscopic degrees of freedom, etc
- condensed matter: phase transitions, superconductivity, etc
- mathematics: integrability, 'vertex operators', etc

Symmetry properties are important both for applications and for our understanding of these theories.

Consider the $\mathcal{N} = 8$ BLG and $\mathcal{N} = 6$ ABJM superconformal matter systems

Question: Can we find new symmetries or algebraic structures in these theories?

Content: M2 branes with 8 or 6 supersymmetries

We will

- review the $\mathcal{N} = 8$ superconformal theory ($N = 2,\ k = 1, 2$): BLG [Bagger, Lambert], [Gustavsson]
  - discuss the gauging of its global symmetries [Gran,N]
  - and how to analyse it in the light-cone gauge [N]

- review the more recent $\mathcal{N} = 6$ version (any N, k): ABJM [Aharony, Bergman, Jafferis, Maldacena] [Benna,Klebanov,Klose,Smedbäck]
  - discuss the topological gauging of these theories [Chu, N], in prep
  - and introduce an infinite dimensional symmetry structure related to generalized Jordan triple systems [N,Palmkvist]

3-dim $\mathcal{N} = 8$ superconformal field theory : field content

Field content of BLG: (M2 branes in 11d)

- scalars X^i_a
- spinors ψ_a
- vector gauge potential $\tilde{A}_\mu{}^a{}_b = A_{\mu cd}\, f^{cda}{}_b$
  - i: $SO(8)$ R-symmetry vector index,
  - ψ_a has a hidden R-symmetry spinor index,
  - a: three-algebra index related to $[T^a, T^b, T^c] = f^{abc}{}_d T^d$
  - conformal dimensions (deduced from their kinetic terms):
    - -1/2 for X^i_a
    - -1 for ψ_a
    - -1 for $\tilde{A}_\mu$ ("kinetic term" = Chern-Simons term) [Schwarz]

3-dim $\mathcal{N} = 8$ superconformal field theory: Lagrangian

The BLG Lagrangian is

$$\begin{aligned}\mathcal{L} \;=\; & -\tfrac{1}{2}(D_\mu X^{ia})(D^\mu X^i{}_a) + \tfrac{i}{2}\bar{\Psi}^a\gamma^\mu D_\mu \Psi_a - \tfrac{i}{4}\bar{\Psi}_b\Gamma_{ij}X^i{}_c X^j{}_d \Psi_a f^{abcd} \\ & -V + \tfrac{1}{2}\varepsilon^{\mu\nu\lambda}\left(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \tfrac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}\right) ,\end{aligned}$$

where $D_\mu = \partial_\mu + \tilde{A}_\mu$ and the potential is

$$V = \tfrac{1}{12}(X^i{}_a X^j{}_b X^k{}_c f^{abcd})(X^i{}_e X^j{}_f X^k{}_g f^{efg}{}_d) .$$

- this assumes a Euclidean metric on the three-algebra => $f^{[abcd]}$
- the gauge theory is Chern-Simons with a split gauge group
- can have (quantized) non-trivial level k on orbifolds (but $k > 2$ unclear): Large k = weak coupling [Lambert,Tong][Distler,Mukhi,Papageorgakis,Van Raamsdonk]
- no other free parameters!

BLG transformation rules

The BLG transformation rules for (global) $\mathcal{N}=8$ supersymmetry are

$$\begin{aligned} \delta X_i^a &= i\epsilon\Gamma_i\Psi^a, \\ \delta\Psi_a &= \tilde{D}_\mu X_a^i\gamma^\mu\Gamma^i\epsilon + \tfrac{1}{6}X_b^i X_c^j X_d^k\Gamma^{ijk}\epsilon f^{bcd}{}_a. \end{aligned}$$

Imposing supersymmetry on the $(Cov.der.)^2$ terms in $\delta\mathcal{L}$ implies

$$\delta\tilde{A}_\mu{}^a{}_b = i\bar{\epsilon}\gamma_\mu\Gamma^i X_c^i\psi_d f^{cda}{}_b$$

and the fundamental identity

$$f^{abc}{}_g f^{efg}{}_d = 3f^{ef[a}{}_g f^{bc]g}{}_d\,,$$

with alternative but equivalent form [Gran, N, Petersson]

$$f^{[abc}{}_g f^{e]fg}{}_d = 0\,.$$

- one finite dim. realization, $\mathcal{A}_4$, with split $SO(4)$ gauge symmetry (with levels $(k,-k)$) [Papadopoulos][Gauntlett,Gutowski]
- ∞ dim'al case: Nambu bracket, $\mathcal{S}Diff(M_3)$ (volume preserving) [Bandos,Townsend], and Witt algebra [Curtright, Fairlie, Zachos]

BLG: more properties

- parity: interchanges the two gauge fields for $SO(4) = SU(2) \times SU(2)$ with levels $(k, -k)$
- BLG describes two M2 branes; attempts to find a similar theory for $\mathcal{N} > 2$ M2 branes have used
  - degenerate metrics with no Lagrangian (any Lie group possible), and not totally antisymmetric structure constants
    [Gran, N,Petersson]
    if the scalar in the degenerate direction is frozen it can be obtained from a Lagrangian
  - Lagrangian exists also if a Lorentzian metric is used but may be just D2 branes in disguise [Sen, Verlinde, Schwarz, ...]
- The field equations for the Chern-Simons gauge field is

$$\tilde{F}_{\mu\nu}{}^{b}{}_{a} + \epsilon_{\mu\nu\rho}(X^i_c \partial^\rho X^i_d + \tfrac{i}{2}\bar{\Psi}_c \gamma^\rho \Psi_d) f^{cdb}{}_{a} = 0$$

i.e. not dynamical. In the light-cone gauge one can even solve for the gauge field!

3-dim $\mathcal{N} = 8$ superconformal gravity

Can the global symmetries of the BLG theory be gauged?
If yes, can it be done without adding new degrees of freedom?

- Off-shell field content of 3-dim. $\mathcal{N} = 8$ conformal supergravity :

  $$e_\mu{}^\alpha \ [0],\ \chi^i_\mu \ [-1/2],\ B^{ij}_\mu \ [-1],\ b_{ijkl} \ [-1],\ \rho_{ijk} \ [-3/2],\ c_{ijkl} \ [-2],$$

  ([scaling dimension])[Howe,Izquierdo,Papadopoulos,Townsend]
- On-shell the Lagrangian = three Chern-Simons-like terms ([Gran,N]) (compare $\mathcal{N} = 1$ [Deser,Kay(1983)], [van Nieuwenhuizen], and for any $\mathcal{N}$ [Lindström,Roček])

$$\mathcal{L} = \tfrac{1}{2}\epsilon^{\mu\nu\rho} Tr_\alpha(\tilde{\omega}_\mu \partial_\nu \tilde{\omega}_\rho + \tfrac{2}{3}\tilde{\omega}_\mu\tilde{\omega}_\nu\tilde{\omega}_\rho)$$

$$-ie^{-1}\epsilon^{\alpha\mu\nu}(\tilde{D}_\mu \bar{\chi}_\nu \gamma_\beta\gamma_\alpha \tilde{D}_\rho \chi_\sigma)\epsilon^{\beta\rho\sigma} - \epsilon^{\mu\nu\rho} Tr_i(B_\mu \partial_\nu B_\rho + \frac{2}{3} B_\mu B_\nu B_\rho),$$

- supercovariant spin connection: $\tilde{\omega}_{\mu\alpha\beta}(e_\mu{}^\alpha, \chi^i_\mu)$
- CS terms are of 3rd, 2nd and 1st order in derivatives, respectively

Symmetries of 3-dim $\mathcal{N} = 8$ superconformal gravity

- 3-dim diff's and local $SO(8)$ R-symmetry
- local $\mathcal{N} = 8$ supersymmetry (f^ν is the spin 3/2 field strength)

$$\delta e_\mu{}^\alpha = i\bar{\epsilon}(x)\gamma^\alpha\chi_\mu, \quad \delta\chi_\mu = \tilde{D}_\mu\epsilon(x),$$
$$\delta B_\mu^{ij} = -\tfrac{i}{2}\bar{\epsilon}(x)\Gamma^{ij}\gamma_\nu\gamma_\mu f^\nu,$$

- local scale invariance

$$\delta_\Delta e_\mu{}^\alpha = -\phi(x)e_\mu{}^\alpha, \quad \delta_\Delta\chi_\mu = -\tfrac{1}{2}\phi(x)\chi_\mu, \tag{1}$$
$$\delta_\Delta B_\mu^{ij} = 0,$$

- and local $\mathcal{N} = 8$ superconformal symmetry

$$\delta_S e_\mu{}^\alpha = 0, \quad \delta_S\chi_\mu = \gamma_\mu\eta(x),$$
$$\delta_S B_\mu^{ij} = \tfrac{i}{2}\bar{\eta}(x)\Gamma^{ij}\chi_\mu.$$

Symmetries of 3-dim $\mathcal{N}=8$ superconformal gravity: Fierz

A proof a la Deser-Kay requires some nice *Fierzing*! [Gran, N]

Typical expressions that arise multiplying each other are

- the supercovariant dual spin connection

$$\delta\tilde{\omega}_{\mu}^{*\alpha} = -2i(\bar{\epsilon}\gamma_{\mu}f^{\alpha} - \tfrac{1}{2}e_{\mu}{}^{\alpha}\bar{\epsilon}\gamma_{\nu}f^{\nu})$$

- and the triple dual of the Riemann tensor

$$\tilde{R}_{\mu}^{***} = i\bar{\chi}_{\nu}\gamma_{\mu}f^{\nu}$$

- giving the Fierz basis (Gran's GAMMA is useful)

$$\begin{array}{ll}
(-) & (\bar{\epsilon}\chi_{\mu})(\bar{f}_{\nu}f_{\rho})\epsilon^{\mu\nu\rho} = 0, \\
(-) & (\bar{\epsilon}\chi_{\alpha})(\bar{f}^{\beta}\gamma^{\alpha}f_{\beta}) = 0, \\
(1) & (\bar{\epsilon}\chi_{\alpha})(\bar{f}^{\alpha}\gamma^{\beta}f_{\beta}), \\
(2) & (\bar{\epsilon}\gamma^{\alpha}\chi_{\alpha})(\bar{f}^{\beta}f_{\beta}), \\
(3) & \ldots.
\end{array}$$

Topologically gauged BLG theory

- This supergravity theory has no propagating degrees of freedom!
  - clear in the light-cone gauge: all non-zero field components (plus ∂_+ on them) can be solved for [N] => "topologically gauged BLG"
- couplings have been checked to order $(Cov.der.)^3$ and $(Cov.der.)^2$ [Gran,N]
- parity: a problem
- a new level k ([Horne,Witten])
- coordinate dependent parameters also on $\mathbf{R} \times S^2$ [Ali-Akbari]

Many questions concerning interpretation:

- AdS/CFT (compare [Liu,Tseytlin] for AdS_5/CFT_4)
- curved M2's, topologically twisted BLG [Lee,Lee,Park], quantization (compare [Polyakov],[Brink,Di Vecchia,Howe])

Topologically gauged BLG theory: details

The Lagrangian: (up to a number of scalar and fermionic interaction terms; see gauged ABJM later)

$$L^{top}_{BLG} \quad = \quad L^{conf}_{grav} + L^{cov}_{BLG}$$

$$+\tfrac{1}{\sqrt{2}} ie\bar{\chi}_\mu \Gamma^i \gamma^\nu \gamma^\mu \Psi^a (\tilde{D}_\nu X^{ia} - \tfrac{i}{2\sqrt{2}} \bar{\chi}_\nu \Gamma^i \Psi^a)$$

$$+\tfrac{1}{6\sqrt{2}} ie\bar{\chi}_\mu \gamma^\mu \Gamma^{ijk} \Psi_a (X^i_b X^j_c X^k_d) f^{abcd}$$

$$-\tfrac{i}{4}\epsilon^{\mu\nu\rho}\bar{\chi}_\mu \Gamma^{ij} \chi_\nu (X^i_a \tilde{D}_\rho X^j_a) + \tfrac{i}{\sqrt{2}} \bar{f}^\mu \Gamma^i \gamma_\mu \Psi_a X^i_a$$

$$-\tfrac{e}{16} X^2 \tilde{R} + \tfrac{i}{16} X^2 \bar{f}^\mu \chi_\mu$$

Topologically gauged BLG theory: details

The transformation rules are

$$\begin{aligned}
\delta e_\mu{}^\alpha &= i\sqrt{2}\bar{\epsilon}\gamma^\alpha\chi_\mu\,, \\
\delta\chi_\mu &= \sqrt{2}\tilde{D}_\mu\epsilon, \\
\delta B^{ij}_\mu &= -\tfrac{i}{\sqrt{2}}\bar{\epsilon}\Gamma^{ij}\gamma_\nu\gamma_\mu f^\nu - \tfrac{i}{2}\bar{\Psi}_a\gamma_\mu\Gamma^{[i}\epsilon X^{j]}_a \\
&\quad -\tfrac{i}{2\sqrt{2}}\bar{\chi}_\mu\Gamma^{k[i}\epsilon X^{j]}_a X^k_a - \tfrac{i}{16}\bar{\Psi}_a\Gamma^k\Gamma^{ij}\gamma_\mu\epsilon X^k_a + \tfrac{i}{16\sqrt{2}}\bar{\epsilon}\Gamma^{ij}\chi_\mu X^2, \\
\delta X^a_i &= i\epsilon\Gamma_i\Psi^a, \\
\delta\Psi_a &= (\tilde{D}_\mu X^i_a - \tfrac{1}{\sqrt{2}}\bar{\chi}_\mu\Gamma^i\Psi_a)\gamma^\mu\Gamma^i\epsilon + \tfrac{1}{6}X^i_b X^j_c X^k_d\Gamma^{ijk}\epsilon f^{bcd}{}_a, \\
\delta\tilde{A}_\mu{}^a{}_b &= i\bar{\epsilon}\gamma_\mu\Gamma^i X^i_c\Psi_d f^{cda}{}_b - \tfrac{i}{\sqrt{2}}\bar{\chi}_\mu\Gamma^{ij}\epsilon X^i_c X^j_d f^{cda}{}_b.
\end{aligned}$$

BLG on the light-cone: components

$\mathcal{N}=4$ SYM in 3d can be proven perturbatively finite using the light-cone gauge [Brink,N,Lindgren],[Mandelstam]
Can this be done for BLG/ABJM theories?

- The light-cone BLG [N]: bosonic sector with $A_-=0$

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2}X^I_A(-2\partial_-\partial_+ + \partial^2)X^I_A - X^I_A\tilde{A}^{AB}_+\partial_-X^I_B + X^I_A\tilde{A}^{AB}\partial X^I_B\\ &+ \tfrac{1}{2}X^I_A(\tilde{A}^2)^{AB}X^I_B - A^{AB}_+\partial_-\tilde{A}_{AB} - V\end{aligned}$$

- => the other two components of A_μ can be integrated out
- adding the fermions, also half the spinor field can be integrated out by using the matrix ($H=XX\delta$)

$$M_{AB}{}^{CD} = \begin{pmatrix} 0 & \delta^{CD}_{AB} & 0\\ \delta^{CD}_{AB} & H_{AB}{}^{CD} & -i\delta^C_{[A}\Psi^{(-)}_{B]}\\ 0 & i\delta^{[C}_A\Psi^{D](-)} & i\sqrt{2}\delta^C_A\partial_- \end{pmatrix},$$

- which is defined from $\mathcal{L}=\frac{1}{2}\mathcal{A}^\dagger M\mathcal{A}+\mathcal{A}^\dagger\mathcal{J}$ and

$$\mathcal{A} = (\partial_-A^{AB}_+\ \tilde{A}^{AB},\ \Psi^{(+)}_A),$$

BLG on the light-cone: superspace

- Light-cone superfield $\Phi(x^\mu, \theta^m)$ with θ in **4** of $SU(4)$

$$\begin{aligned} d_m d_n \Phi &= \tfrac{1}{2}\epsilon_{mnpq}\bar{d}^p \bar{d}^q \bar{\Phi} \\ \Phi|_0 &= \tfrac{1}{\partial_-}A, \ \ d_m\Phi|_0 = \sqrt{2}\tfrac{1}{\partial_-}\chi_m, \ \ d_m d_n \Phi|_0 = 2\sqrt{2} i C_{mn}, \end{aligned}$$

- Superspace Lagrangian (surpressing the three-algebra indices)

$$\begin{aligned} \mathcal{L}_2 &= A\Box\bar{A} + \tfrac{1}{4}C_{mn}\Box\bar{C}^{mn} - \tfrac{i}{\sqrt{2}}\bar{\chi}^m \tfrac{\Box}{\partial_-}\chi_m \\ &= -2^{-7}\int d^4\theta d^4\bar{\theta}(\Phi\frac{\Box}{\partial_-^2}\bar{\Phi}) \end{aligned}$$

- interaction terms are more complicated (six-point not done yet)
- should work the same way for ABJM but possibly more close to 4d $\mathcal{N} = 4$ SYM.

3-dim $\mathcal{N} = 6$ superconformal field theory: ABJM basics

How to describe stacks with more than two M2's?

- More than two M2-branes seems to require less susy than $\mathcal{N} = 8$

ABJM: $\mathcal{N} = 6$, any k and gauge group $U(N) \times U(N)$ or $SU(N) \times SU(N)$ (and some other possible choices)

- scalar fields now complex Z^A: in bifundamental, A an $SU(4)$ fund. index (after breaking the R-symmetry to $SU(4) \times U(1)$)
- supersymmetry parameters are ϵ^{AB} in a 6 of $SU(4)$
- the ABJM version uses no three-algebra f symbols,
- but f can be reinstated [Bagger,Lambert]: then the f's are only antisymmetric separately in the first and second pair of indices
- $U(2) \times U(2)$ BLG requires 't Hooft (vortex) operators ($k = 1, 2$) [Klebanov,Klose,Murugan],[Borokhov,Kapustin,Wu], also for $N > 2$?

3-dim $\mathcal{N}=6$ superconformal field theory: more structure

It is natural to use [N,Palmkvist]

- $f^{ab}{}_{cd}$
- $(Z^A_a)^* = Z^a_A$, $(\Psi_{Aa})^* = \Psi^{Aa}$

Then

$$\begin{aligned}
\mathcal{L} \;=\;& -\tfrac{1}{2}(D_\mu Z^A_a)(D^\mu \bar{Z}^a_A) + \tfrac{i}{2}\bar{\Psi}^{Aa}\gamma^\mu D_\mu \Psi_{Aa} \\
& -\tfrac{i}{2} f^{ab}{}_{cd}\bar{\Psi}^{Ad}\Psi_{Aa} Z^B_b \bar{Z}^c_B + i f^{ab}{}_{cd}\bar{\Psi}_{Aa}\Psi_{Bb} Z^B_c \bar{Z}^d_A \\
& -\tfrac{i}{4}\epsilon_{ABCD} f^{ab}{}_{cd}\bar{\Psi}^{Ac}\Psi^{Bd} Z^C_a Z^D_b - \tfrac{i}{4}\epsilon^{ABCD} f^{ab}{}_{cd}\bar{\Psi}_{Aa}\Psi_{Bb}\bar{Z}^c_C \bar{Z}^d_B \\
& -V + \tfrac{1}{2}\epsilon^{\mu\nu\lambda}\big(f^{ab}{}_{cd} A_{\mu b}^{d}\partial_\nu A_{\lambda a}^{c} + \tfrac{2}{3} f^{bd}{}_{gc} f^{gf}{}_{ae} A_{\mu b}^{a} A_{\nu d}^{c} A_{\lambda f}^{e}\big)\,,
\end{aligned}$$

and the fundamental identity

$$f^{e[a}{}_{dc} f^{b]d}{}_{gh} = f^{ab}{}_{d[g} f^{ed}{}_{h]c}\,.$$

Topologically gauged $\mathcal{N} = 6$ superconformal ABJM

Since ABJM is much less rigid than BLG it should be easier to gauge

- The complete Lagrangian is easy (but tedious) to derive [Chu, N], in prep
  - it has a number of new scalar interaction terms with one or no structure constant
  - and new Yukawa like terms with no structure constants
- the transformation rules change compared to the BLG situation above
  - new terms in $\delta\Psi_{Aa}$
  - new transformation rules for a special $U(1)$

The generalized Jordan triple structure

- The index properties in ABJM suggest a connection to generalized Jordan triple systems [N,Palmkvist]
  - examples of triple systems (finite gradings) : Jordan, Kantor, Freudenthal
- triple systems may be based on a graded involution τ mapping between g_1 and g_{-1} in some n-graded Lie algebra:

$$(abc) = [[a, \tau(b)], c]$$

- all triple systems obey:

$$(ab(xyz)) - (xy(abz)) = ((abx)yz) \pm (x(bay)z)$$

with
  - minus sign=>the generalized Jordan identity,
  - plus sign=> Freudenthal (5-graded only)

plus additional relations if the Lie algebra is finite dimensional.

The connection to generalized Jordan triple systems (GJTS)

GJTS:

- $\hat{f}^{a}{}_{b}{}^{c}{}_{d}$ are structure constants of the triple system

ABJM/BL :

- $f^{ab}{}_{cd}$ have two antisymmetric pairs of indices:
- imposing this property on the triple systems =>
  - generalized Jordan identity = fundamental identity
  - no additional relations =>
    - the whole Lie algebra is infinitely graded (the BLG/ABJM gauge symmetry is here in g_0)
    - could be either Kac-Moody or Borcherds-like

Summary

We have discussed

- local symmetries of BLG and ABJM theories: topological gauging
- infinite dimensional Lie algebra structures in ABJM/BLG: GJTS
- light-cone methods to understand the physics and superspace

Future goals: to better understand 3-dim superconformal field theories

- ABJM -> BLG symmetry enhancement, 't Hooft (or monopole) operators, quantization, integrability, mirror symmetry, etc
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AND MIKE:

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AND MIKE: CONGRATULATIONS!!