putting the hawking back into the noise

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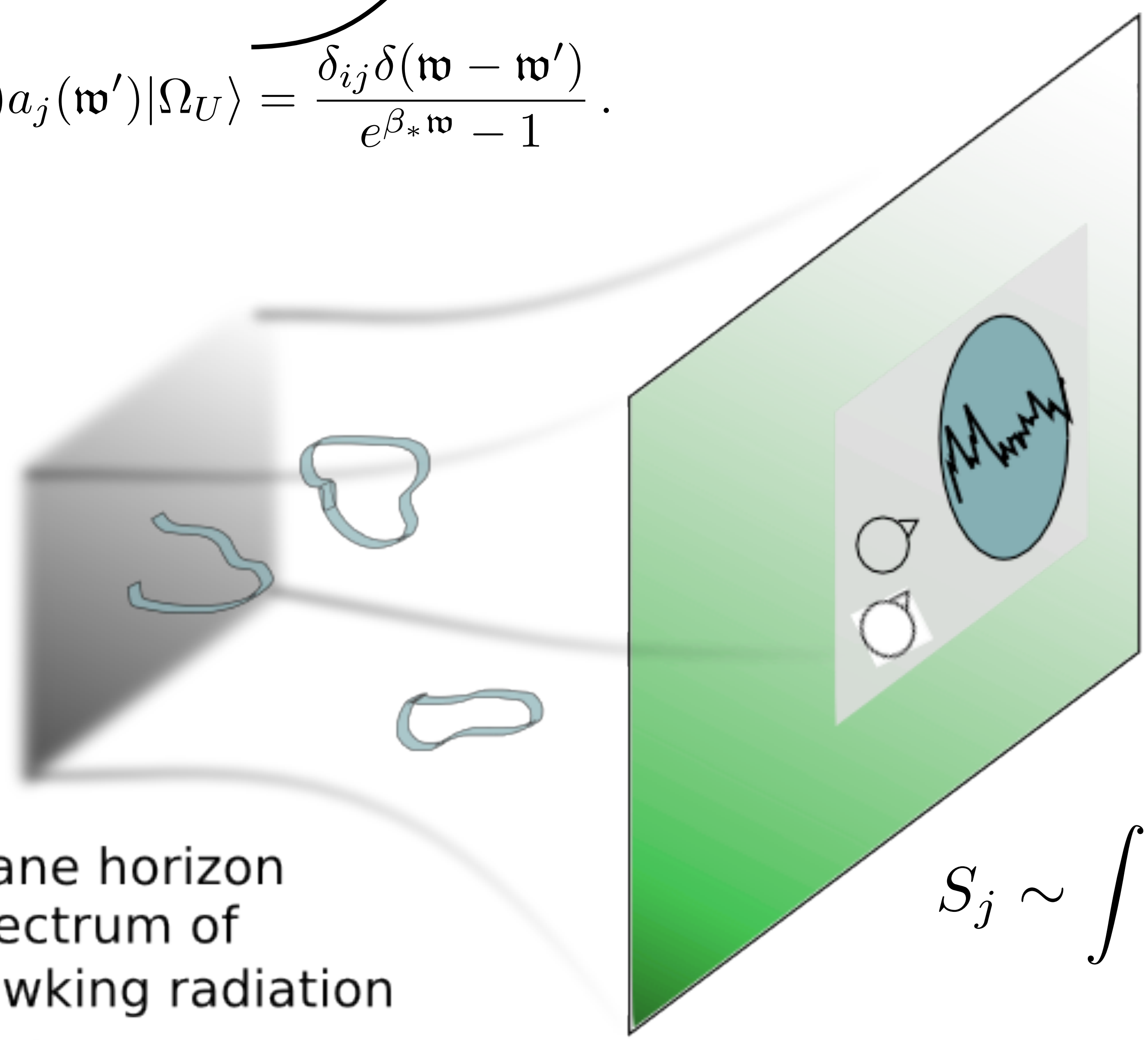
Kavli Centre @ Chicheley Hall
24 April 2012

one-slide summary

$$\langle \xi_i(t)\xi_j(t')\rangle = \kappa_{ij}(t,t')$$

$$\langle \Omega_U | a_i^\dagger(\mathfrak{w}) a_j(\mathfrak{w}') | \Omega_U \rangle = \frac{\delta_{ij}\delta(\mathfrak{w}-\mathfrak{w}')}{e^{\beta_* \mathfrak{w}} - 1} \,.$$



$$S_j \sim \int \mathrm{Tr}\,\kappa(\omega) d\omega = T_* \Phi(T/T_*)$$

brane horizon
spectrum of
Hawking radiation

boundary E field
spectrum of
current noise

losing your equilibrium

going away from equilibrium

- almost all results on transport (conductivity, viscosity, ...) rely on smallness of departure from equilibrium, so that a **Kubo**-type formula can be applied

$$H \to H_0 + \delta H(t)$$

time dependent disturbance

$$\langle \mathcal{O}(t,x) \rangle = \mathrm{Tr}\rho(t)\mathcal{O}(x)$$

time dependent expectation value

$$\begin{aligned} \delta\langle \mathcal{O}(t,x)\rangle &= -i\mathrm{Tr}\rho_0 \int dt' \left[\mathcal{O}(t,x)\,,\,\delta H(t')\right] + \cdots \\ &= -i \int d\mathbf{x}' dt' \langle \big[\mathcal{O}(t,x), \mathcal{O}^F(t',x')\big]\rangle \mu(t',\mathbf{x}') \end{aligned}$$

causal correlation function with respect to **equilibrium** density matrix

- example: (linear) conductivity is given in terms of current-current auto-correlation (a.k.a. retarded two-point function)

transport in ads/cft

- we saw that a quantum Boltzmann approach can overcome some of these difficulties: 1/N expansion makes it technically possible to compute σ_{NL}

- recall: balance of Schwinger pair production and scattering relaxation produces current-driven **steady state** (CDSS). consequence of **large N**!

- in ads/cft **linear response** is well established (GKPW formula). Son & Starinets showed how to do Lorentzian case

- non-equilibrium Lorentzian dynamics maps to (generally) time-dependent solutions of Einstein's equations (see J. Bhaseen)
  $\Rightarrow$ such problems can be addressed using numerical relativity

- Can we find a similar implementation of large N to produce (and solve) transport & noise in CDSS in ads/cft?

a holographic $z=1$ QCP

the brane setup

- consider the 2+1 intersection of N D3 branes and M D5 branes
  [see e.g. de Wolfe et al., Erdmenger et al.]

	0	1	2	3	4	5	6	7	8	9
D3	●	●	●	●	○	○	○	○	○	○
D5	●	●	●	○	●	●	●	○	○	○

- The (non-) extremal D3 brane metric

$$
\begin{aligned}
G_{mn}dx^m dx^n &= \frac{u^2}{R^2}\left(-f(u)dt^2 + d\vec{x}^2\right) + \frac{R^2}{f(u)u^2} + R^2 d\Omega_5^2 \\
f(u) &= 1 - \frac{u_h^4}{u^4}
\end{aligned}
$$

- With temperature $\boxed{\pi T = \frac{u_h}{R^2}}$

metallic AdS/dCFT

[Karch & O'Bannon, 2007]

- Taking M << N we can treat the D5 branes as **probes** governed by DBI action; wrap AdS_4 x S^2 submanifold (angle θ gives position of $S^2 \subset S^5$: mass operator) ➠ Field theory (in this limit) lives on 2+1 intersection

$$\boxed{S_{D_5} = -\mathcal{N}_5 \int d^6\xi \sqrt{-det\left(g_{\rm ind} + F\right)} + S_{\rm WZ}}$$

- To take system away from equilibrium apply **strong electric field** in x direction

$$A = \left(-Et - A_x(u)\right) dx\,.$$

- Embedding described by a set of non-linear ODEs determining $A_x(u)$ and θ(u) Taking θ(u) =0 (zero mass deformation) always a solution

- Coupled M << N degrees of freedom to field: non-equilibrium **steady state**

non-linear conductivity

- $A_x(u)$ enters action as a cyclic coordinate. So it has a first integral

$$A_x'(u) = C\sqrt{\frac{G_{uu}\left(E^2 + G_{tt}G_{xx}\right)}{G_{tt}\left(C^2 + G_{yy}G_{tt}\right)}}$$

- Vanishing point of denominator defines special point u_*

$$u_*^4 = u_h^4 + E^2 R^{-4}$$

- Reality demands that the numerator also vanish at this point

$$\boxed{E = C = j_x/\tilde{\mathcal{N}}_5}$$

- Non-linear conductivity in d=2 is a **constant**.

non-linear current noise

open string metric; D5 black hole

- In order to study the current noise in the system, need to look at **fluctuations** around the non-linear current steady state

$$S^{(2)} = \tilde{\mathcal{N}_5} \int du dt \sqrt{-\alpha} \alpha^{ab} \left(\partial_a a^{\parallel} \partial_b a^{\parallel} + Z(u)^{\perp} \partial_a a^{\perp} \partial_b a^{\perp} + Z^t(u) \partial_a a^t \partial_b a^t \right)$$

- These propagate in the OSM (open string metric). Fluctuations see a black-hole **horizon** at u_*

$$ds^2 = -\frac{u^4 - u_*^4}{R^2 u^2} d\tau^2 + \frac{u^2 R^2}{u^4 - u_*^4} du^2$$

- Which has a temperature (look at surface gravity)

$$\boxed{\pi T_* = \left[E^2 R^{-4} + (\pi T)^4 \right]^{1/4}}$$

- Note this is not the same as the background ‘bath’ ➠ **non-equilibrium**

current noise

[see also de Boer et al., 2008, Teaney et al., 2009]

- The noise in this system is characterised by a Langevin equation

$$\frac{d\mathbf{j}}{dt} + \int G_R(t,t')\mathbf{j}(t')dt' = \xi(t)$$

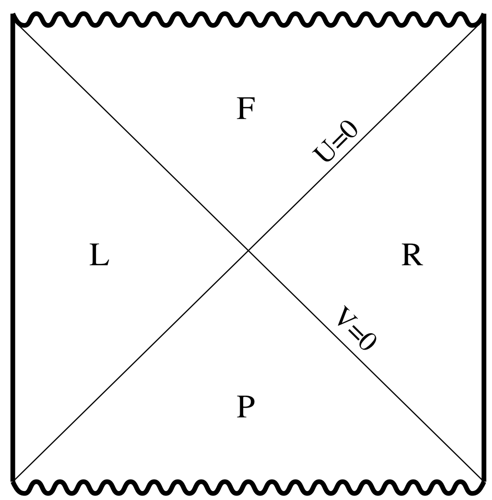
- The stochastic noise term is governed by the Keldysh two-point function

$$\langle \xi^i(\omega)\xi^j(-\omega)\rangle = G^{ij}_{\mathrm{sym}}(\omega)$$

- So we need to calculate the retarded and **symmetrised** Green function of the probe degrees of freedom (Schwinger-Keldysh two-time formalism)

- This can be done by thinking carefully about the (1/2) whole Kruskal plane

current noise



- Generating function in Lorentzian AdSBH in fact gets contributions from **both** asymptotic boundaries

- **Analyticity** arguments at the horizon relate positive frequency modes in L and R and we find

$$G^{ij}_{\rm sym}(\omega) = -(1+2n_*(\omega))\mathrm{Im}G^{ij}_R(\omega)$$

- n_* is a thermal factor at the **effective temperature** T_*

- Thus Langevin equation is fully determined by retarded correlation function, which we can obtain exactly (in k=0 sector)

$$\boxed{G^{ij}_R(\omega) = -i\delta^{ij}\sigma\frac{\omega}{\pi T_*}}$$

noise power

- A common way to characterise the noise in the system is **noise power**:

$$
\begin{aligned}
S_j &= -\int_{-\infty}^{\infty} d\omega d\mathbf{k} \coth(\omega/2T_*) \mathrm{Im}\langle \mathbf{j}(\omega)\cdot\mathbf{j}(-\omega)\rangle_R \\
&= 4\sigma T_* = 4\sigma\frac{\sqrt{E}}{R\pi}\left[1+\frac{(R\pi T)^4}{E^2}\right]^{1/4}
\end{aligned}
$$

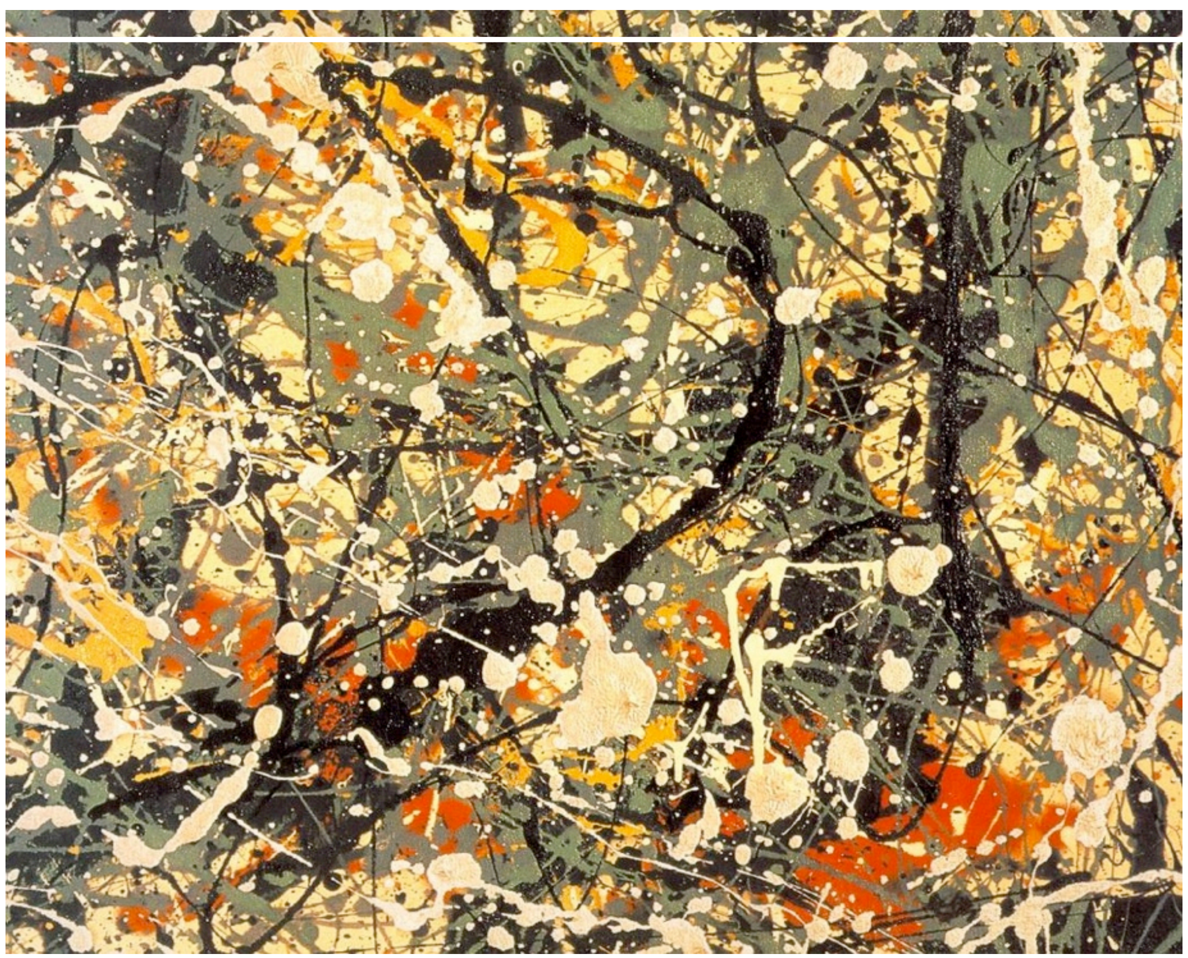
- Compare with result obtained using quantum Boltzmann at SI transition:

$$S_j = 4\sigma_E\sqrt{E}$$ large field

$$S_j = 4\sigma_T T$$ zero field

- We obtain agreement in limits. Can we measure the interpolating function in experiment? Discriminating factor for holographic dual?

CONCLUSIONS



CONCLUSIONS

sometimes noise
is more important
than the
background

conclusions

- **Near-equilibrium** calculations encode interesting features of quantum matter, such as Fermi surfaces, hydrodynamics, **Non-equilibrium** is next frontier
- Large N in ads/cft establishes CDSS. Non-linear conductivity easy to get!
- Horizon of the OSM encode subtle **out-of equilibrium** physics. Use this to calculate current noise
- Fluctuations about CDSS are **precisely thermal** at effective temperature T*
- **Stochastic noise** term related to **quantum** effect (Hawking radiation): Can we turn the tables and measure Hawking spectrum in a solid-state device (or cold atoms)?

  (mental) health warning:
  large-N may turn out to be problematic! Opens a can of worms: BH information paradox, coarse graining, ...