Gravitational aspects of String Theory
(Effective Supergravity, compactification/KK reduction;
black holes, p-branes as gravitational objects, etc.) ⟶
non-perturbative gravity, cosmology .....

AdS/CFT correspondence ⇕ vs.

Quantum field theory aspects of String Theory
(spectrum of quantised strings on orbifolds/orientifolds,
w/D- branes & D-Instantons) ⟶ particle physics implications

[Recent review:R.Blumenhagen, M.C., S.Kachru,T.Weigand,
``D-brane Instantons in Type II String Theory,'' arXiv:0902.3251 [hep-th] &
work in progress w/I.Garcia-Etxebarria,R.Richter]

Charged Black Holes in Gauged Supergravity: Old and New

To honor one of many important contributions Mike Duff has made in the field of Supergravity and String Theory!

Actually, related to one paper (out of two) we co-authored (with 8 other co-authors!):

M. C., M.J. Duff, P. Hoxha , J. T. Liu , H.Lu , J.X. Lu, R. Martinez-Acosta, C.N. Pope, H. Sati, T. A. Tran, ”Embedding AdS black holes in ten-dimensions and eleven-dimensions”, Nucl.Phys.B558:96-126,1999, hep-th/9903214

Charged Black Holes in (Un-)Gauged Supergravities:

I. Old: Overview of black holes in (maximally) supersymmetric supergravities:

a) Abelian U(1) charged solutions in D=4 and D=5

b) Asymptotically Minkowski (ungauged SG) & anti-deSitter space-time (gauged SG)
(no Taub-NUT, c.f. Chen,Lu,Pope'06 ; no asymptotically lense-spaces, c.f. Lu, Mei,Pope'08.; no rings, Emparan,Reall '01)

c) General spinning solutions: [(D-1)/2]-angular momenta

d) Supersymmetric (BPS) and other Extreme black hole limits

(Incomplete: apologies to experts in the audience)

II. New:

a) Novel microscopic interpretation - ``Kerr/CFT correspondence''
Guica,Hatman,Song,Strominger, arXiv:0809.4266 . . .
for entropy of extreme charged spinning black holes in D-dimensions
D.D.K. Chow, M.C., H.Lu & C.N.Pope,arXiv:0812.2918

b) Non-Abelian Black Holes in D=5 maximally supersymmetric supergravity
M.C.,H.Lu&C.N.Pope, UPR-1206-T (to appear)

Charged black holes in D=4 ungauged supergravities

Prototype black hole solutions of N=4 (N=8) supersymmetric ungauged SG in D=4 can be obtained as a toroidal reduction of Heterotic String (Type IIA String) on $T^{(10\text{-}D)}$ (D=4).
In the former case D=4, N=4 SG, w/ global symmetry O(6,22) x SL(2,R).

The relevant subsector can also viewed as D=4 N=2 SG coupled to three vector super-multiplets.

The four-dimensional Lagrangian for the bosonic sector of the N = 2 (ungauged) supergravity coupled to three vector multiplets

$$\begin{aligned}\mathcal{L}_4 &= R*\mathbb{1} - \tfrac{1}{2}{*d\varphi_i}\wedge d\varphi_i - \tfrac{1}{2}e^{2\varphi_i}\,{*d\chi_i}\wedge d\chi_i - \tfrac{1}{2}e^{-\varphi_1}\Big(e^{\varphi_2-\varphi_3}\,{*\hat F_{(2)1}}\wedge \hat F_{(2)1}\\ &\quad + e^{\varphi_2+\varphi_3}\,{*\hat F_{(2)2}}\wedge \hat F_{(2)2} + e^{-\varphi_2+\varphi_3}\,{*\hat{\mathcal{F}}^1_{(2)}}\wedge \hat{\mathcal{F}}^1_{(2)} + e^{-\varphi_2-\varphi_3}\,{*\hat{\mathcal{F}}^2_{(2)}}\wedge \hat{\mathcal{F}}^2_{(2)}\Big)\\ &\quad -\chi_1\,(\hat F_{(2)1}\wedge \hat{\mathcal{F}}^1_{(2)} + \hat F_{(2)2}\wedge \hat{\mathcal{F}}^2_{(2)})\,,\end{aligned}$$

dilatons φ_i and axions χ_i ranges over $1 \le i \le 3$.

4-U(:1)

$$\begin{aligned}\hat F_{(2)1} &= d\hat A_{(1)1} - \chi_2\, d\hat{\mathcal{A}}^2_{(1)}\,,\\ \hat F_{(2)2} &= d\hat A_{(1)2} + \chi_2\, d\hat{\mathcal{A}}^1_{(1)} - \chi_3\, d\hat A_{(1)1} + \chi_2\,\chi_3\, d\hat{\mathcal{A}}^2_{(1)}\\ \hat{\mathcal{F}}^1_{(2)} &= d\hat{\mathcal{A}}^1_{(1)} + \chi_3\, d\hat{\mathcal{A}}^2_{(1)}\,,\\ \hat{\mathcal{F}}^2_{(2)} &= d\hat{\mathcal{A}}^2_{(1)}\,.\end{aligned}$$

The four-dimensional Lagrangian itself has an O(2, 2) ~ SL(2,R) × SL(2,R) global symmetry, which enlarges at the level of the equations of motion to include SL(2,R) factor when electric/magnetic S-duality transformations are included.

w/Youm hep-th/9507090

Static Charged Black Holes in D=4 (toroidally compactified N =1 SG):

Metric Ansatz: $g_{\mu\nu}dx^\mu dx^\nu = \lambda dt^2 - \lambda^{-1}dr^2 - R(d\theta^2 + \sin^2\theta d\phi^2)$

Gauge fields: $\mathcal{F}^i_{tr} = \frac{e^\phi}{R}[M_{ij}\tilde{Q}_j + \Psi(ML)_{ij}P_j], \quad \mathcal{F}^i_{\theta\phi} = P_i \sin\theta,$

Supersymmetric (BPS) solution ($\delta\hat{\psi}_\mu$ = $\delta\lambda$= $\delta\chi^I$ = 0) ; U(1)^4:

$$\lambda = r^2/[(r-\eta_P P_1^{(1)})(r-\eta_P P_1^{(2)})(r-\eta_Q Q_2^{(1)})(r-\eta_Q Q_2^{(2)})]^{\frac{1}{2}},$$

$$R = [(r-\eta_P P_1^{(1)})(r-\eta_P P_1^{(2)})(r-\eta_Q Q_2^{(1)})(r-\eta_Q Q_2^{(2)})]^{\frac{1}{2}},$$

$$e^\phi = \left[\frac{(r-\eta_P P_1^{(1)})(r-\eta_P P_1^{(2)})}{(r-\eta_Q Q_2^{(1)})(r-\eta_Q Q_2^{(2)})}\right]^{\frac{1}{2}},$$

$$g_{11} = \left(\frac{r-\eta_P P_1^{(2)}}{r-\eta_P P_1^{(1)}}\right), \; g_{22} = \left(\frac{r-\eta_Q Q_2^{(1)}}{r-\eta_Q Q_2^{(2)}}\right), \; g_{mm} = 1 \quad (m \neq 1,2)$$

$$\eta_Q \mathrm{sign}(Q_2^{(1)} + Q_2^{(2)}) = -1 \qquad \eta_P \mathrm{sign}(P_1^{(1)} + P_1^{(2)}) = -1$$

ADM Mass: $$M_{BPS} = |P_1^{(1)} + P_1^{(2)}| + |Q_2^{(1)} + Q_2^{(2)}|$$

Entropy Finite: $$S = \pi\sqrt{|P_1^{(1)} P_1^{(2)} Q_2^{(1)} Q_2^{(2)}|}$$

Temperature: $T_H = 0$

Special cases:

All four charges equal: Extreme (BPS) Reissner-Nordstrőm BH (As BPS solution of SG first shown by Gibbons&Hull'84)

Pairwise equal charges: Dyonic black holes of Kallosh et al.'93

One (a=√3),, two (a=1),three (a=1/√3) equal non-zero charges: One charge dilatonic black holes (c.f.Gibbons& Wiltshire'90,Townsend'96; Duff&Rahmfeld96..)

Special properties of BPS black-holes:

a) Entropy INDEPENDENT of the values of scalar fields – explicitly first demonstrated w/Tseytlin hep-th/9512031

(in a general context: attractor mechanism of Ferrara&Kalosh'06)

b) Entropy expressed in terms of combination of couplings invariant under the non-compact symmetry of the theory $\longrightarrow$
for toroidally compactified heterortic string: O(6,22)x Sl(2,R):

S= $\pi\left[\frac{1}{2}\mathcal{L}_{ac}\mathcal{L}_{bd}(\vec{v}^{aT}L\vec{v}^{b})(\vec{v}^{cT}L\vec{v}^{d})\right]^{\frac{1}{2}}$ w/Tseytlin hep-th/9512031

for toroidally compactified Type II string (M-theory) E7 invariant Kol & Kalosh'96

(generalizations to other dims. Ferrara&Kallosh'06, w/Hull'96)

c) Use the generators of the non-compact symmetry to populate all the charges from a GENERATING Solution. Four charge BH is not enough $\longrightarrow$

w/Tseytlin hep-th/9512031

Five charge BPS generating solution:

$$\lambda = \frac{r^2}{\left[(r+Q_1)(r+Q_2)(r+P_1)(r+P_2) - q^2[r+\frac{1}{2}(P_1+P_2)]^2\right]^{\frac{1}{2}}}$$

$$e^{2\Phi'} = \frac{(r+P_1)(r+P_2)}{\left[(r+Q_1)(r+Q_2)(r+P_1)(r+P_2) - q^2[r+\frac{1}{2}(P_1+P_2)]^2\right]^{\frac{1}{2}}}$$

$$\Psi = \frac{q(P_2-P_1)}{2(r+P_1)(r+P_2)} \ ,$$

$$G_{11} = \frac{r+P_2}{r+P_1}, \quad G_{22} = \frac{r+Q_1}{r+Q_2}, \quad G_{12} = -B_{12} = \frac{q[r+\frac{1}{2}(P_1+P_2)]}{(r+Q_2)(r+P_1)}$$

$$S = \pi\left[(Q_1Q_2 - q^2)P_1P_2 - \frac{1}{4}q^2(P_1-P_2)^2\right]^{\frac{1}{2}}$$

General Multi-Charged Spinning Black Holes (of D=4 SG) →
Solution generating technique (c.f. Gibbons, Sen)

a) Starting with an uncharged four-dimensional solution that has a timelike Killing vector $\partial/\partial t$; Kerr black hole solution

b) Reducing it to three dimensions on t direction.
Yields a three-dimensional theory with an addiitional $O(4, 4)$ global symmetry

c) By acting with an $O(1, 1)^4$ subgroup of $O(4, 4)$ on the dimensionally reduced solution → generate new solutions involving four parameters δ_i characterising $O(1, 1)^4$.

d) Upon lifting back to $D = 4$, we there by arrive at spinning solutions carrying 4 electromagnetic charges, parameterised by the δ_i.

w/Youm hep-th/9603147

also: w/Lu& Pope: hep-th/0411045

Starting point, D=4 Kerr black hole:

$$ds^2 = -\frac{r^2 + l^2\cos^2\theta - 2mr}{r^2 + l^2\cos^2\theta}dt^2 + \frac{r^2 + l^2\cos^2\theta}{r^2 + l^2 - 2mr}dr^2 + (r^2 + l^2\cos^2\theta)d\theta^2$$
$$+ \frac{\sin^2\theta}{r^2 + l^2\cos^2\theta}[(r^2 + l^2)(r^2 + l^2\cos^2\theta) + 2ml^2r\sin^2\theta]d\phi^2 - \frac{4mlr\sin^2\theta}{r^2 + l^2\cos^2\theta}dtd\phi$$

m-mass, l-angular momentum

Impose 4 O(1,1) boosts on D=3 solution, lift back to D=4:

Four charge spinning black hole:

$$g_{11} = \frac{(r + 2m\sinh^2\delta_{p2})(r + 2m\sinh^2\delta_{e2}) + l^2\cos^2\theta}{(r + 2m\sinh^2\delta_{p1})(r + 2m\sinh^2\delta_{e2}) + l^2\cos^2\theta},$$

$$g_{12} = \frac{2ml\cos\theta(\sinh\delta_{p1}\cosh\delta_{p2}\sinh\delta_{qe1}\cosh\delta_{e2} - \cosh\delta_{p1}\sinh\delta_{p2}\cosh\delta_{e1}\sinh\delta_{e2})}{(r + 2m\sinh^2\delta_{p1})(r + 2m\sinh^2\delta_{e2}) + l^2\cos^2\theta},$$

$$g_{22} = \frac{(r + 2m\sinh^2\delta_{p1})(r + 2m\sinh^2\delta_{e1}) + l^2\cos^2\theta}{(r + 2m\sinh^2\delta_{p1})(r + 2m\sinh^2\delta_{e2}) + l^2\cos^2\theta},$$

$$B_{12} = -\frac{2ml\cos\theta(\sinh\delta_{p1}\cosh\delta_{p2}\cosh\delta_{e1}\sinh\delta_{e2} - \cosh\delta_{p1}\sinh\delta_{p2}\sinh\delta_{e1}\cosh\delta_{e2})}{(r + 2m\sinh^2\delta_{p1})(r + 2m\sinh^2\delta_{e2}) + l^2\cos^2\theta},$$

$$e^{\varphi} = \frac{(r + 2m\sinh^2\delta_{p1})(r + 2m\sinh^2\delta_{p2}) + l^2\cos^2\theta}{\Delta^{\frac{1}{2}}},$$

$$\begin{aligned} ds_E^2 = {} & \Delta^{\frac{1}{2}}[-\frac{r^2 - 2mr + l^2\cos^2\theta}{\Delta}dt^2 + \frac{dr^2}{r^2 - 2mr + l^2} + d\theta^2 + \frac{\sin^2\theta}{\Delta}\{(r + 2m\sinh^2\delta_{p1} \\ & \times (r + 2m\sinh^2\delta_{p2})(r + 2m\sinh^2\delta_{e1})(r + 2m\sinh^2\delta_{e2}) + l^2(1 + \cos^2\theta)r^2 + W \\ & + 2ml^2r\sin^2\theta\}d\phi^2 - \frac{4ml}{\Delta}\{(\cosh\delta_{p1}\cosh\delta_{p2}\cosh\delta_{e1}\cosh\delta_{e2} \\ & - \sinh\delta_{p1}\sinh\delta_{p2}\sinh\delta_{e1}\sinh\delta_{e2})r + 2m\sinh\delta_{p1}\sinh\delta_{p2}\sinh\delta_{e1}\sinh\delta_{e2}\}\sin^2\theta dt d\phi], \end{aligned}$$

D=4 four-charge spinning black hole - continued

$$\begin{aligned}
\Delta &\equiv (r+2m\sinh^2\delta_{p1})(r+2m\sinh^2\delta_{p2})(r+2m\sinh^2\delta_{e1})(r+2m\sinh^2\delta_{e2}) \\
&+ (2l^2r^2+W)\cos^2\theta, \\
W &\equiv 2ml^2(\sinh^2\delta_{p1}+\sinh^2\delta_{p2}+\sinh^2\delta_{e1}+\sinh^2\delta_{e2})r \\
&+ 4m^2l^2(2\cosh\delta_{p1}\cosh\delta_{p2}\cosh\delta_{e1}\cosh\delta_{e2}\sinh\delta_{p1}\sinh\delta_{p2}\sinh\delta_{e1}\sinh\delta_{e2} \\
&- 2\sinh^2\delta_{p1}\sinh^2\delta_{p2}\sinh^2\delta_{e1}\sinh^2\delta_{e2} - \sinh^2\delta_{p2}\sinh^2\delta_{e1}\sinh^2\delta_{e2} \\
&- \sinh^2\delta_{p1}\sinh^2\delta_{e1}\sinh^2\delta_{e2} - \sinh^2\delta_{p1}\sinh^2\delta_{p2}\sinh^2\delta_{e2} - \sinh^2\delta_{p1}\sinh^2\delta_{p2}\sinh^2\delta_{e1}) \\
&+ l^4\cos^2\theta.
\end{aligned}$$

Charges and angular momentum:

$$\begin{aligned}
Q_2^{(1)} &= 4m\cosh\delta_{e1}\sinh\delta_{e1}, \qquad Q_2^{(2)} = 4m\cosh\delta_{e2}\sinh\delta_{e2}, \\
P_1^{(1)} &= 4m\cosh\delta_{p1}\sinh\delta_{p1}, \qquad P_1^{(2)} = 4m\cosh\delta_{p2}\sinh\delta_{p2}, \\
J &= 8lm(\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_{p1}\cosh\delta_{p2} - \sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_{p1}\sinh\delta_{p2})
\end{aligned}$$

ADM mass: $M = 4m(\cosh^2\delta_{e1}+\cosh^2\delta_{e2}+\cosh^2\delta_{p1}+\cosh^2\delta_{p2}) - 8m$

$G_N = 1/8$

Entropy: S= $= 16\pi[m^2(\prod_{i=1}^4\cosh\delta_i + \prod_{i=1}^4\sinh\delta_i) + \{m^4(\prod_{i=1}^4\cosh\delta_i - \prod_{i=1}^4\sinh\delta_i)^2 - J^2\}^{1/2}]$

Suggestive of microscopic structure !

Extreme (BPS) limit: $m \to 0$, $\delta_i \to \infty$, $J \to 0$

Entropy: S = $2\pi[P_1^{(1)}P_1^{(2)}Q_2^{(1)}Q_2^{(2)}]^{\frac{1}{2}}$

Other Extreme limits when: m=l

D=5 black holes in ungauged supergravities

Prototype black holes of N=4 (N=8) supersymmetric ungauged SG in D=5 can be obtained as a toroidal reduction of Heterotic String (Type IIA String) on $T^{(10\text{-}D)}$ (D=5).
Former D=5, N=4 SG, w/ global symmetry O(5,21) xO(1,1).

The relevant subsector can also viewed as D=5 N=2 SG coupled to vector super-multiplets:

$$e^{-1}\,\mathcal{L} = R - \tfrac{1}{2}\partial\vec{\varphi}^2 - \tfrac{1}{4}\sum_{i=1}^{3} X_i^{-2}\,(F^i)^2 + 4g^2\sum_{i=1}^{3} X_i^{-1} + \tfrac{1}{24}|\epsilon_{ijk}|\,\epsilon^{\mu\nu\rho\sigma\lambda}F^i_{\mu\nu}\,F^j_{\rho\sigma}\,A^k_\lambda\,,$$

$$X_1 = e^{-\frac{1}{\sqrt{6}}\varphi_1 - \frac{1}{\sqrt{2}}\varphi_2}\,,\qquad X_2 = e^{-\frac{1}{\sqrt{6}}\varphi_1 + \frac{1}{\sqrt{2}}\varphi_2}\,,\qquad X_3 = e^{\frac{2}{\sqrt{6}}\varphi_1}\,.$$

two scalar fields; three U(1)

Employing generating technique:

a) Reduce D=5 stationary solution-Kerr BH with two angular momenta
to D=3 on t and one angular direction

b) D=3 Largrangian has O(3,3) symmetry

c) Acting with an $O(1, 1)^3$ subgroup of O(3, 3) transformations on t
the dimensionally reduced solution to generate
generate new solutions with three parameters δ_i .

d) Upon lifting back to D = 5, arrive at spinning solutions
with two angular momenta & three charges parameterised by the
three δ_i.

w/Youm hep-th/9603100

D=5 Kerr Solution:

$$
\begin{aligned}
ds^2 = &-\frac{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta-2m}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}dt^2+\frac{r^2(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)}{(r^2+l_1^2)(r^2+l_2^2)-2mr^2}dr^2\\
&+(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)d\theta^2+\frac{4ml_1l_2\sin^2\theta\cos^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}d\phi d\psi\\
&+\frac{\sin^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}[(r^2+l_1^2)(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)+2ml_1^2\sin^2\theta]d\phi^2\\
&+\frac{\cos^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}[(r^2+l_2^2)(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)+2ml_2^2\cos^2\theta]d\psi^2\\
&-\frac{4ml_1\sin^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}dtd\phi-\frac{4ml_2\cos^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}dtd\psi.
\end{aligned}
$$

m-mass; l_{12}=two angular momenta

Myers&Perry'86

Scalar and gauge fields:

$$g_{11} = \frac{r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta}{r^2 + 2m\sinh^2\delta_{e2} + l_1^2\cos^2\theta + l_2^2\sin^2\theta},$$

$$e^{2\varphi} = \frac{(r^2 + 2m\sinh^2\delta_e + l_1^2\cos^2\theta + l_2^2\sin^2\theta)^2}{(r^2 + 2m\sinh^2\delta_{e2} + l_1^2\cos^2\theta + l_2^2\sin^2\theta)(r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta)},$$

$$A^{(1)}_{t1} = \frac{m\cosh\delta_{e1}\sinh\delta_{e1}}{r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta},$$

$$A^{(1)}_{\phi 1} = m\sin^2\theta\frac{l_1\sinh\delta_{e1}\sinh\delta_{e2}\cosh\delta_e - l_2\cosh\delta_{e1}\cosh\delta_{e2}\sinh\delta_e}{r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta},$$

$$A^{(1)}_{\psi 1} = m\cos^2\theta\frac{l_1\cosh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e - l_2\sinh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e}{r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta},$$

$$A^{(2)}_{t1} = \frac{m\cosh\delta_{e2}\sinh\delta_{e2}}{r^2 + 2m\sinh^2\delta_{e2} + l_1^2\cos^2\theta + l_2^2\sin^2\theta},$$

$$A^{(2)}_{\phi 1} = m\sin^2\theta\frac{l_1\cosh\delta_{e1}\sinh\delta_{e2}\cosh\delta_e - l_2\sinh\delta_{e1}\cosh\delta_{e2}\sinh\delta_e}{r^2 + 2m\sinh^2\delta_{e2} + l_1^2\cos^2\theta + l_2^2\sin^2\theta},$$

$$A^{(2)}_{\psi 1} = m\cos^2\theta\frac{l_1\sinh\delta_{e1}\cosh\delta_{e2}\sinh\delta_e - l_2\cosh\delta_{e1}\sinh\delta_{e2}\cosh\delta_e}{r^2 + 2m\sinh^2\delta_{e2} + l_1^2\cos^2\theta + l_2^2\sin^2\theta},$$

$$\begin{aligned}
B_{t\phi} = & -2m\sin^2\theta(l_1\sinh\delta_{e1}\sinh\delta_{e2}\cosh\delta_e - l_2\cosh\delta_{e1}\cosh\delta_{e2}\sinh\delta_e)(r^2 + l_1^2\cos^2\theta \\
& + l_2^2\sin^2\theta + m\sinh^2\delta_{e1} + m\sinh^2\delta_{e2})/[(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e1}) \\
& \times(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e2})],
\end{aligned}$$

$$\begin{aligned}
B_{t\psi} = & -2m\cos^2\theta(l_2\sinh\delta_{e1}\sinh\delta_{e2}\cosh\delta_e - l_1\cosh\delta_{e1}\cosh\delta_{e2}\sinh\delta_e)(r^2 + l_1^2\cos^2\theta \\
& + l_2^2\sin^2\theta + m\sinh^2\delta_{e1} + m\sinh^2\delta_{e2})/[(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e1}) \\
& \times(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e2})],
\end{aligned}$$

$$B_{\phi\psi} = \frac{2m\cosh\delta_e\sinh\delta_e\cos^2\theta\sin^2\theta(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + m\sinh^2\delta_{e1} + m\sinh^2\delta_{e2})}{(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e1})(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e2})},$$

Metric:

$$
\begin{aligned}
ds_E^2 = \bar{\Delta}^{\frac{1}{3}} \Bigg[& -\frac{(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta)(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta - 2m)}{\bar{\Delta}} dt^2 \\
& + \frac{r^2}{(r^2 + l_1^2)(r^2 + l_2^2) - 2mr^2} dr^2 + d\theta^2 + \frac{4m\cos^2\theta\sin^2\theta}{\bar{\Delta}} [l_1 l_2 \{(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta) \\
& - 2m(\sinh^2\delta_{e1}\sinh^2\delta_{e2} + \sinh^2\delta_e\sinh^2\delta_{e1} + \sinh^2\delta_e\sinh^2\delta_{e2})\} + 2m\{(l_1^2 + l_2^2) \\
& \times \cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e - 2l_1 l_2\sinh^2\delta_{e1}\sinh^2\delta_{e2}\sinh^2\delta_e\}] d\phi d\psi \\
& - \frac{4m\sin^2\theta}{\bar{\Delta}} [(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta)(l_1\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e - l_2\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e) \\
& + 2ml_2\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e] d\phi dt - \frac{4m\cos^2\theta}{\bar{\Delta}} [(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta) \\
& \times (l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e - l_1\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e) + 2ml_1\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e] d\psi dt \\
& + \frac{\sin^2\theta}{\bar{\Delta}} [(r^2 + 2m\sinh^2\delta_e + l_1^2)(r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta)(r^2 + 2m\sinh^2\delta_{e2} \\
& + l_1^2\cos^2\theta + l_2^2\sin^2\theta) + 2m\sin^2\theta\{(l_1^2\cosh^2\delta_m - l_2^2\sinh^2\delta_m)(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta) \\
& + 4ml_1 l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e - 2m\sinh^2\delta_{e1}\sinh^2\delta_{e2} \\
& \times (l_1^2\cosh^2\delta_e + l_2^2\sinh^2\delta_e) - 2ml_2^2\sinh^2\delta_e(\sinh^2\delta_{e1} + \sinh^2\delta_{e2})\}] d\phi^2 \\
& + \frac{\cos^2\theta}{\bar{\Delta}} [(r^2 + 2m\sinh^2\delta_e + l_2^2)(r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta)(r^2 + 2m\sinh^2\delta_{e2} \\
& + l_1^2\cos^2\theta + l_2^2\sin^2\theta) + 2m\cos^2\theta\{(l_2^2\cosh^2\delta_e - l_1^2\sinh^2\delta_e)(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta) \\
& + 4ml_1 l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e - 2m\sinh^2\delta_{e1}\sinh^2\delta_{e2} \\
& \times (l_1^2\sinh^2\delta_e + l_2^2\cosh^2\delta_e) - 2ml_1^2\sinh^2\delta_e(\sinh^2\delta_{e1} + \sinh^2\delta_{e2})\}] d\psi^2 \Bigg],
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{\Delta} \equiv & (r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta)(r^2 + 2m\sinh^2\delta_{e2} + l_1^2\cos^2\theta + l_2^2\sin^2\theta) \\
& \times (r^2 + 2m\sinh^2\delta_e + l_1^2\cos^2\theta + l_2^2\sin^2\theta),
\end{aligned}
$$

Somewhat more compact form:

w/Chong, Lu & Pope: hep-th/06006213

with the scalars and gauge potentials given by

$$H_i = 1 + \frac{2ms_i^2}{x+y}\,,$$

$$\begin{aligned} X_i &= H_i^{-1}\,(H_1H_2H_3)^{1/3}\,, \\ A^1 &= \frac{2m}{x+y}\,H_1^{-1}\,\{s_1c_1dt + s_1c_2c_3[abd\chi + (y-a^2-b^2)d\sigma] + c_1s_2s_3(abd\sigma - yd\chi)\} \end{aligned}$$

Metric:

$$ds_5^2 = (H_1H_2H_3)^{1/3}\,(x+y)\,d\hat{s}_5^2\,, \qquad d\hat{s}_5^2 = -\Phi\,(dt+\mathcal{A})^2 + ds_4^2$$

$$ds_4^2 = \left(\frac{dx^2}{4X} + \frac{dy^2}{4Y}\right) + \frac{U}{G}\left(d\chi - \frac{Z}{U}\,d\sigma\right)^2 + \frac{XY}{U}\,d\sigma^2$$

$$\begin{aligned} Y &= -(a^2-y)(b^2-y)\,, \\ G &= (x+y)(x+y-2m)\,, \\ U &= yX - xY\,, \qquad Z = ab(X+Y)\,, \quad X = (x+a^2)(x+b^2) - 2mx \\ \Phi &= \frac{G}{(x+y)^3\,H_1H_2H_3}\,, \\ \mathcal{A} &= \frac{2mc_1c_2c_3}{G}[(a^2+b^2-y)d\sigma - abd\chi] - \frac{2ms_1s_2s_3}{x+y}\,(abd\sigma - yd\chi) \end{aligned}$$

Solution specified by three charges, mass, two angular momenta:

$$
\begin{aligned}
M &= 2m(\cosh^2\delta_{e1} + \cosh^2\delta_{e2} + \cosh^2\delta_e) - 3m \\
&= \sqrt{m^2 + (Q_1^{(1)})^2} + \sqrt{m^2 + (Q_1^{(2)})^2} + \sqrt{m^2 + Q^2}, \\
J_\phi &= 4m(l_1 \cosh\delta_{e1} \cosh\delta_{e2} \cosh\delta_e - l_2 \sinh\delta_{e1} \sinh\delta_{e2} \sinh\delta_e), \\
J_\psi &= 4m(l_2 \cosh\delta_{e1} \cosh\delta_{e2} \cosh\delta_e - l_1 \sinh\delta_{e1} \sinh\delta_{e2} \sinh\delta_e).
\end{aligned}
$$

Entropy:

$$
\begin{aligned}
S \quad &= 4\pi \Big[\{2m^3(\prod_{i=1}^{3} \cosh\delta_i + \prod_{i=1}^{3} \sinh\delta_i)^2 - \tfrac{1}{16}(J_\phi - J_\psi)^2\}^{1/2} \\
&\quad + \{2m^3(\prod_{i=1}^{3} \cosh\delta_i - \prod_{i=1}^{3} \sinh\delta_i)^2 - \tfrac{1}{16}(J_\phi + J_\psi)^2\}^{1/2} \Big]
\end{aligned}
$$

Extreme (BPS) limit: m → 0, δ_i → ∞ , $J_\Phi = -J_\Psi$ $\quad G_N = 1/4$

$S = \pi \quad [Q_1^{(1)} Q_1^{(2)} Q - \frac{1}{4} J^2]^{\frac{1}{2}}$ Microscopics: Breckenridge et al.'96

Other Extreme limits: $(l_1^2 - l_2^2)^2 + 4m(m - l_1^2 - l_2^2) = 0$

Higher dimensional Embedding of Spinning BH's

Lift to D=6: Spinning Dyonic String

w/Larsen hep-th/9805097

$$
\begin{aligned}
ds_6^2 \;=\; & \frac{1}{\sqrt{H_1H_2}}\Bigg[-(1-\frac{2mf_D}{r^2})d\tilde{t}^2 + d\tilde{y}^2 + H_1H_2 f_D^{-1}\frac{r^4}{(r^2+l_1^2)(r^2+l_2^2)-2mr^2}dr^2 \\
- & \frac{4mf_D}{r^2}\cosh\delta_1\cosh\delta_2(l_2\cos^2\theta d\psi + l_1\sin^2\theta d\phi)d\tilde{t} \\
- & \frac{4mf_D}{r^2}\sinh\delta_1\sinh\delta_2(l_1\cos^2\theta d\psi + l_2\sin^2\theta d\phi)d\tilde{y} \\
+ & \left((1+\frac{l_2^2}{r^2})H_1H_2r^2 + (l_1^2-l_2^2)\cos^2\theta(\frac{2mf_D}{r^2})^2\sinh^2\delta_1\sinh^2\delta_2\right)\cos^2\theta d\psi^2 \\
+ & \left((1+\frac{l_1^2}{r^2})H_1H_2r^2 + (l_2^2-l_1^2)\sin^2\theta(\frac{2mf_D}{r^2})^2\sinh^2\delta_1\sinh^2\delta_2\right)\sin^2\theta d\phi^2 \\
+ & \frac{2mf_D}{r^2}(l_2\cos^2\theta d\psi + l_1\sin^2\theta d\phi)^2 + H_1H_2r^2f_D^{-1}d\theta^2\Bigg] \;,
\end{aligned}
$$

$$
\begin{aligned}
H_i &= 1+\frac{2mf_D\sinh^2\delta_i}{r^2} \quad ; i=1,2 \\
f_D^{-1} &= 1+\frac{l_1^2\cos^2\theta}{r^2}+\frac{l_2^2\sin^2\theta}{r^2} \;,
\end{aligned}
$$

$$
\begin{aligned}
d\tilde{t} &= \cosh\delta_0 dt - \sinh\delta_0 dy \\
d\tilde{y} &= \cosh\delta_0 dy - \sinh\delta_0 dt
\end{aligned}
$$

Near horizon metric- BTZ BH (AdS_3) x S^3

(gravitry/gauge theory duality (AdS/CFT) – identify 2-dim boundary conformal field theory (CFT)

and get the precise microscopic interpretation of

$$S = 4\pi \Big[\{2m^3(\prod_{i=1}^{3}\cosh\delta_i + \prod_{i=1}^{3}\sinh\delta_i)^2 - \tfrac{1}{16}(J_\phi - J_\psi)^2\}^{1/2} + \{2m^3(\prod_{i=1}^{3}\cosh\delta_i - \prod_{i=1}^{3}\sinh\delta_i)^2 - \tfrac{1}{16}(J_\phi + J_\psi)^2\}^{1/2} \Big]$$

For $\delta_{12} \rightarrow \infty$, but the third charge parameter and angular mom. parameters general (dilute gas approximation);

$$S = \pi\sqrt{Q_1 Q_2}\left[\sqrt{2m - (l_1 - l_2)^2}\, e^{\delta_0} + \sqrt{2m - (l_1 + l_2)^2}\, e^{-\delta_0}\right]$$

Further progress: a) higher gravity corrections Cardoso et al.

b) detailed mcroscopics focus on BPS (topological fileld theory Vafa et al.,)

c) generalizations to general N=2 BH solutions (not just toroidally compactified string theory)

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Black holes in asymptotically AdS space-time (gauged SG)

(general background for addressing AdS/CFT correspondence)

Proto-type: BH in D=5

The bosonic sector of the relevant $\mathcal{N}=2$ theory can be derived from the Lagrangian

$$e^{-1}\,\mathcal{L} = R - \tfrac{1}{2}\partial\vec{\varphi}^2 - \tfrac{1}{4}\sum_{i=1}^{3} X_i^{-2}\,(F^i)^2 + 4g^2\sum_{i=1}^{3} X_i^{-1} + \tfrac{1}{24}|\epsilon_{ijk}|\,\epsilon^{\mu\nu\rho\sigma\lambda}F^i_{\mu\nu}\,F^j_{\rho\sigma}\,A^k_\lambda\,, \qquad (1)$$

$$X_1 = e^{-\frac{1}{\sqrt{6}}\varphi_1 - \frac{1}{\sqrt{2}}\varphi_2}\,, \qquad X_2 = e^{-\frac{1}{\sqrt{6}}\varphi_1 + \frac{1}{\sqrt{2}}\varphi_2}\,, \qquad X_3 = e^{\frac{2}{\sqrt{6}}\varphi_1}\,.$$

Potential for scalar fields

Consistent truncation of N=8 gauged supergravity that can be obtained as a consistent compactification of Type IIB string theory on S^5 (five-sphere);
vacuum solution AdS_5.

w/Behrndt &Sabra, hep-th/9810227

Static Solution w./ three charges:

$$ds^2 = -(H_1H_2H_3)^{-2/3} f dt^2 + (H_1H_2H_3)^{1/3}\left(f^{-1}dr^2 + r^2 d\Omega_{3,k}\right) ,$$

$$f = 1 - \frac{\mu}{r^2} + g^2 r^2 H_1H_2H_3 \quad , \quad X^I = H_I^{-1}(H_1H_2H_3)^{1/3} \quad , \quad F^I_{r0} = -\frac{1}{2} \ (H_I)^{-2}\partial_r \tilde{H}_I$$

$$H_I = h_I + \frac{q_I}{r^2} \qquad \tilde{H}_I = 1 + \frac{\tilde{q}_I}{r^2}$$

$$q_I = \tilde{\mu}\sinh^2\beta_I \quad , \quad \tilde{q}_I = \tilde{\mu}\sinh\beta_I\cosh\beta_I \qquad (\tilde{\mu} \equiv \frac{\mu}{k})$$

$\mu=0$ – extreme (BPS) limit (singular) Behrndt,Chamseddine&Sabra hep-th/9807187

Global space-time involved, it depends on the size of g-cosmological constant
analysis for ``large'' BH's with regular horisons $\longrightarrow$ generalization of Hawking-Page
phase transitions for charged AdfS BH's w/Gubser hep-th/990219

[D=4, N=8, four charge AdS black holes: Duff&Liu'99; also Minasian,Liu'98]

Embedding into D=10 Type IIB (D=11 SG) Duff et 9 co-authors'99

General D=5 AdS spinning charged black hole solutions

Important role as AdS/CFT. Prototype:

cosmol. const. ; mass , 3 – charges & 2 ang. mom. ;

D=4 boundary CFT ; scaling dim. , 3 – R charges & spins of dual FT operators;

Prototype solution parameterized with:
mass, 2-angular momenta, three independent charges – not obtained yet!

Problem: for gauged supergravities (typically obtained from sphere compactification of effective string theories) - non-compact symmetries (there for toroidal compactifications) are absent. $\longrightarrow$

No generating techniques to obtain charged solutions from uncharged ones

Only special cases
obtained with incremental progress with two angular momenta:equal
w/Lu&Pope hep-th/0406196
&educated Ansatz, matches onto ungauged examples & known BPS limits

Status of charged AdS BH's w/ two unequal angular momenta :

3-equal charges w/Chong, Lü & Pope hep-th/0506029

2 -equal charges hep-th/0505112

1 charge hep-th/0606213

2 equal charges 3rd unequal Mei&Pope arXiv:0709.0559

General supersymmetric black holes Kunduri, Lucietti, Reall '06

Involved global space-time & thermodynamics: analysis of the global space time, thermodynamics, new BPS BH limits & BPS solitons (regular solutions)

w/ Gary Gibbons, H. Lü and C. Pope, hep-th/0504080

[D=4, with 2 pairwise equal charges; D=7, 2-charges, with 3 equal ang. mom.

w/Z. Chong, H. Lü & C. Pope, hep-th/0411045;0412094

D= 6 , one charge, 2 unequal ang. mom, D. Chow arXiv:0808.2728]

D=5 , 3-equal charge, 2 ang. mom. AdS BH: hep-th/0506029

Metric:
(co-homegeneity 2)

$$ds^2 = -\frac{\Delta_\theta\left[(1+g^2r^2)\rho^2 dt + 2q\nu\right] dt}{\Xi_a\,\Xi_b\,\rho^2} + \frac{2q\,\nu\omega}{\rho^2} + \frac{f}{\rho^4}\left(\frac{\Delta_\theta\, dt}{\Xi_a\Xi_b} - \omega\right)^2 + \frac{\rho^2 dr^2}{\Delta_r} + \frac{\rho^2 d\theta^2}{\Delta_\theta} + \frac{r^2+a^2}{\Xi_a}\sin^2\theta d\phi^2 + \frac{r^2+b^2}{\Xi_b}\cos^2\theta d\psi^2$$

Gauge pot.:

$$A = \frac{\sqrt{3}q}{\rho^2}\left(\frac{\Delta_\theta\, dt}{\Xi_a\,\Xi_b} - \omega\right),$$

$$\nu = b\sin^2\theta d\phi + a\cos^2\theta d\psi\,,$$

$$\omega = a\sin^2\theta\frac{d\phi}{\Xi_a} + b\cos^2\theta\frac{d\psi}{\Xi_b}\,,$$

$$\Delta_\theta = 1 - a^2g^2\cos^2\theta - b^2g^2\sin^2\theta\,,$$

$$\Delta_r = \frac{(r^2+a^2)(r^2+b^2)(1+g^2r^2) + q^2 + 2abq}{r^2} - 2m\,,$$

$$\rho^2 = r^2 + a^2\cos^2\theta + b^2\sin^2\theta\,,$$

$$\Xi_a = 1 - a^2g^2\,,\quad \Xi_b = 1 - b^2g^2\,,$$

$$f = 2m\rho^2 - q^2 + 2abqg^2\rho^2\,.$$

Mass:

$$E = \frac{m\pi(2\Xi_a + 2\Xi_b - \Xi_a\,\Xi_b) + 2\pi qabg^2(\Xi_a + \Xi_b)}{4\Xi_a^2\,\Xi_b^2}$$

3-equal charges:

$$Q = \frac{\sqrt{3}\,\pi q}{4\Xi_a\,\Xi_b}$$

2 ang. mom.:

$$J_a = \frac{\pi[2am + qb(1+a^2g^2)]}{4\Xi_a^2\,\Xi_b}$$

$$J_b = \frac{\pi[2bm + qa(1+b^2g^2)]}{4\Xi_b^2\,\Xi_a}$$

Thermodynamics:

$$S=\frac{\pi^2[(r_+^2+a^2)(r_+^2+b^2)+abq]}{2\Xi_a\Xi_b r_+}$$

$$T_H=\frac{r_+^4[(1+g^2(2r_+^2+a^2+b^2)]-(ab+q)^2}{2\pi\; r_+\,[(r_+^2+a^2)(r_+^2+b^2)+abq]}$$

Supersymmetric (BPS) limit: $E-gJ_a-gJ_b-\sqrt{3}\,Q=0 \quad \Longleftrightarrow \quad q=\frac{m}{1+(a+b)g}$

Regular Solution
(no naked CTC);

$$gm=(a+b)(1+ag)(1+bg)(1+ag+bg)$$

$$\begin{aligned}
E &= \frac{\pi(a+b)}{4g(1-ag)^2(1-bg)^2}\Big((1-ag)(1-bg) \\
&\qquad +(1+ag)(1+bg)(2-ag-bg)\Big)\,, \\
S &= \frac{\pi^2(a+b)\sqrt{a+b+abg}}{2g^{3/2}(1-ag)(1-bg)}\,, \\
J_a &= \frac{\pi(a+b)(2a+b+abg)}{4g(1-ag)^2(1-bg)}\,, \\
J_b &= \frac{\pi(a+b)(a+2b+abg)}{4g(1-ag)(1-bg)^2}\,, \\
Q &= -\frac{\pi\sqrt{3}(a+b)}{4g(1-ag)(1-bg)}\,.
\end{aligned}$$

$T_H=0$

Other extreme limits

Spin off:
(digression)

New Einstrein-Sasaki from Kerr-deSitter

Special case of D=5 neutral rotating AdS BH's
&
Hunter, Hawking & Taylor'98

finding BPS (scaling) limit ($l_1 = l_2$)
w/Gao& Simón hep-th/0504136

⇓

Euclidean regime BPS Kerr AdS BH's

New Einstein-Sasaki spaces $L^{a,b,c}$ ($l_1 \neq l_2$)
local &gobal analysis (regularity), topology /dual FT
w/ Lü, Page & Pope hep-th/0504225; 0505223
(independently Sparks&Martelli hep-th/0505027)

[it generalizes $Y^{p,q}$ ($l_1 = l_2$) Gauntlett,Martelli,Sparks&Waldram'04]

[generalizations to ES in D=2n+1 (n>2) & regular Einstein sp... Chen, Lu &Pope....]

BPS limit of D=5 Kerr-de Sitter BH ($a \neq b$): w/Lü,Page&Pope hep-th/0504225; 0505223

$$\Xi_a \equiv 1-g^2a^2 = \epsilon\alpha,\ \Xi_b \equiv 1-g^2b^2 = \epsilon\beta,\ m = \mu\epsilon^3\ ,\ g^2r^2 = (1-x^2\epsilon)\ ,\ \epsilon \to 0$$

Solution saturates Bogomol'nyi bound: $E - gJ_a - gJ_b = 0 \quad \{E, J_a \neq J_b\}\ -\ \text{finite}$

(¼ supersymmetry)

Euclidean regime $\quad t \to \mathrm{i}\tau\,,\quad g \to \frac{\mathrm{i}}{\sqrt{\lambda}}\,,\quad a \to \mathrm{i}a\,,\quad b \to \mathrm{i}\,\mathrm{b}\,,$

$$\lambda\, ds_5^2 = (d\tau + \sigma)^2 + ds_4^2$$

New Einstein-Sasaki spaces!

Kähler-Einstein

$$\begin{aligned} ds_4^2 &= \frac{\rho^2\,dx^2}{4\Delta_x} + \frac{\rho^2\,d\theta^2}{\Delta_\theta} + \frac{\Delta_x}{\rho^2}\Big(\frac{\sin^2\theta}{\alpha}d\phi + \frac{\cos^2\theta}{\beta}d\psi\Big)^2 \\ &+ \frac{\Delta_\theta\sin^2\theta\cos^2\theta}{\rho^2}\Big(\frac{\alpha-x}{\alpha}d\phi - \frac{\beta-x}{\beta}d\psi\Big)^2\,, \end{aligned}$$

$J = \frac{1}{2}\,d\,\sigma\ -\ \text{Kähler form}$

$$\begin{aligned} \sigma &= \frac{(\alpha-x)\sin^2\theta}{\alpha}d\phi + \frac{(\beta-x)\cos^2\theta}{\beta}d\psi\,, \\ \Delta_x &= x(\alpha-x)(\beta-x)-\mu\,,\quad \rho^2 = \Delta_\theta - x\,, \\ \Delta_\theta &= \alpha\,\cos^2\theta + \beta\,\sin^2\theta\,. \end{aligned}$$

local metric w/different method

Sparks&Martelli,hep-th/0505027

New Developments: Microscopics of extreme spinning black holes (defined by $T_H = 0$) in D-dimensions

The near horizon metric of extreme spinning solutions as a manifold M fibration over AdS_2:

$$ds^2 = A\left(-(1+r^2)d\tau^2 + \frac{dr^2}{1+r^2}\right) + h_{\alpha\beta}\, dy^\alpha\, dy^\beta + \tilde{g}_{ij}\, \tilde{e}^i \tilde{e}^j$$

$$\tilde{e}^i = d\phi_i + k_i r\, d\tau\,,$$

[D-1]/2] commuting U(1) diffeomorphisms:

$$\zeta^i_m = -e^{-\mathrm{i}m\phi_i}\frac{\partial}{\partial\phi_i} - \mathrm{i}m\,r\,e^{-\mathrm{i}m\phi_i}\frac{\partial}{\partial r}\,,$$

that generate {(D-1)/2] l commuting Virasoro algebra (of chargess $1/(8\pi)\int_{\partial\Sigma} k^i_{(n)}$;)
with central charge:s c_i: $\frac{1}{8\pi}\int_{\partial\Sigma} k_{\zeta^i_{(m)}}[\mathcal{L}_{\zeta^i_{(-m)}} g, g] = -\frac{\mathrm{i}}{12}(m^3 + \alpha m)c_i$

Barnich,Brandt'01

$$k_\zeta[h,g] = \tfrac{1}{2}\Big[\zeta_\nu \nabla_\mu h - \zeta_\nu \nabla_\sigma h_\mu{}^\sigma + \zeta_\sigma \nabla_\nu h_\mu{}^\sigma + \tfrac{1}{2} h \nabla_\nu \zeta_\mu - h_\nu{}^\sigma \nabla_\sigma \zeta_\mu + \tfrac{1}{2} h_{\nu\sigma}(\nabla_\mu \zeta^\sigma + \nabla_\sigma \zeta_\mu)\Big] * (dx^\mu \wedge dx^\nu)\,.$$

$$c_i = \frac{3k_i \mathcal{A}}{2\pi}, \qquad \mathcal{A} = \int \sqrt{h\,\tilde{g}}\, d^p y \int \prod_i d\phi_i$$ Volume of manifold M

$$k_i = \frac{1}{2\pi T_i}; \qquad T_i$$ Frolov-Thorne Temperatures

Precisely reproduces black hole entropy (via Cardy's formula)

$$S_{BH} = \frac{\pi^2}{3} c_i T_i$$

First calculated explicitly for a specific case of extreme Kerr BH's in D=4,

Guica,Hartman,Song,Strominger 0809.4266

and for neutral spinning BH's in D=5,6 Lu,Mei,Pope 0811.2225

The general argument of the previous page given in

w/Lu,Pope 0812.2918

& shown to be true for most known charged spinning BH's in D≥ 4 !

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Further developments: Non-Abelian black holes in gauged SG

The only explicitly known example in D=4 N=4 gauged
SG example Chamseddine, Volkov (BPS) monopole

Recent results for t'Hooft- Polyakov type (BPS) monopoles
of D=4 N=2 ungauged SG Ortin et al. '08-'09

[in D=7, BPS non-Abelian BH Gauntlett et al.'04)

D=5 - little known about SG embedding of non-Abelian BH's

Some progress for N=8 gauged SG w/Lu&Pope, UPR-1206-T, to appear

Type IIB compactified on five-sphere: a consistent truncation w/ SO(6) gauge symmetry and scalars in symmetric, unimodular rep. of SO(6) w/Lu,Pope&Tran'98

$$\mathcal{L}_5 = R * 1\!\!1 - \tfrac{1}{4} T_{ij}^{-1} * DT_{jk} \wedge T_{k\ell}^{-1} DT_{\ell i} - \tfrac{1}{4} T_{ik}^{-1} T_{j\ell}^{-1} * F^{ij} \wedge F^{k\ell} - V * 1\!\!1$$
$$- \tfrac{1}{48} \epsilon_{i_1 \cdots i_6} \left(F^{i_1 i_2} F^{i_3 i_4} A^{i_5 i_6} - g F^{i_1 i_2} A^{i_3 i_4} A^{i_5 j} A^{j i_6} + \tfrac{2}{5} g^2 A^{i_1 i_2} A^{i_3 j} A^{j i_4} A^{i_5 k} A^{k i_6} \right)$$

$$V = \tfrac{1}{2} g^2 \left(2 T_{ij} T_{ij} - (T_{ii})^2 \right)$$

Consistent truncation to SU(2) x SU(2):

Non-Abelian gauge fields and scalar fields
(truncation to SU(2) x SU(2) and one scalar):

$$A^{12} = A^3\,, \quad A^{23} = A^1\,, \quad A^{31} = A^2\,, \qquad A^{45} = \tilde{A}^3\,, \quad A^{56} = \tilde{A}^1\,, \quad A^{64} = \tilde{A}^2$$
$$T^{11} = T^{22} = T^{33} = X\,, \qquad T^{44} = T^{55} = T^{66} = X^{-1}\,,$$

$$\mathcal{L} = R{*}\mathbb{1} - \tfrac{3}{2}X^{-2}{*}dX \wedge dX - \tfrac{1}{2}X^{-2}{*}F^i \wedge F^i - \tfrac{1}{2}X^2{*}\widetilde{F}^i \wedge \widetilde{F}^i$$
$$+\tfrac{3}{2}g^2(X^2 + X^{-2} + 6)\,,$$

a constraint

$$F^i \wedge \widetilde{F}^j = 0\,.$$

Metric and SU(2) x SU(2) (magnetic field Ansatz):

$$ds_5^2 = -\alpha^2 dt^2 + d\rho^2 + \tfrac{1}{4}\beta^2(\sigma_1^2 + \sigma_2^2 + \sigma_3^2)$$
$$A^i = g^{-1}\gamma\sigma_i\,, \qquad \tilde{A}^i = g^{-1}\tilde{\gamma}\sigma_i\,,$$ Magnetic SU(2) x SU(2) Ansatz

w/ constraint: $\gamma\tilde{\gamma} = k(\gamma - 1)(\tilde{\gamma} - 1)$ (k-integration constant)

Special explicit solution:

X=1 (constant scalar), $k=1,\ \gamma=\tilde{\gamma}=1/2$ [SU(2) x SU(2) $\longrightarrow$ SU(2)]

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + \tfrac{1}{4} r^2 (\sigma_1^2 + \sigma_2^2 + \sigma_3^2)\,,$$

$$f = 1 + g^2 r^2 - \frac{M + \log(gr)}{g^2 r^2}\,.$$

Extreme solution: $M = \dfrac{1+\sqrt{5}+\log(8(7+3\sqrt{5}))}{8g^2}$

Near-horizon geometry: $\mathrm{AdS}_2 \times S^3$

$$ds^2 = \alpha^2 ds_2^2 + \beta^2 d\Omega_3^2\,,$$
$$\alpha^2 = \frac{5-\sqrt{5}}{40g^2}\,, \qquad \beta^2 = \frac{\sqrt{5}-1}{4g^2}\,.$$

Supersymmetric?

Other explicit solutions with running scalar=work in progress...

Black Holes of gauged supergravity

Rich structure and important implications

for Gravity/Field Theory Duality

Still more to come!

Appreciation for Mike's vision and support!

Happy Birthday Mike!