FLUXES, GENERALIZED COMPLEX GEOMETRY AND DUALITIES

work with D. ANDRIOT AND M.PETRINI

SUPERSYMMETRIC BACKGROUNDS

10D string theory with

- the metric: $ds_{10}^2 = e^{2A(y)} \mathrm{d}s_4^2(M_4) + \mathrm{d}s_6^2$
- the fluxes: $F^{(10)} = F + \mathrm{vol}_4 \wedge \lambda(*F)$ (where $\lambda(F_n) = (-1)^{\mathrm{Int}[n/2]} F_n$)

$$\text{Equations of motion} \Leftrightarrow \left\{ \begin{array}{ll} \bullet & \text{pure spinor equations} \\ & \mathrm{d}\left(e^{3A}\Phi_1\right) = 0 \qquad \Leftrightarrow \qquad \text{Gen. CY structure} \\ & \mathrm{d}\left(e^{2A}\mathrm{Re}\Phi_2\right) = 0 \\ & \mathrm{d}\left(e^{4A}\mathrm{Im}\Phi_2\right) = e^{4A} e^{-B} * \lambda(F) \\ \bullet & \text{Bianchi identities} \\ & (\mathrm{d} - H\wedge)F = \delta(\text{source}) \end{array} \right.$$

Φ_1 and Φ_2 are even/odd poly-forms for IIA/B: IIA $\to$ $\begin{array}{l} \Phi_1 = \Phi_+ \\ \Phi_2 = \Phi_- \end{array}$ IIB $\to$ $\begin{array}{l} \Phi_1 = \Phi_- \\ \Phi_2 = \Phi_+ \end{array}$

PURE SPINORS and GCG

$$\Phi_+ = 8\, e^{-\phi} e^{-B} |\eta^1_+ \otimes \eta^{2\,\dagger}_+|_{\text{norm}} \qquad \longmapsto \qquad e^{i\theta_+} e^{-\phi} e^{-B}\, e^{-iJ}$$
$$\Phi_- = 8\, e^{-\phi} e^{-B} |\eta^1_+ \otimes \eta^{2\,\dagger}_-|_{\text{norm}} \qquad \longmapsto \qquad -ie^{i\theta_-} e^{-\phi} e^{-B} \Omega$$

(use: $\eta^1_+ \otimes \eta^{2\,\dagger}_\pm = \frac{1}{8} \sum_{k=0}^{6} \frac{1}{k!} \left(\eta^{2\,\dagger}_\pm \gamma_{m_k \dots m_1} \eta^1_+ \right) \gamma^{m_1 \dots m_k}$ for spinors

IIA $\quad \rightarrow \quad \begin{aligned} \epsilon_1 &= \zeta_+ \otimes \eta^1_+ + \zeta_- \otimes \eta^1_- \\ \epsilon_2 &= \zeta_+ \otimes \eta^2_- + \zeta_- \otimes \eta^2_+ \end{aligned}$ $\qquad$ IIB $\quad \rightarrow \quad \begin{aligned} \epsilon_1 &= \zeta_+ \otimes \eta^1_+ + \zeta_- \otimes \eta^1_- \\ \epsilon_2 &= \zeta_+ \otimes \eta^2_+ + \zeta_- \otimes \eta^2_- \end{aligned}$)

On Generalized tangent bundle: $\qquad 0 \longrightarrow T^*M \longrightarrow E \xrightarrow{\pi} TM \longrightarrow 0\,,$

$$\Phi_\pm \in L \otimes \Lambda^{\text{even/odd}} T^*M\,.$$

Sections of E:

$$X = \begin{pmatrix} v \\ \xi \end{pmatrix} \qquad \longmapsto \qquad X' = e^B X = \begin{pmatrix} \mathbb{I} & 0 \\ B & \mathbb{I} \end{pmatrix} \begin{pmatrix} v \\ \xi \end{pmatrix} = \begin{pmatrix} v \\ \xi - i_v B \end{pmatrix}\,.$$

- RR part is completely fixed by (g, B) (e.g. 32 components of RR flux vs. 42 components of SU(3) intrinsic torsion)
  Pure spinor equations $\Leftrightarrow$ supersymmetry (D=6!)

- $\int \langle (d - H\wedge)F,\ e^{3A}\mathrm{Im}\Phi_2 \rangle = ... = \int e^{4A}\,\langle F,\ *\lambda(F)\rangle$ has definite sign

  - ◇ Projection into singlet $\Leftrightarrow \mathrm{Tr}(\delta\mathcal{L}^{(4)}/\delta g_{\mu\nu}) \quad \longleftarrow$ TADPOLE
  - ◇ Need for negative charge sources $\quad \longrightarrow$ O-planes
  - ◇ Individual terms may correspond both to O-planes and D-branes
  - ◇ Sources given by:

$$(d - H\wedge)F_{p-3} \equiv c_i \eta^i = Q_i(\mathrm{source})\ \mathrm{vol}^i$$

- 4+6 split lifts the Self-Duality of RR fields

$$F_n^{10} = (-)^{\mathrm{Int}[n/2]} *_{10} F_{10-n}^{10} \Rightarrow F + \mathrm{vol}_4 \wedge \lambda(*F)$$

(But Pure Spinors are SD: $\quad \not{\Phi}_\pm\,\gamma = i\ \cancel{\lambda(*\Phi_\pm))}$

ORIENTIFOLDS

Orientifold action:

- O3/O7 and O6: $\Omega_{\rm WS}(-)^{F_L}\sigma$
- O5/O9 and O4/O8 $\Omega_{\rm WS}\sigma$
- IIA: $\sigma I = -I$, IIB: $\sigma I = I$

For SU(3) structure:

IIA: $\sigma\Omega_3 = \mp\bar{\Omega}_3 \quad \sigma e^{-iJ} = e^{iJ}$,

IIB: $\sigma\Omega_3 = \mp\Omega_3 \quad \sigma e^{-iJ} = e^{-iJ}$.

In general:

O3/O7	O5	06
$\sigma(\Phi_+) = -\lambda(\bar{\Phi}_+)$	$\sigma(\Phi_+) = \lambda(\bar{\Phi}_+)$	$\sigma(\Phi_+) = -\lambda(\Phi_+)$
$\sigma(\Phi_-) = \lambda(\Phi_-)$	$\sigma(\Phi_-) = -\lambda(\Phi_-)$	$\sigma(\Phi_-) = \lambda(\bar{\Phi}_-)$

The phases in the pure spinors get fixed. For type IIB the compact backgrounds are:

- ◇ Type B: $\pi/2$ (mod π)
- ◇ Type C: 0 (mod π)

3-STEP CONSTRUCTION OF SUPERSYMMETRIC BACKGROUNDS

	NON-COMPACT	COMPACT
STEP 1 (Twisted) GCY structure	$d_H\Phi_1 = 0$	Φ_1 is compatible with involution σ $d_H\Phi_1 = 0$
STEP 2 Metric $SU(3,3) \to S(3) \times SU(3)$	Φ_2 compatible with Φ_1 $d_H\Phi_2 = *F^{\rm RR}$	Φ_2 compatible with Φ_1 and σ $d_H\Phi_2 = *F^{\rm RR}$
STEP 3 Tadpole	compute $F^{\rm RR}$ $d_H F^{\rm RR} = 0$	compute $F^{\rm RR}$ $d_H F^{\rm RR} = sources$

EXAMPLE: CONFORMAL CY BACKGROUND

- X_6 - conformally CY
- $F_3 + \tau H_3$ - (2,1) and primitive type B
- $F_5 \sim *_6 dA$ (A - warp factor)
- The tadpole: $dF_5 = F_3 \wedge H_3 + sources \neq 0$
  - ◇ Need O3's and D3's
  - ◇ The tadpole is a top-form (singlet)!
- X_6 can be chosen to be (conformally) T^6
- Can use the isometries of the background to perform T-dualities:
  ≪NEW VACUA≫ ('02-'03)

  Ex: Two T-dualities with $F_3, F_5 \rightarrow \tilde{F}_3$ (O3,D3 $\rightarrow$ O5,D5) and a new manifold:

$$\begin{tikzcd} T^2 \arrow[r, hook] & M_6 \arrow[d] \\ & T^4 \end{tikzcd}$$

Nilmanifold - "Twisted torus" type C

Negative curvature - Not CY

TWISTED TORI - NIL(SOLV)MANIFOLDS

Definition: d-dimensional parallelisable manifolds

- $\exists\ d$ globally defined 1-forms

$$de^a = \frac{1}{2} f^a_{bc} e^b \wedge e^c$$

  (or dual vectors: $[E_b, E_c] = f^a{}_{bc}\, E_a$)

- f^a_{bc} - constants $\Rightarrow$ homogeneous spaces

- Jacobi identities:

$$d^2 e^a = 0 \quad \Rightarrow \quad f^a_{[bc} f^e_{d]a} = 0$$

  f^a_{bc} - structure constants of a real Lie algebra $\mathcal{G}$

- "Twisted" indentifications

$$M = \frac{G}{\Gamma} = \frac{\text{Lie Group}}{\text{discrete maximal subgroup}}$$

  Nilpotent (solvable) groups $\Rightarrow$ Nil (Solv) manifolds

- ALL Nilmanifolds - compact and admit GCY structure.

SOLVABLE ALGEBRAS

- $\exists$ a complete classification of solvable algebras in D=6 by dim_N, the dimension of nilradical $N(\mathcal{G})$:

	Nil	Solv
	$\mathcal{G}^0 = \mathcal{G}$ $\mathcal{G}_s \equiv [\mathcal{G}_{s-1}, \mathcal{G}]$ $\exists k \vert \quad \mathcal{G}_k = \{0\}$ k – nilpotency index	$\mathcal{G}^0 = \mathcal{G}$ $\mathcal{G}^s \equiv [\mathcal{G}^{s-1}, \mathcal{G}^{s-1}]$ $\exists k \vert \quad \mathcal{G}^k = \{0\}$
Number of algebras in D=6	34	$\mathrm{dim}_N = 3 \;\rightarrow\; 9$ $(all \;\; decomp)$ $\mathrm{dim}_N = 4 \;\rightarrow\; 40$ $\mathrm{dim}_N = 5 \;\rightarrow\; 99$ $\mathrm{dim}_N = 6 \;\rightarrow\; 34$

SUMMARY OF SOLUTIONS

- Minkowski vacua
  - **T-duals** of conformal Calabi Yaus $\longrightarrow$ full solutions including the **warp factor**
  - Multi-source solutions $\longrightarrow$ delocalised solution (**large volume** only)

		IIA			IIB	
	algebras	**O4**	**O6**		**O5**	
		type 12	type 30	type 12	type 30	type 12
n	**(0,0,0,12,13,23)**		456			
n	**(0,0,0,12,23,14-35)**				**(45 + 26)***	
n	**(0,0,0,0,12,14+23)**				56	56
n	**(0,0,0,0,12,34)**				56	56
n	**(0,0,0,0,12,13)**				56	56
n	**(0,0,0,0,13 + 42,14+23)**				56	56
n	**(0,0,0,0,0,12+34)**	6			56	56
n	**(0,0,0,0,0,12)**	6				
s	**(25,- 15,$\pm$45,$\mp$35,0,0)**		(136 + 246)* (146 + 236)*	(136 + 246)* (146 + 236)*		(13 + 24)* (14 + 23)*

- Type 3/0 solutions on IIB with O5 planes - balanced manifolds ($\mathrm{d}J^2 = 0$)

SOLUTIONS

- Type 3 and type 0 pure spinors with O5 planes

$$\Phi_- = -i\frac{ab}{8}\Omega \qquad \Phi_+ = \frac{a\bar{b}}{8}e^{-iJ}$$

- The only allowed flux is F_3. The equations are:

$$\begin{aligned} d(e^A\Omega) &= 0 = dJ^2 \\ \phi - 2A &= 0 = H \\ d(e^{2A}J) &= -e^{4A} * F_3 \qquad \Rightarrow \qquad d(e^{4A} * F_3) = 0 \\ dF_3 &= 2i\partial\bar{\partial}(e^{-2A}J) = \delta(D5) - \delta(O5) \end{aligned}$$

Bianchi identity

- A (T-dual) example:
  - ◇ the manifold (0,0,0,0,12,14+23)
    ($de^1 = 0$ for $i = 1, \ldots, 4$; $de^5 = e^1 \wedge e^2$; $de^6 = e^1 \wedge e^4 + e^2 \wedge e^3$)
  - ◇ orientifold: (-,-,-,-,+,+)
  - ◇ pure spinors

$$\begin{aligned} \Omega &= e^{-A}(e^1 + ie^2)(e^3 - ie^4)(e^5 + ie^6)\ , \\ J &= ie^{-2A}(e^1 \wedge e^2 - e^3 \wedge e^4) + e^{2A}e^5 \wedge e^6 \end{aligned}$$

All equations are solved, and the Bianchi Identity (moduli are fixed by hand) is

$$dF_3 = \left(6 + \nabla_-^2 (e^{-4A})\right) e_-^1 \wedge e_-^2 \wedge e_-^3 \wedge e_-^4$$

$dF_3 \wedge J$ shows the source terms has an overall O-plane sign.

T-duality:

$$\begin{aligned} F_5^D &= e^{4A} * d(e^{-4A}) \\ *F_3^D &= -*_2 dJ = H_3^D \end{aligned}$$

After coordinate transformation:

- forms ($e_i = i$)

$$H = 125 - 126 - 135 + 136 - 245 + 346$$

$$F_3 = 125 - 136 - 245 - 246 - 345 - 346$$

- pure spinors

$$\Omega = (1 + i4) \wedge (2 + i3) \wedge (5 + i6)$$

$$e^{-iJ} = e^{-i(14+23+56)}$$

- Bianchi identitiy: $dF_5 = H \wedge F + Q_s \nabla^2 e^{-4A} vol_{T^6}$

The conformal T^6 solution with self-dual 3-form flux $G_3 = F_3 - iH_3$

DELOCALIZED SOURCE SOLUTIONS

- The nilmanifold (0,0,0,12,23,14-35):

$$\begin{tikzcd} S^1_{\{6\}} \arrow[r, hook] & M_6 \arrow[d] \\ T^2_{\{4,5\}} \arrow[r, hook] & M_5 \arrow[d] \\ & T^3_{\{1,2,3\}} \end{tikzcd}$$

- The metric is given by: $(B=0)$

$$\begin{aligned} \Omega_3 &= (e^1 - ie^3) \wedge (e^2 + i\tau e^6) \wedge (e^4 + ie^5) \\ J &= -t_1 e^1 \wedge e^3 + t_2 \tau_r e^2 \wedge e^6 + t_3 e^4 \wedge e^5 \end{aligned}$$

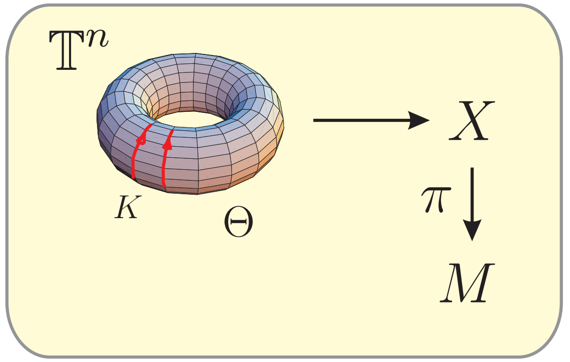
- RR 3-form: $F_3 = -(\tau_i e^2 - |\tau|^2 e^6) \wedge \left(t_2 (e^1 \wedge e^4 - e^3 \wedge e^5) + \frac{t_3}{\tau_r} (e^1 \wedge e^5 + e^3 \wedge e^4) \right)$
- Moduli: $\tau = \tau_r + i\tau_i$ - CS, t_1, t_2, t_3 - "Kähler" (get fixed by BI)
  $t_i > 0$ and $1 + |\tau|^2 \geq 2|\tau_r|$ - for positive definite metric
- The tadpole: $dF_3 = -2\,|\tau|^2 \left(\frac{t_3}{\tau_r^2 t_1 t_2}\ \mathrm{vol1}^{1236} + \frac{t_2}{t_1 t_3}\ \mathrm{vol2}^{1345} \right)$
  O5-planes along 45 26 - intersecting sources

- Not T-dual, but ... can be connected to other solutions!

COORDINATE-DEPENDENT O(n,n) TRANSFORMATIONS

$\mathbb{T}^n$ action

- Principal torus bundle $\mathbb{T}^n \hookrightarrow X \xrightarrow{\pi} M$:



- A globally well-defined smooth 1-form Θ on X with values in $\mathfrak{t} := \mathrm{Lie}\,\mathbb{T}^n \cong \mathbb{R}^n$.
- Isometries: $\imath_K \Theta = \mathbb{I} \in \mathfrak{t}^* \otimes \mathfrak{t}$ ($\mathcal{L}_K \Theta = 0$) and $\mathcal{L}_K H = 0$
- 3-form H:

$$H = \pi^* H_3 + \langle \pi^* H_2, \Theta \rangle + \frac{1}{2} \langle \pi^* H_1, \Theta \wedge \Theta \rangle + \frac{1}{6} \langle \pi^* H_0, \Theta \wedge \Theta \wedge \Theta \rangle$$

(but $B_\alpha = B_{2\alpha} + \langle B_{1\alpha}, \Theta \rangle + \frac{1}{2} \langle B_{0\alpha}, \Theta \wedge \Theta \rangle$ (No π^*!))

$\longrightarrow$ $H_j \in \Omega^j(M; \Lambda^{3-j}\mathfrak{t})$ for $j = 0, 1, 2, 3$

$\longrightarrow$ $\langle \cdot, \cdot \rangle$: the natural pairing $\mathfrak{t}^* \otimes \mathfrak{t} \longrightarrow \mathbb{R}$

$\longrightarrow$ $dH = 0 \quad \Rightarrow \quad dH_j + \langle H_{j-1}, F \rangle = 0$

$\mathbb{T}^n$ action & Courant

- Courant bracket:
  $[(v,\rho),(w,\lambda)]_H = [v,w] + \left\{\mathcal{L}_v\lambda - \mathcal{L}_w\rho - \frac{1}{2}\,\mathrm{d}(\imath_v\lambda - \imath_w\rho) + \imath_v\imath_w H\right\}$

- $v = v_M + \langle K, f\rangle$ and $\rho = \rho_M + \langle \phi, \Theta\rangle$
  $\mathcal{L}_K v = 0$ and $\mathcal{L}_K\rho = 0$, $\Rightarrow f \in \Omega^0(M,\mathfrak{t})$ and $\phi \in \Omega^0(M,\mathfrak{t}^*)$. ($\mathbb{T}^n$-invariant section of TX can be written as an element $(v_M, f) \in TM \oplus \mathfrak{t}$, while a $\mathbb{T}^n$-invariant section of TX^* can be written as $(\rho_M,\phi) \in T^*M \oplus \mathfrak{t}^*$.)

- Courant with $\mathbb{T}^n$ action:

$$
\begin{aligned}
&[(v_M, f;\rho_M,\phi),\ (w_M, g;\lambda_M,\omega)]_H = [(v_M;\rho_M),\ (w_M;\lambda_M)]_{H_3} + \\
&\qquad\Big(\underbrace{0}_{\text{vector}},\underbrace{\mathcal{L}_{v_M}g - \mathcal{L}_{w_M}f}_{\text{function}};\underbrace{\langle\omega, df\rangle - \langle\phi, dg\rangle - d(\langle\omega, f\rangle - \langle\phi, g\rangle)/2}_{1-\text{form}},\underbrace{\mathcal{L}_{v_M}\omega - \mathcal{L}_{w_M}\phi}_{\text{function}}\Big) + \\
&\qquad\Big(0, \imath_{v_M}\imath_{w_M}F; \langle\omega, \imath_{v_M}F\rangle + \langle\imath_{v_M}F_\#, g\rangle - \langle\imath_{w_M}F_\#, f\rangle - \langle\phi, \imath_{w_M}F\rangle, \imath_{v_M}\imath_{w_M}F_\#\Big) -
\end{aligned}
\tag{1}
$$

where $F^I_\# := \imath(\frac{\partial}{\partial\theta_I})H = H^I_2 + (-H^{IJ}_1 \wedge \Theta_J + \frac{1}{2}\,H^{IJK}_0\Theta_J \wedge \Theta_K)$ and $dF^I_\# = 0$

Automorphisms of the bracket:

◇ Constant $O(n,n)$ transformations on $\mathfrak{t}\oplus\mathfrak{t}^*$:

$$S_{\mathfrak{t}}(X) = \left(\begin{array}{cc|cc} \mathbb{I} & 0 & 0 & 0 \\ 0 & A & 0 & B \\ \hline 0 & 0 & \mathbb{I} & 0 \\ 0 & C & 0 & D \end{array}\right) \left(\begin{array}{c} v \\ f \\ \hline \rho \\ \phi \end{array}\right)$$

◇ Generalized B-transforms

$$X \mapsto e^{\hat{B}} X = \Big(v, f + \imath_v U; \rho + \imath_v b^{\mathcal{B}} + \langle b, f\rangle + \langle \phi, U\rangle, \phi + \imath_v b\Big)^T$$

with closed two-form $b^{\mathcal{B}}$ and one-forms b and U.

- The automorphism (T-duality):

$$[S_{\mathfrak{t}}(e^{\hat{B}} X), S_{\mathfrak{t}}(e^{\hat{B}} Y)] = S_{\mathfrak{t}}(e^{\hat{B}}[X,Y])$$

(one-forms b and U are coordinate dependent now)

Automorphisms with coordinate-dependent $O(n,n)$ $\longrightarrow$ “TWIST DUALITY”

“TWIST DUALITY” and PURE SPINORS

◇ $\Phi_\pm \quad \longmapsto \quad \Phi'_\pm = O^\pm \cdot \Phi_\pm$

◇ $O = e^{i\theta_c^\pm} \frac{1}{\sqrt{\det A}} e^{-y_{mn} dx^m \wedge dx^n} \, e^{a^m{}_n dx^n \wedge \iota_{\partial_m}} \, e^{x_{mn} dx^m \wedge dx^n} = e^{i\theta_c^\pm} O_f$

O is a combination of

- B-transform
- a scaling transformation of (parts of) the metric $\Rightarrow$ a shift in dilaton
- a (pair of) $U(1)$ rotation(s)
- a change in the connection - twist of $\mathbb{T}^n$

For $\Phi'_\pm \Rightarrow$ $\left\{ \begin{array}{l} \mathrm{d}\,(e^{3A}\Phi'_1) = 0 \qquad \Leftrightarrow \qquad \text{Gen. CY structure} \\ \mathrm{d}\,(e^{2A}\mathrm{Re}\Phi'_2) = 0 \\ \mathrm{d}\,(e^{4A}\mathrm{Im}\Phi'_2) = R' \end{array} \right.$

◇ $\mathrm{d}(O_f)\,\Phi_1 = 0$

◇ New integrability defect from RR part $R = e^{4A} e^{-B} * \lambda(F)$:
$R' = \cos(\theta_c^+) O_f\, R + \sin(\theta_c^+) \mathrm{d}(e^{2A} O_f)\, e^{2A}\mathrm{Re}\Phi_2 + \cos(\theta_c^+) \mathrm{d}(O_f)\, e^{4A}\mathrm{Im}\Phi_2$

TWISTED TORUS BACKGROUNDS FROM TWIST DUALITY

- $\mathbb{T}^4 \times \mathbb{T}^2 \quad \Rightarrow \quad \mathbb{T}^2 \hookrightarrow M \xrightarrow{\pi} \mathbb{T}^4$

- $SU(3)$ structure after the twist:

$$J' = J_M + \frac{i}{2} g'_{z\bar{z}} \, \Theta \wedge \overline{\Theta}$$

$$\Omega' = \sqrt{g'} \, \omega_M \wedge \Theta$$

- PS equations $\Rightarrow$ conditions on curvature $\pi^* F = \mathrm{d}\Theta$:

$$\left. \begin{array}{l} F \wedge J_M = \overline{F} \wedge J_M = 0 \\ F \wedge \omega_M = 0 \end{array} \right\} \quad \Rightarrow \quad F = F^{2,0} + F^{1,1}_{-}$$

- Five solutions:

$(0,0,0,0,12,34)$	$M = N_3 \times N_3$	$b_2(M) = 8$	$b_3(M) = 10$
$(0,0,0,0,13,14)$	$M = S^1 \times M_5$	$b_2(M) = 9$	$b_3(M) = 8 + 2$
$(0,0,0,0,2 \times 13, 14 + 23)$	$M = N_6^{(1)}$	$b_2(M) = 8$	$b_3(M) = 9$
$(0,0,0,0,13 + 42, 14 + 23)$	$M = N_6^{(2)}$	$b_2(M) = 8$	$b_3(M) = 10$
$(0,0,0,0,0,14 - 23)$	$M = S^1 \times N_5$		$b_1(M) = 5$

ITERATING THE TWIST

- ◇ $S^1 \times M_5 \quad \Rightarrow \quad S^1 \hookrightarrow M \xrightarrow{\pi'} M_5$
- ◇ Conditions on curvature :

$$\begin{aligned} F \wedge (e^3 + ie^4) \wedge (e^5 + ie^6) &= 0 \\ F \wedge (e^1 \wedge e^3 \wedge e^4 + e^1 \wedge e^5 \wedge e^6) &= 0 \end{aligned}$$

- ◇ $M : (0, F, 0, 0, 13, 14) \cong (0, 0, 0, 12, 23, 14 - 35), \qquad b_1(M) = 3$
- ◇ Bianchi Identity

$$g_s \mathrm{d}F_3 = 2i\partial\bar{\partial}(e^{-2A}J) = \delta(D5) - \delta(O5)$$

with intersecting sources.

OPEN QUESTIONS:

- Torsional heterotic backgrounds (Het. string and GCG?)
- More general twists (cover solvmanifolds? more? ... non-geom backgrounds?)
- Non-compact backgrounds