Higher derivative corrections in five-dimensional supergravity

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JTL and P. Szepietowski, arXiv:0806.1026

S. Cremonini, K. Hanaki, JTL and P. Szepietowski, arXiv:0812.3572, arXiv:0903.3244

Why explore higher derivative corrections?

- Stringy corrections — the α' expansion

  low energy effective field theory

$$e^{-1}\mathcal{L} = R + \alpha' R^2 + \alpha'^2 R^3 + \alpha'^3 R^4 + \cdots$$

- Black hole physics

  black hole entropy, OSV, small black holes

- AdS/CFT finite 't Hooft coupling corrections ($\alpha' = L^2/\sqrt{\lambda}$)

  entropy: $$s = \frac{3}{4} s_0 \left[1 + \frac{15}{8}\zeta(3)\lambda^{-3/2} + \cdots\right] \qquad s_0 = \frac{2\pi^2}{3} N^2 T^3$$

  shear viscosity: $$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 + 15\zeta(3)\lambda^{-3/2} + \cdots\right]$$

Higher derivative gravity

- Consider a pure gravity theory

$$e^{-1}\mathcal{L} = R + \alpha_1 R^2 + \alpha_2 R_{\mu\nu}^2 + \alpha_3 R_{\mu\nu\rho\sigma}^2 + \cdots$$

- This generally gives rise to several unpleasant features

  non-unitary propagation, propagation outside the lightcone
  ill posed Cauchy problem, no generalized Gibbons-Hawking surface term

- These are related to the fourth (or higher) order equation of motion

  One exception — the Gauss-Bonnet combination

$$e^{-1}\mathcal{L}_{\mathrm{GB}} = \alpha(R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2)$$

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- However: Not a problem as an effective field theory

Field redefinitions

- One complication — we may perform a field redefinition of the form

$$g_{\mu\nu} \to g_{\mu\nu} + aR_{\mu\nu} + bg_{\mu\nu}R$$

For pure gravity with $\Lambda = 0$ this is an on-shell field redefinition

- At linear order in α_i, only $\alpha_3 R^2_{\mu\nu\rho\sigma}$ is physical

The $\alpha_1 R^2$ and $\alpha_2 R^2_{\mu\nu}$ terms can be shifted away by an appropriate field redefinition

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- The above field redefinition can be generalized in the presence of matter

For Maxwell-Einstein gravity

$$g_{\mu\nu} \to g_{\mu\nu} + aR_{\mu\nu} + bg_{\mu\nu}R + cF_\mu{}^\lambda F_{\nu\lambda} + dg_{\mu\nu}F^2$$

Five dimensional supergravity

- Motivation comes from

  $D = 5$ black holes and attractors (ungauged supergravity)

  AdS/CFT (gauged supergravity)

- Minimal $\mathcal{N} = 2$ gauged supergravity

$$e^{-1}\mathcal{L}_0 = R - \frac{1}{4}F_{\mu\nu}^2 + 12g^2 + \frac{1}{12\sqrt{3}}\epsilon^{\mu\nu\rho\lambda\sigma}F_{\mu\nu}F_{\rho\lambda}A_\sigma + \text{fermi}$$

- For higher derivative corrections

  Also includes graviphoton (F^4) and mixed (RF^2) terms

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- This is in general a non-trivial task

  Many potential terms to consider, even at the four derivative level

Supersymmetric completion of $A \wedge \mathrm{Tr}\, R \wedge R$

- Natural application of susy tensor calculus

  $i)$ off-shell formulation of $D = 5$, $\mathcal{N} = 2$ sugra

  $ii)$ conformal supergravity (Weyl multiplet) coupled to
  $n_V + 1$ conformal vector multiplets

  $iii)$ add a conformal compensator (hypermultiplet)

  $iv)$ create invariants using superconformal tensor calculus

- At the four derivative level

$$\mathcal{L} = \underbrace{\mathcal{L}(V \cdot L[H^2])}_{\text{hyper}} - \tfrac{1}{2}\underbrace{\mathcal{L}(V \cdot L[V^2])}_{\text{vector}} + \underbrace{\mathcal{L}(V \cdot L[W^2])}_{\text{four-derivative}}$$

K. Hanaki, K. Ohashi and Y. Tachikawa, hep-th/0611329

Minimal supergravity up to four derivatives

- The four derivative terms are parameterized by a single coefficient c_2
- After integrating out auxiliary fields (and working to linear order in c_2)

$$
\begin{aligned}
e^{-1}\mathcal{L} \quad = \quad & R - \frac{1}{4}F_{\mu\nu}^2 + 12g^2 + \frac{1}{12\sqrt{3}}\Big(1 - \boxed{c_2}\frac{}{6}g^2\Big)\epsilon^{\mu\nu\rho\lambda\sigma}F_{\mu\nu}F_{\rho\lambda}A_\sigma \\
& + \frac{\boxed{c_2}}{24}\Big[\frac{1}{16\sqrt{3}}\epsilon^{\mu\nu\rho\lambda\sigma}R_{\mu\nu\alpha\beta}R_{\rho\lambda}{}^{\alpha\beta}A_\sigma + \frac{1}{8}C_{\mu\nu\rho\sigma}^2 \\
& \qquad + \frac{5}{64}F_{\mu\nu}F^{\nu\rho}F_{\rho\lambda}F^{\lambda\mu} - \frac{41}{2304}(F_{\mu\nu}^2)^2 \\
& \qquad + \frac{1}{16}C_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} + \frac{1}{3}R_{\mu\nu}F^{\mu\lambda}F^{\nu}{}_{\lambda} - \frac{1}{48}RF_{\mu\nu}^2 \cdots\Big]
\end{aligned}
$$

- What physical significance can we give to c_2?

The c_2 parameter in the ungauged case

- Consider M-theory (11-dimensional supergravity) on CY_3
- In 11 dimensions M.J. Duff, JTL and R. Minasian, hep-th/9506126

$$e^{-1}\mathcal{L}_{11} = \cdots + C_3 \wedge \left(\frac{1}{6} F_4 \wedge F_4 + \frac{1}{768} (\mathrm{Tr}\, R^2)^2 - \frac{1}{192} \mathrm{Tr}\, R^4 \right)$$

- After compactifying on CY_3, with $C_3 = A^I \wedge \omega_I$

$$e^{-1}\mathcal{L}_5 = \cdots + \frac{1}{6} c_{IJK} A^I \wedge F^J \wedge F^K + \frac{1}{96} c_{2\,I} A^I \wedge \mathrm{Tr}\, R \wedge R + \cdots$$

$c_{IJK} =$ triple intersection number

$c_{2\,I} =$ second Chern class on appropriate cycles of CY_3

- The parameters of the five-dimensional supergravity are given by topological data of the CY_3

The c_2 parameter in the gauged case

- IIB theory on S^5 gives $c_2 = 0$, but non-vanishing c_2 can come from other sources such as branes
- We can use input from AdS/CFT

  $i)$ The holographic Weyl anomaly $\Rightarrow C^2_{\mu\nu\rho\sigma}$

  $ii)$ The R-current anomaly $\Rightarrow A \wedge \mathrm{Tr}\, R \wedge R$

- For the CFT$_4$ Weyl anomaly
  D.M. Capper and M. J. Duff, Nuovo Cimento, 23A (1974) 173
  M. J. Duff, Nucl. Phys. B125 (1977) 334

$$g^{\mu\nu}\langle T_{\mu\nu}\rangle = b\left(C^2_{\mu\nu\rho\sigma} + \frac{2}{3}\Box R\right) + b'\left(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma}\right)$$

- Matching with the holographic AdS$_5$ calculation gives

$$\frac{c_2}{24} = -\frac{b+b'}{b'g^2} \qquad \text{or} \qquad \frac{c_2}{24} = \frac{c-a}{ag^2} \quad \text{with} \quad b = \frac{c}{16\pi^2} \text{ and } b' = -\frac{a}{16\pi^2}$$

Holographic thermodynamics

- The b and b' (or a and c) anomaly coefficients depend on the gauge theory

$$b = -b' = \frac{N^2}{64\pi^2} \qquad \text{for } \mathcal{N} = 4 \text{ SYM}$$

More generally, we expect $b + b' = \mathcal{O}(N)$, hence $\dfrac{c_2}{24} = -\dfrac{b+b'}{b'g^2} \sim \mathcal{O}\Big(\dfrac{1}{N}\Big)$

- Holographic thermodynamics may be extracted from the flat horizon black hole solution (to linear order in c_2)

$$ds^2 = -H^{-2} f dt^2 + H\left[\frac{dr^2}{f} + r^2 d\vec{x}^{\,2}\right]$$

$$f = -\frac{\mu}{r^2} + g^2 r^2 H_0^3 + \frac{c_2}{24}\left[\frac{\mu^2}{4r^6 H_0} - \frac{8g^2 Q\mu}{3r^4}\right], \quad H = 1 + \frac{Q}{r^2} + \frac{c_2}{24}\left[-\frac{Q\mu}{3r^6 H_0^2}\right]$$

$$A_t = \sqrt{\frac{3\mu}{Q}}\left(1 - \frac{1}{H_0} + \frac{c_2}{24}\left[\frac{Q\mu}{2r^8 H_0^4}(Q - r^2)\right]\right)$$

Temperature and entropy

- The temperature is related to the surface gravity at the horizon (or, equivalently, from demanding the absence of a conical singularity)

$$T = \frac{g^2 r_0 (2-q)(1+q)^{1/2}}{2\pi}\left[1 - \frac{\bar{c}_2}{8}\frac{10 - 59q - 4q^2 - 3q^3}{(2-q)^2}\right]$$

- We use Wald's formula for the entropy density

$$s = \frac{(g r_0)^3 (1+q)^{3/2}}{4G_5}\left[1 + \frac{\bar{c}_2}{8}\frac{21 + 14q - 3q^2}{2-q}\right]$$

Note that we have defined $\bar{c}_2 = -\frac{b+b'}{b'} = \frac{c-a}{a}, \qquad q = \frac{Q}{r_0^2}$

To lowest order, q is related to the R-charge chemical potential Φ through

$$\Phi = g r_0 \sqrt{3q(1+q)}$$

Holographic hydrodynamics

- We may also extract the shear viscosity using the Kubo formula in the scalar channel

$$\eta = \frac{(gr_0)^3(1+q)^{3/2}}{16\pi G_5}\left[1 + \frac{\overline{c}_2}{8}\frac{5+6q+5q^2}{2-q}\right]$$

- The result simplifies in the ratio

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - \overline{c}_2(1+q)\right] = \boxed{\frac{1}{4\pi}\left[1 + \frac{b+b'}{b'}(1+q)\right]}$$

This was independently obtained in

R.C. Myers, M.F. Paulos and A. Sinha, arXiv:0903.2834

More to be done

- A better understanding of violations of $\eta/s \geq 1/4\pi$

  Generic large N theories with a gravitational dual have $(b+b')/b' \leq 0$

  Adding R-charge only increases the violation

- Universality of η/s including higher derivatives
- Going beyond $\mathcal{O}(1/N)$

  Obtaining ‘exact’ solutions to higher derivative gravity is a challenge

- What about the $1/\lambda$ corrections?

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Much remains to be explored in higher derivative gravity