$\mathcal{N} = 8$ Supergravity on the Light Cone

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Based on: RK, arXiv:0903.4630

Supersymmetry, Branes and M-Theory: A Meeting in Celebration of Michael Duff's 60th Birthday, Imperial College London, 1st-2nd April 2009

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- By performing a Fourier transform into the light-cone chiral coordinate superspace we find that the quantum corrections to the superfield amplitudes with n legs are non-local in transverse directions for the diagrams with the number of loops smaller than $n(n-1)/2+1$. This suggests the reason why UV infinities, which are proportional to local vertices, cannot appear at least before 7 loops in the light-cone supergraph computations.

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- By combining the $E_{7(7)}$ symmetry with the supersymmetric recursion relations we argue that the light-cone supergraphs predict all loop finiteness of d=4 $\mathcal{N}$=8 SG.

- We suggest a list of "things to do" to validate this prediction.

Outline of the Talk

1. For My Friends Who Remember the 1st Wave of $\mathcal{N}=8$
   - What we did long time ago
   - New Wave
2. 1st wave + New Wave
3. $\mathcal{N}$=8 SG on the Light Cone
   - Chiral Scalar Light-Cone Superfield
   - Light-Cone Generating Functional
   - $\mathcal{N}$=8 Equivalence Theorem
   - 4-Point 3-Loop Amplitude
4. $\mathcal{N}$=8 Light Cone Supergraph Predictions
   - 4-Point Amplitude is Divergence-Free Till 7 loop
   - n-Point Amplitude is Divergence-Free Till $\frac{n(n-1)+2}{2}$ loop
   - General Helicity Structures in n-Point Amplitudes
   - $E_{7(7)}$ symmetry : $n \to \infty$, $\mathcal{N}$=8 SG is Divergence-Free Till $L \to \infty$ loop order
5. $\mathcal{N}$=8 Black Holes and QFT
   - Credit for the slide to A. Marrani
   - Continuos or discrete?
6. $\mathcal{N}=8$ Black Holes, Michael Duff and Quantum Computing
   - Credit for the slide to A. Marrani

For My Friends Who Remember the 1st Wave of $\mathcal{N} = 8$

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- We constructed the linearized 3-loop counterterm:

$$\frac{\kappa^4}{\epsilon}\int d^4x\; d^{16}\theta_B\; W^4(x,\theta_B) = \kappa^4\int d^4x R_{\alpha\beta\gamma\delta}(x) R_{\dot\alpha\dot\beta\dot\gamma\dot\delta}(x) R^{\alpha\beta\gamma\delta}(x) R^{\dot\alpha\dot\beta\dot\gamma\dot\delta}(x) + \ldots\,.$$

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- We constructed non-linear L-loop counterterms with $L \geq 8$, for example

$$\frac{\kappa^{14}}{\epsilon} \int d^4x \; d^{32}\theta \; \mathrm{BerE} \; \mathrm{T}_{\mathrm{ijk}\,\alpha}(\mathrm{x}, \theta) \overline{\mathrm{T}}^{\mathrm{ijk}\,\dot\alpha}(\mathrm{x}, \theta) \mathrm{T}_{\mathrm{mnl}}{}^{\alpha}(\mathrm{x}, \theta) \overline{\mathrm{T}}^{\mathrm{mnl}}{}_{\dot\alpha}(\mathrm{x}, \theta)$$

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- End of story! UV divergences in all loops (not quite clear onset of divergences). We all agreed that they will never stop.

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- One result from my work in 2007 relevant to the current result: I constructed the manifestly $\mathcal{N}$=8 supersymmetric finite 1-loop box amplitude (supported by few recent papers of W. Siegel on projective superspace). I used the same linearized short superfields which we all used for the 3-loop counterterms.

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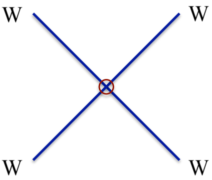
- New Wave: Helicity Formalism, Twistors, Recursion Relations, $\mathcal{N}$=8 SG=Simplest QFT, 3-loop Superfiniteness, Indications of All Loop Finiteness

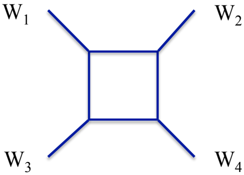
In N = 8 SG the 3-loop counterterm was predicted but did not show up



$$\frac{C}{\epsilon} \int d^4x d^{16}\theta_B \; W^4(x, \theta_B)$$

$$C = 0$$

1-loop finite box amplitude is well represented by our old short superfield



$$\int d^4x_1 d^4x_2 d^4x_3 d^4x_4 \, d^{16}\theta_B \, \frac{W_1 W_2 W_3 W_4}{x_{12}^2 x_{23}^2 x_{34}^2 x_{41}^2}$$

In pure gravity, the two loop counterterm was predicted and did show up

$$\frac{209\kappa^2}{(4\pi)^4 \, 2880 \, \epsilon} \int d^4x \; R_{\mu\nu}^{\lambda\delta} R_{\lambda\delta}^{\eta\xi} R_{\eta\xi}^{\mu\nu}$$

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$$S^{\text{box}}_{\mathcal{N}=8} \sim \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 \; d^{16}\theta_B \; \frac{W_1 W_2 W_3 W_4}{x_{12}^2 x_{23}^2 x_{34}^2 x_{41}^2} \; ,$$

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$$S^{\rm box} = \int \frac{d^4x_1 d^4x_2 d^4x_3 d^4x_4}{x_{12}^2 x_{23}^2 x_{34}^2 x_{41}^2} (R_{\alpha\beta\gamma\delta}(x_1) R^{\alpha\beta\gamma\delta}(x_2) R_{\dot\alpha\dot\beta\dot\gamma\dot\delta}(x_3) R^{\dot\alpha\dot\beta\dot\gamma\dot\delta}(x_4) + sym.)$$

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- The 4-graviton MHV amplitude in helicity formalism follows:

$$\langle 12\rangle^4 [34]^4 (F^{\text{box}}(s,t) + \text{sym})$$

$$F^{\text{box}} = \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 (q-p_1)^2 (q-p_1-p_2)^2 (q+p_4)^2}$$

- There is an UNCONSTRAINED CHIRAL SCALAR superfield $\phi(x, \theta^a)$ describing only physical degrees of freedom

$$\frac{1}{\partial_+^2} h(x) + \theta^a \frac{1}{\partial_+^{3/2}} \bar{\psi}_a(x) + \theta^{ab} \frac{1}{\partial_+} \bar{B}_{ab}(x) + \theta^{abc} \frac{1}{\partial_+^{1/2}} \bar{\chi}_{abc}(x) + \theta^{abcd} \phi_{abcd}(x) + \ldots + \tilde{\theta} \partial_+^2 \bar{h}(x)$$

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- The unitary path integral for the QFT of a scalar superfield is a "piece of cake"

$$e^{iW[\phi_{in}\Box]}=\int d\phi\; e^{i(S_{cl}[\phi(x,\theta)]+\int d^4x d^8\theta \phi_{in}(x,\theta)\Box\phi(x,\theta))}$$

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- The light-cone superfield in Fourier

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- Miracle # 1: all nasty Lorentz non-covariant terms vanished

$$\boxed{\Phi(p,\eta)=\bar h(p)+\eta_a\psi^a(p)+\eta_{ab}B^{ab}(p)+\eta_{abc}\chi^{abc}(p)++\eta_{abcd}\phi^{abcd}(p)+\ldots+\tilde\eta h(p)}$$

Light-Cone Generating Functional

- The generating functional for connected n-point amplitudes in chiral Fourier superspace is

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$$\boxed{\mathcal{A}_n^{lc}(p_1, ...p_n; \eta_1, ..., \eta_n) = \left(\sum_{l=1}^{l=n} \frac{p_{\perp l}}{(p_l^+)^{1/2}} \eta_l \right)^8 \mathcal{P}(p_1, ...p_n; \eta_1, ..., \eta_n)}$$

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- The $\mathcal{P}$ -factor is totally symmetric, has mass dimension -4 and helicity +2 at each point

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- Here the first Lorentz non-covariant 8-dimensional δ-function originates from the Fourier transform from θ-space. The second Lorentz non-covariant 8-dimensional δ-function is the first factor in the amplitude $\mathcal{A}_n^{lc}$
- The equivalence theorem: the second factor in the light-cone superfield amplitude $\mathcal{A}_n^{lc}(p_i;\eta_i) = \left(\sum_{l=1}^{l=n} \frac{p_{\perp l}}{(p_l^+)^{1/2}}\eta_l\right)^8 \mathcal{P}(p_i;\eta_i)$, namely $\mathcal{P}(p_i;\eta_i)$, must be Lorentz covariant and coincide with the corresponding "all plus amplitude" in the covariant Nair's generating functional for $\mathcal{N}$=8 SG

$$\boxed{\mathcal{P}^{lc}(p_1,...p_n;\eta_1,...,\eta_n) = \mathcal{P}^{cov}(p_1,...p_n;\eta_1,...,\eta_n)}$$

Relation to the Square of the Bel-Robinson Tensor

- To get the log divergence in the 4-graviton amplitude one has to take the light-cone superfield amplitude in the form

$$\boxed{(\mathcal{A}_4^{lc})_{UV}^{3loop} \sim \kappa^4 \ln \Lambda} \left(\sum_{l=1}^{4} \lambda_l^2 \eta_l \right)^8 \frac{[34]^4}{\langle 12 \rangle^4}$$

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- By performing all fermionic integration over $\eta_1^8, \eta_2^8, \eta_3^8, \eta_4^8$ one finds the effective 4-graviton action

$$\kappa^4 \ln \Lambda \prod_{i=1}^{4} (\int d^4 p_i)\ \langle 12 \rangle^4 [34]^4 \bar{h}(p_1) \bar{h}(p_2) h(p_3) h(p_4)$$

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- The gravitational amplitude seem to correspond to a legitimate local counterterm. This is why for 25 years we were not sure about the UV status of the 3-loop amplitude

$$\kappa^4 \ln \Lambda \int d^4 x (R_{\alpha\beta\gamma\delta} \bar{R}_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}})^2$$

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$$\kappa^4 \ln \Lambda \prod_{i=1}^{4} (\int d^4 p_i)\ \langle 12\rangle^4 [34]^4 \bar{h}(p_1)\bar{h}(p_2)h(p_3)h(p_4)$$

- The gravitational amplitude seem to correspond to a legitimate local counterterm. This is why for 25 years we were not sure about the UV status of the 3-loop amplitude

$$\kappa^4 \ln \Lambda \int d^4x (R_{\alpha\beta\gamma\delta}\bar{R}_{\dot\alpha\dot\beta\dot\gamma\dot\delta})^2$$

- However, the light-cone superfield amplitude is non-local in transverse directions: supergraphs cannot have such log divergence: NO 3-LOOP UV DIVERGENCE!

$$\mathcal{A}_4^{lc})_{UV}^{3loop} = \kappa^4 \ln \Lambda \left(\sum_{l=1}^{l=4} \frac{p_{\perp l}}{(p_l^+)^{1/2}} \eta_l\right)^8 \left(\frac{\bar{p}_{\perp 3} p_4^+ - \bar{p}_{\perp 4} p_3^+}{p_{\perp 1} p_2^+ - p_{\perp 2} p_1^+}\right)^4 \left(\frac{p_1^+ p_2^+}{p_3^+ p_4^+}\right)^2$$

4-Point Amplitude is Divergence-Free Till 7 loop

- The $\mathcal{P}$-factor in the 4-point superfield amplitude has dimension -4 and helicity +2 at each point. The first expression which has no inverse dependence on transverse momenta starts at the 7 loop order

$$\mathcal{P}^0_{7loop}(++++) \sim \kappa^{14} stu(s^4+t^4+u^4)\mathcal{P}^0_{tree}(++++) \sim \kappa^{12}(s^4+t^4+u^4)\frac{[ij]^4}{\langle i'j'\rangle^4}$$

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- We may perform an analogous analysis for the n-point amplitude

n-Point Amplitude is Divergence-Free Till $\frac{n(n-1)+2}{2}$ loop

- The famous Berends, Giele, Kuijf explicit formula (1988) for n-point MHV tree amplitudes is

$$\mathcal{P}^0_{tree}(1^+, 2^+, 3^+, 4^+, ..., n^+) = \frac{[12][n-2\, n-1]}{\langle 1 n-1\rangle N(n)} \times f(p)\ .$$

Here $N(n) = \prod_{i=1}^{i=n-1} \prod_{j=i+1}^{n} \langle jk\rangle$ and $f(p)$ is some polynomial function of momenta and angular brackets.

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- Using a procedure analogous to 4-point amplitude we infer that to remove the non-locality we have to multiply the tree amplitude on a dimensional combination: $\kappa^{n(n-1)+2}[1n-1]\rangle N^*(n)$. This means that the n-point MHV amplitude is predicted to be divergence-free before loop order $\frac{n(n-1)+2}{2}$.

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- This means that the proliferation of the singularities between the n-point amplitude and the $n-1$-point amplitude is $[n(n-1)-2] - [(n-1)(n-2)-2] = 2(n-1)$. the n-point generic amplitude is predicted to be divergence-free before loop order $\frac{n(n-1)+2}{2}$.

n-Point Amplitude With New Helicity Structures Comparative to Tree Amplitudes

- In loop amplitudes some new helicity structures are possible, which are not necessarily proportional to tree amplitudes. Consider a case when at some loop order L a certain n-point amplitude can be given in the form of a local expression, without non-local factors, and therefore may serve as a candidate for a divergence.

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- Now we may try to consider at the same loop order the amplitude with $m+n$ external legs. Since we are not changing the loop level, we have to keep the dimension of the amplitude without change, however, we have to increase the helicity of the amplitude by a factor of $+2$ at each of the n new legs.

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- The difference with a simpler case when the loop amplitudes repeat the helicity structure of the tree amplitudes is the following. When $M^{Lloop} \sim M^{tree} f(s_{ij})$ we had explicit information on the properties of the amplitudes, like the delay of divergence to the $L_{cr} = \frac{n(n-1)+2}{2}$. In more general case we do not have explicit formula, however, we only need the fact that the delay of divergences increases with the number of legs.

$E_{7(7)}$ symmetry : $n \to \infty$, $\mathcal{N}$=8 SG is Divergence-Free Till $L \to \infty$ loop order

- The coset part of the $E_{7(7)}$ symmetry is non-linear in superfields. Symbolically,

$$\delta_{E_{7(7)}} \phi(x, \theta) = \sum_{n=0}^{\infty} (f^n_{abcd} \Sigma^{abcd} + f_n^{abcd} \bar{\Sigma}_{abcd}) \phi^n(x, \theta)$$

Here f^n_{abcd} and f_n^{abcd} depend on θ and $\frac{\partial}{\partial \theta}$ and ∂_+ and ∂_+^{-1}. Σ^{abcd} and $\bar{\Sigma}_{abcd}$ are the 70 parameters of the $\frac{E_{7(7)}}{SU(8)}$ coset transformations. Together with 63 linearly realized $SU(8)$ transformations they form the 133-parameter $E_{7(7)}$ symmetry. Therefore it relates the m-point amplitudes with $m + n$-point amplitudes at a given loop order, $n \to \infty$.

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- If $E_{7(7)}$ symmetry is anomaly-free, the delay of divergences for the $m+n$-point superfield amplitudes at a given loop order, for $n \to \infty$ pushes the infinities out of any finite loop order.

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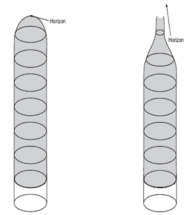
- There is a long list of "things to do" to validate this prediction. Before the set of arguments above can be fully trusted, one should critically analyze each argument and look for examples/counterexamples via specific computations. This will help to rule out or improve and confirm each argument one by one.

Credit for the slide to A. Marrani

Black Holes and Attractor Mechanism

First discovered in d=4 N=2 SG: stabilization of scalars in terms of charges at the horizon of extremal black holes (Ferrara, RK, Strominger)



Recent Renaissance, due to the discovery of non-supersymmetric classes of attractors (Trivedi et al, RK, Sen,…)

Effective BH potential formalism (Ferrara, Gibbons, RK) and entropy function formalism (Sen)

Charge Orbits of U-duality and Moduli Spaces of Attractors (Ferrara, Marrani, Trigiante, Andrianopoli, D'Auria, et al)

d=4 scalar flows and d=3 geodesic motion: a way to complete integration of flows (Gaiotto, Bergshoeff, Trigiante, Nicolai, Stelle, et al)

Multi-center attractors and split attractor flows (Bates, Denef, Gaiotto, RK, …), First Order Formalism (Ceresole, Dall'Agata, Andrianopoli, D'Auria, Orazi, Trigiante, Perz, Smyth, Van Riet, Vercnocke,…)

Key role of N=8, d=4 SG (Ceresole,Ferrara,Gnecchi,Marrani, forthcoming…)

$E_{(7,7)}(\mathbb{R})$ or $E_{(7,7)}(\mathbb{Z})$??? $\rightarrow$ $E_{(7,7)}(\mathbb{R})$ works for QFT and $E_{(7,7)}(\mathbb{Z})$ works the background

- The assumption/argument that $\mathcal{N}$=8 SG is perturbatively finite leads to a number of puzzles with regard to the U-duality of string theory. The studies of the QFT amplitudes near the Minkowski space suggest that $E_{(7,7)}(\mathbb{R})$ symmetry of the classical $\mathcal{N}$=8 SG may be unbroken in the perturbative QFT computations. Meanwhile, the black hole charges are quantized and they form a fundamental representation 56 of $E_{(7,7)}(\mathbb{Z})$.

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- Any of the exact non-perturbative non-linear solutions can be considered as a background in which we compute quantum corrections. Actually, all divergences can be represented as local expression depending on the background field, e.g. in d=4 pure gravity the 2-loop UV counterterm is $\frac{\kappa^2}{\epsilon(4\pi)^4}\frac{209}{2880}\int d^4x R_{\mu\nu}{}^{\lambda\delta} R_{\lambda\delta}{}^{\eta\xi} R_{\eta\xi}{}^{\mu\nu}$

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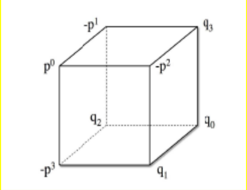
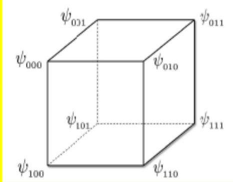
- The absence of UV divergences (which in our analysis requires the unbroken $E_{(7,7)}(\mathbb{R})$ symmetry) does not seem to be affected by the properties of the black hole background, which breaks $E_{(7,7)}(\mathbb{R})$ down to $E_{(7,7)}(\mathbb{Z})$. This is reminiscent of the QFT non-Abelian gauge theories which have the same UV properties independently of the fact that the gauge symmetry may be broken spontaneously

Black Holes and Quantum Information Theory (QIT)

❖ M. J. Duff : **the entropy of the *stu* BH can be expressed through the *Cayley's hyperdeterminant*** $\mathrm{Det}\,\psi$



$$S = \pi\sqrt{|Det\psi|}$$



N=2, d=4 sugra : Electric and magnetic charges of **stu** BH

QIT : 3-tangle of a 3-qubit **[A-B-C systen**

$$Det\psi = (p\cdot q)^2 - 4\left(p^1 q_1 p^2 q_2 + p^1 q_1 p^3 q_3 + p^2 q_2 p^3 q_3\right) + 4p^0 q_1 q_2 q_3 - 4q_0 p^1 p^2 p^3$$

❖ analysis / classification of 2- and 3-qubit systems, relation with *"large"* and *"small"* BHs in **N=2, d=4 sugra**, relevance of twistors, octonions, and relation of Cartan's E_7(7) invariant to Cayley's hyperdeterminant (RK, Linde)

(Levay) **Attractor Mechanism** ⟷ **Optimal local distillation protocol**

❖ BHs in **N=8, d=4 sugra**, entanglement of 7 qu-bits and Fano Plane, relations to Cartan's E_7(7) invariant (Duff, Ferrara)

❖ D3-brane stringy interpretation, novel interpretation of the octonionic grading of the E_7(7) invariant, Freudenthal Triple Systems, QIT and "small" orbits of BHs… **Phys. Rept.** (Borsten, Dahanayake, Duff, Ebrahim, Rubens)