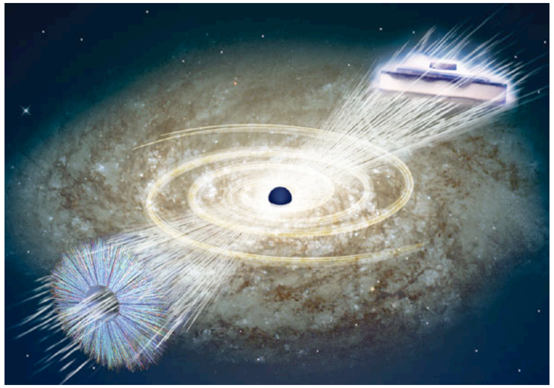
Holographic Fermions

Koenraad Schalm

Institute Lorentz for Theoretical Physics, Leiden University



A. Bagrov, J. Brill, M. Cubrovic, E. Gubankova, B. Overbosch, J. Mydosh, Y. Liu, A. Parnachev, P. Schijven, J. She, Y.-W. Sun, J. Zaanen.

Netherlands Organisation for Scientific Research

Gravity, Black Holes and Condensed Matter
Chicheley Hall, Apr 2012

- Holographic Quantum Critical Fermion Matter
  - Minimally coupled charged Fermion in AdS-RN BH

$$G(\omega,k)=\frac{Z}{\omega-v_F(k-k_F)-e^{i\gamma}\omega^{2\nu_{k_F}}}+\ldots$$

Lee;
Cubrovic, Zaanen, KS;
Faulkner, Liu, McGreevy, Vegh

$$\text{dispersion }\omega\sim(k-k_F)^z\text{ with }z=\begin{cases}1/2\nu_{k_F} & \nu_{k_F}<1/2\\ 1 & \nu_{k_F}>1/2\end{cases}$$

- $\nu_{k_F}\sim\sqrt{m^2+k_F^2-q^2}$ is a free parameter

- Holographic Quantum Critical Fermion Matter
  - Minimally coupled charged Fermion in AdS-RN BH

$$G(\omega,k)=\frac{Z}{\omega-v_F(k-k_F)-e^{i\gamma}\omega^{2\nu_{k_F}}}+\ldots$$

Lee;
Cubrovic, Zaanen, KS;
Faulkner, Liu, McGreevy, Vegh

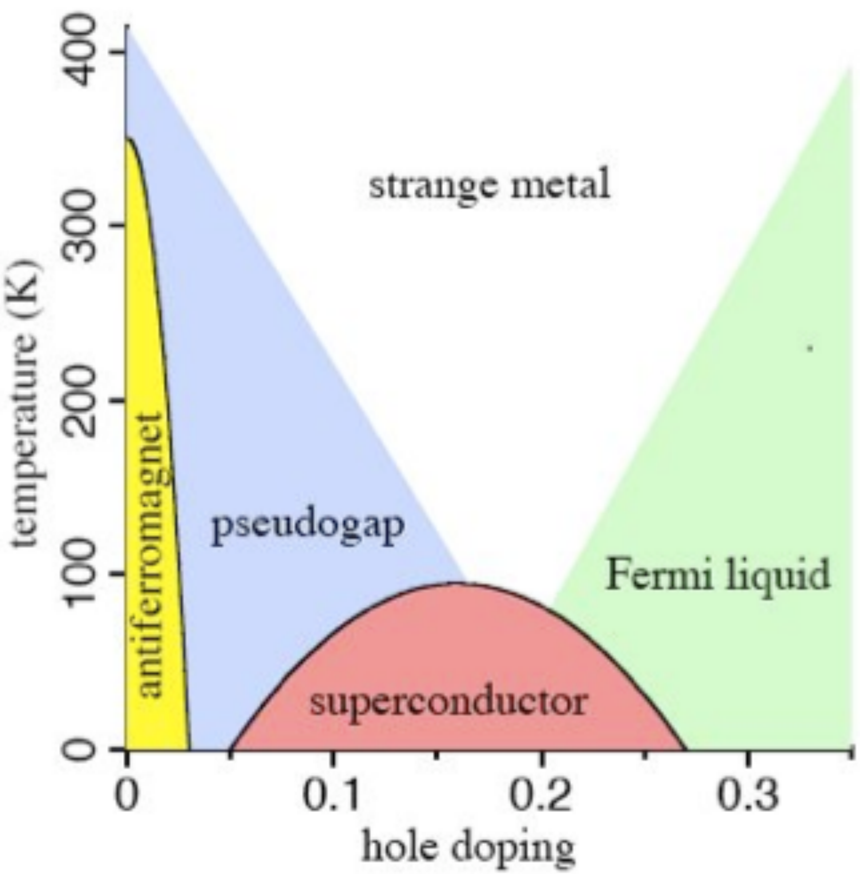
$$\text{dispersion } \omega\sim(k-k_F)^z \text{ with } z=\begin{cases}1/2\nu_{k_F} & \nu_{k_F}<1/2\\ 1 & \nu_{k_F}>1/2\end{cases}$$

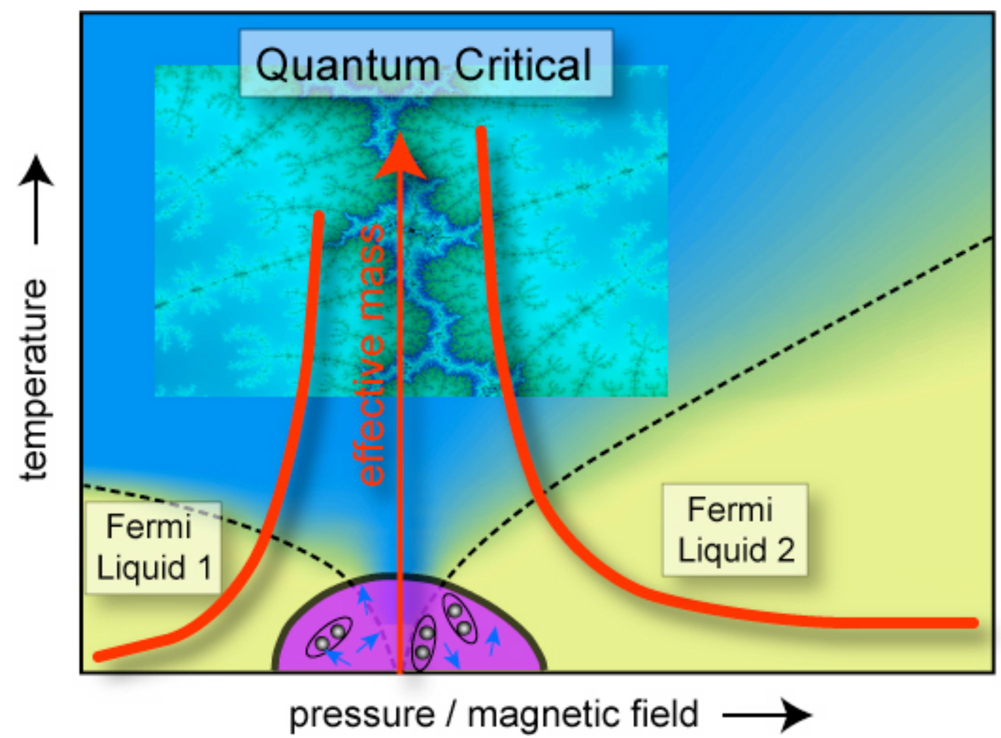
A non-Fermi liquid

- $\nu_{k_F}\sim\sqrt{m^2+k_F^2-q^2}$ is a free parameter

String theory embedding/Top-down construction: DeWolfe, Gubser, Rosen

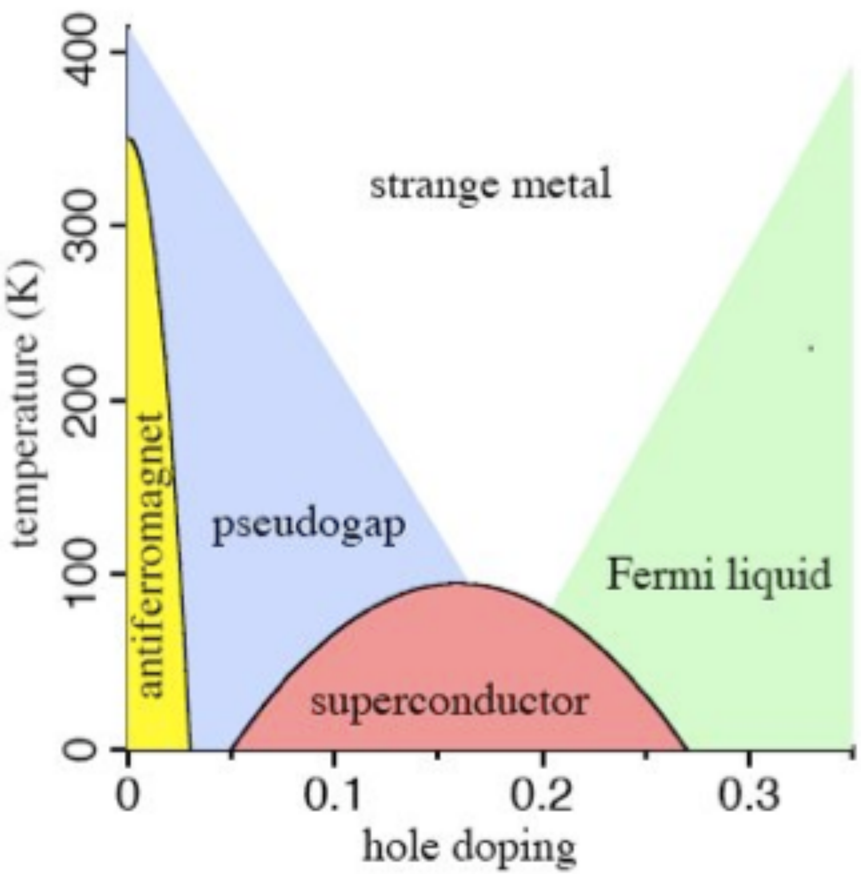
- An explanation of the “strange metal”

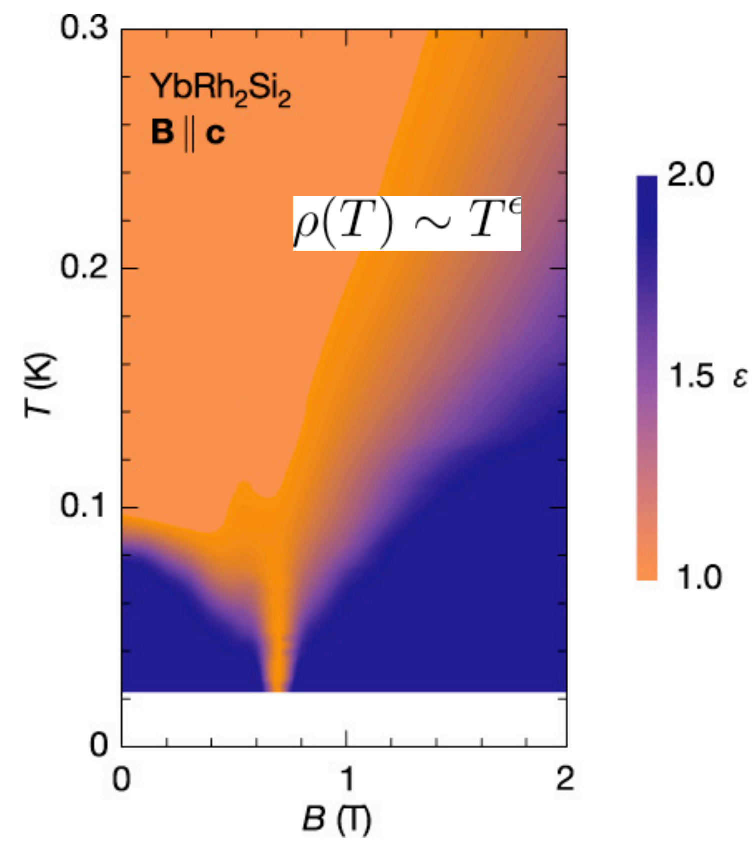




- An explanation of the "strange metal"
  - Marginal Fermi liquid: $\nu_{k_F} = 1/2$ Varma, Littlewood, ...
  - Explains linear resistivity

In AdS/CFT: Faulkner, Liu, McGreevy, Vegh





Custers et al

The puzzles of AdS/CMT...

- Is the near-horizon AdS-RN the quantum critical pt? What are its instabilities?
- What is the zoo of Fermi liquids in the stable ground state? How should we think of the gravitational dual of a single Fermi surface?
- When is large N good? When is it bad?
- What is the AdS2 metal?
- How does the LFL emerge from the strange metal?
- Why is only the marginal Fermi liquid observed?
- Can we explain high Tc?
- What is the pseudogap phase?
- Lattice effects?...

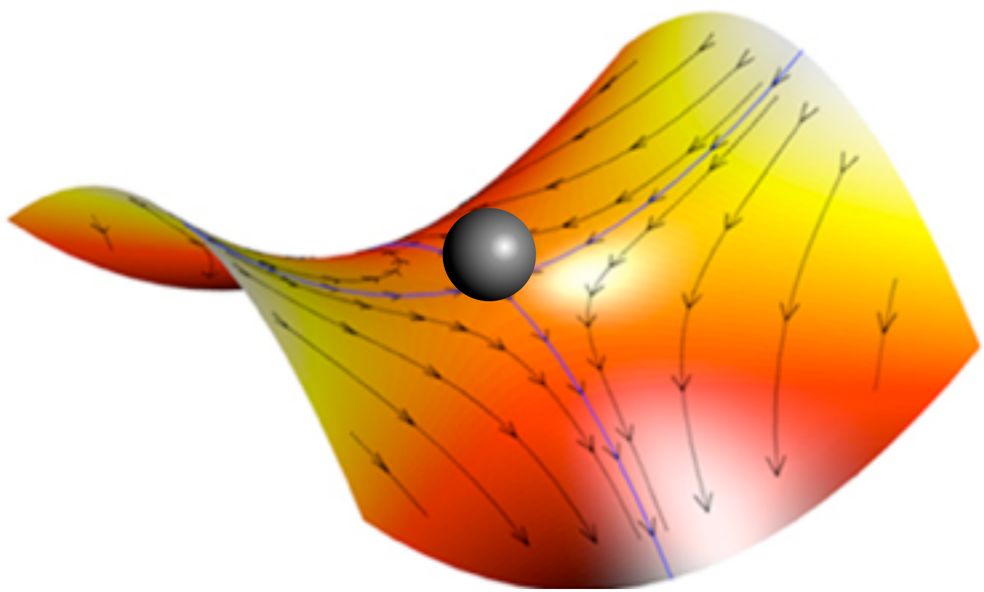
The puzzles of AdS/CMT...

- Is the near-horizon AdS-RN the quantum critical pt? What are its instabilities?
- What is the zoo of Fermi liquids in the stable ground state? How should we think of the gravitational dual of a single Fermi surface?
- When is large N good? When is it bad?
- What is the AdS2 metal?
- How does the LFL emerge from the strange metal?
- Why is only the marginal Fermi liquid observed?
- Can we explain high Tc?
- What is the pseudogap phase?
- Lattice effects?...

Note: we will not discuss defect fermions [Kachru, Karch, Yaida]

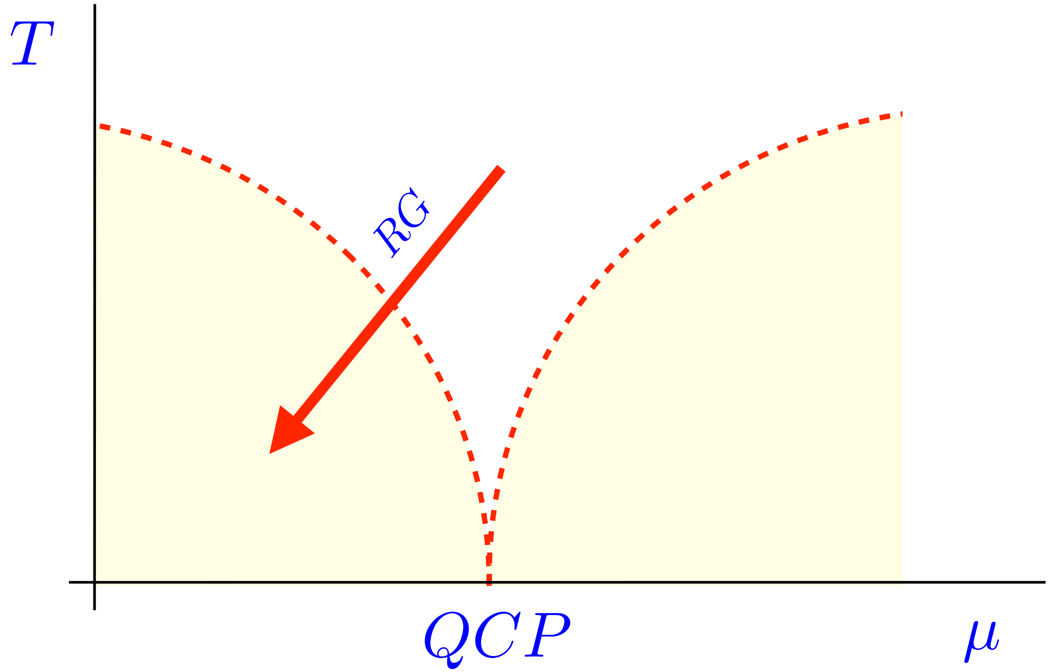
The LQCP of extremal AdS-RN?

- Local Quantum Criticality is an unstable fixed point:
  - T=0 extremal AdS-RN has groundstate entropy.
- What are its instabilities?



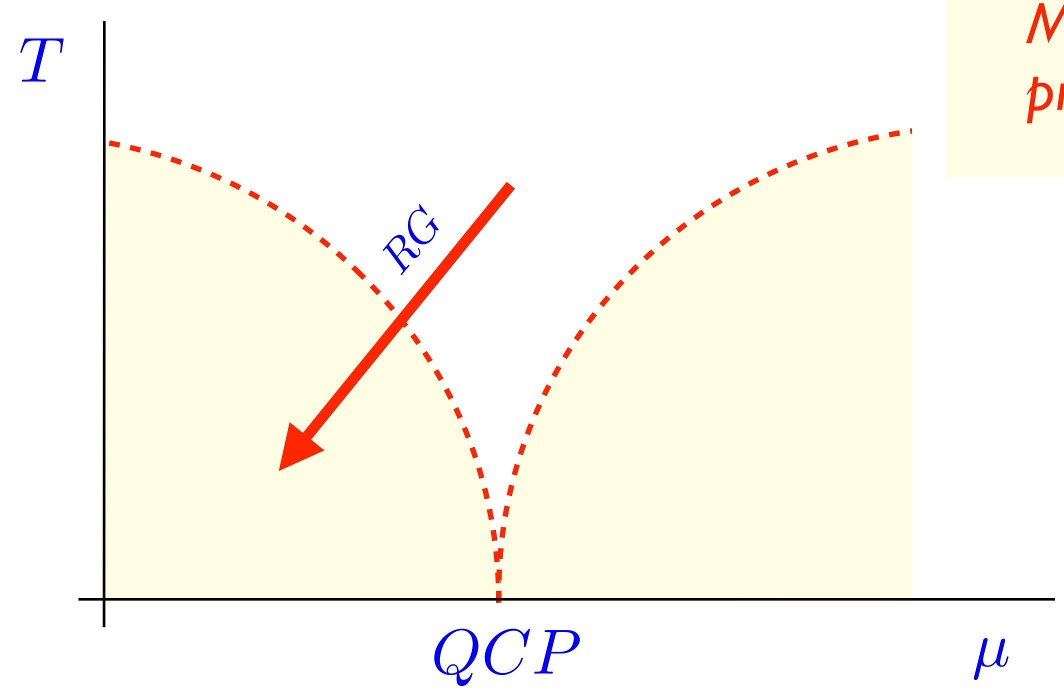
\

- What are its instabilities?



  - Whether the local quantum critical region is reached depends on the true UV.

- What are its instabilities?



Minimal AdS/CFT only probes T/μ

- Whether the local quantum critical region is reached depends on the true UV.

- Charged scalar (holographic superconductor) $T_c \sim \mu$
- Neutral scalar $(q \ll m \ll m_{BF\ AdS_2})$ $T_c \sim \mu e^{-1/\sqrt{m^2}}$
- Charged fermions $T_c \sim \mu e^{-N_c^{2/3}}$
- Neutral fermions/Neutral scalar $(m \gg m_{BF\ AdS_2})$ XXX

The LQCP of extremal AdS-RN?

- Semi-local Quantum Liquid: Bosons
  - Need another scale, e.g. double trace deformation κ

[Faulkner, Horowitz, Roberts]

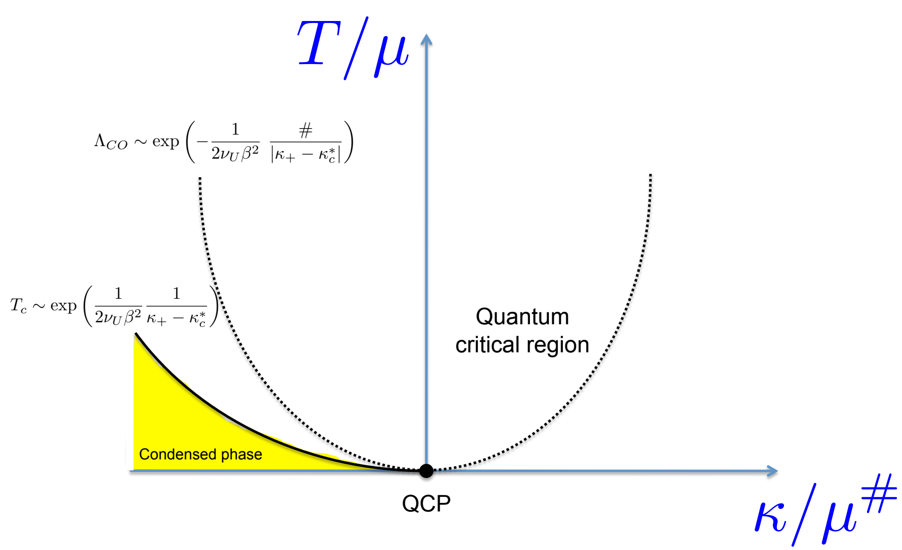


FIG. 15. Finite temperature phase diagram with the quantum critical region for marginal criticality at $u = 0$ and changing $(\kappa_+ - \kappa_+^*)$. The susceptibility in the bowl-shaped quantum critical region is given by (9.16) with the $\omega \gg T$ limit given by (9.8)

[Iqbal, Liu, Mezei]

The LQCP of extremal AdS-RN?

- Semi-local Quantum Liquid: Bosons
  - Need another scale, e.g. double trace deformation κ

[Faulkner, Horowitz, Roberts;...]

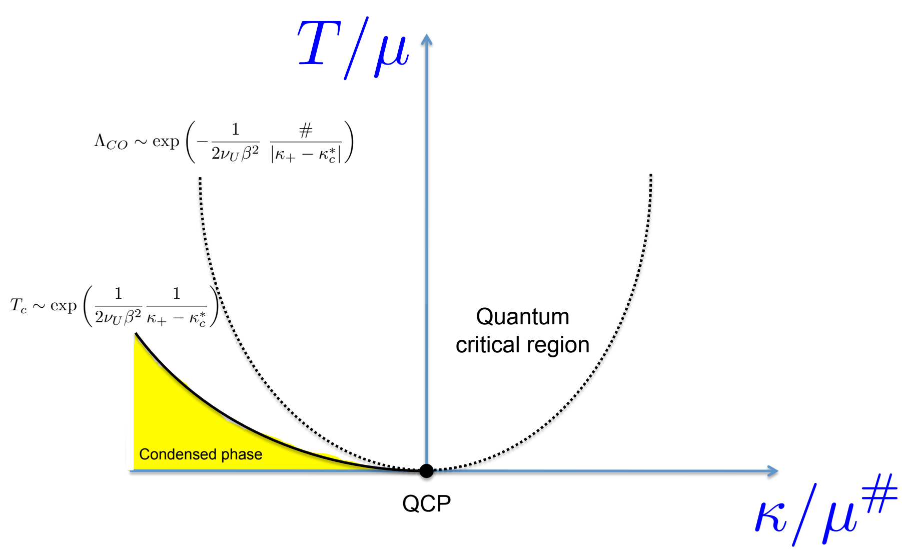
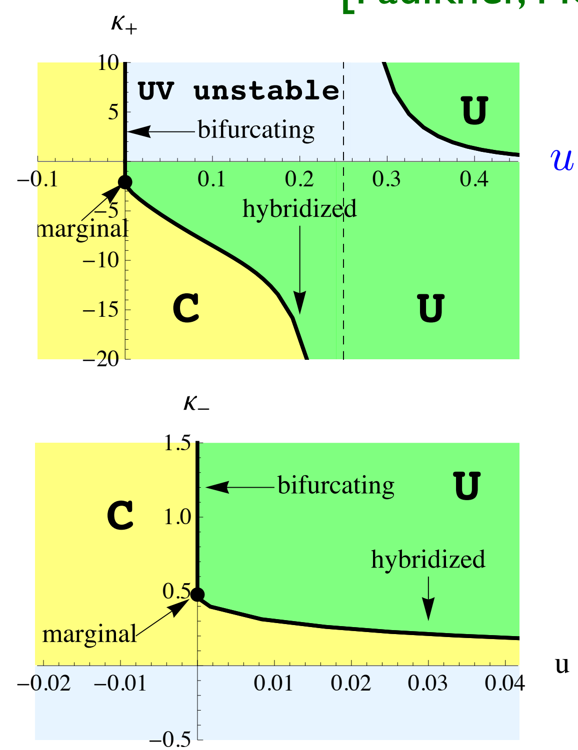


FIG. 15. Finite temperature phase diagram with the quantum critical region for marginal criticality at $u = 0$ and changing $(\kappa_+ - \kappa_+^*)$. The susceptibility in the bowl-shaped quantum critical region is given by (9.16) with the $\omega \gg T$ limit given by (9.8)



$$u = \sqrt{m^2 + \frac{1}{4} - q^2}$$

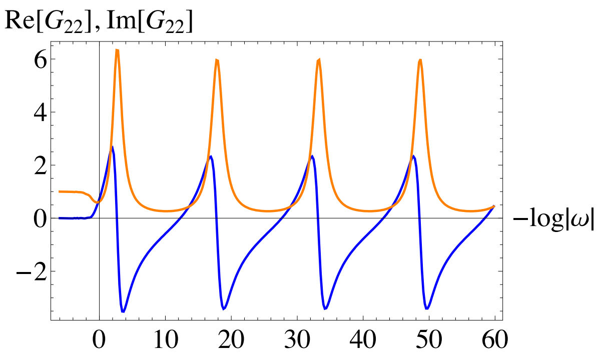
Tuning Δ

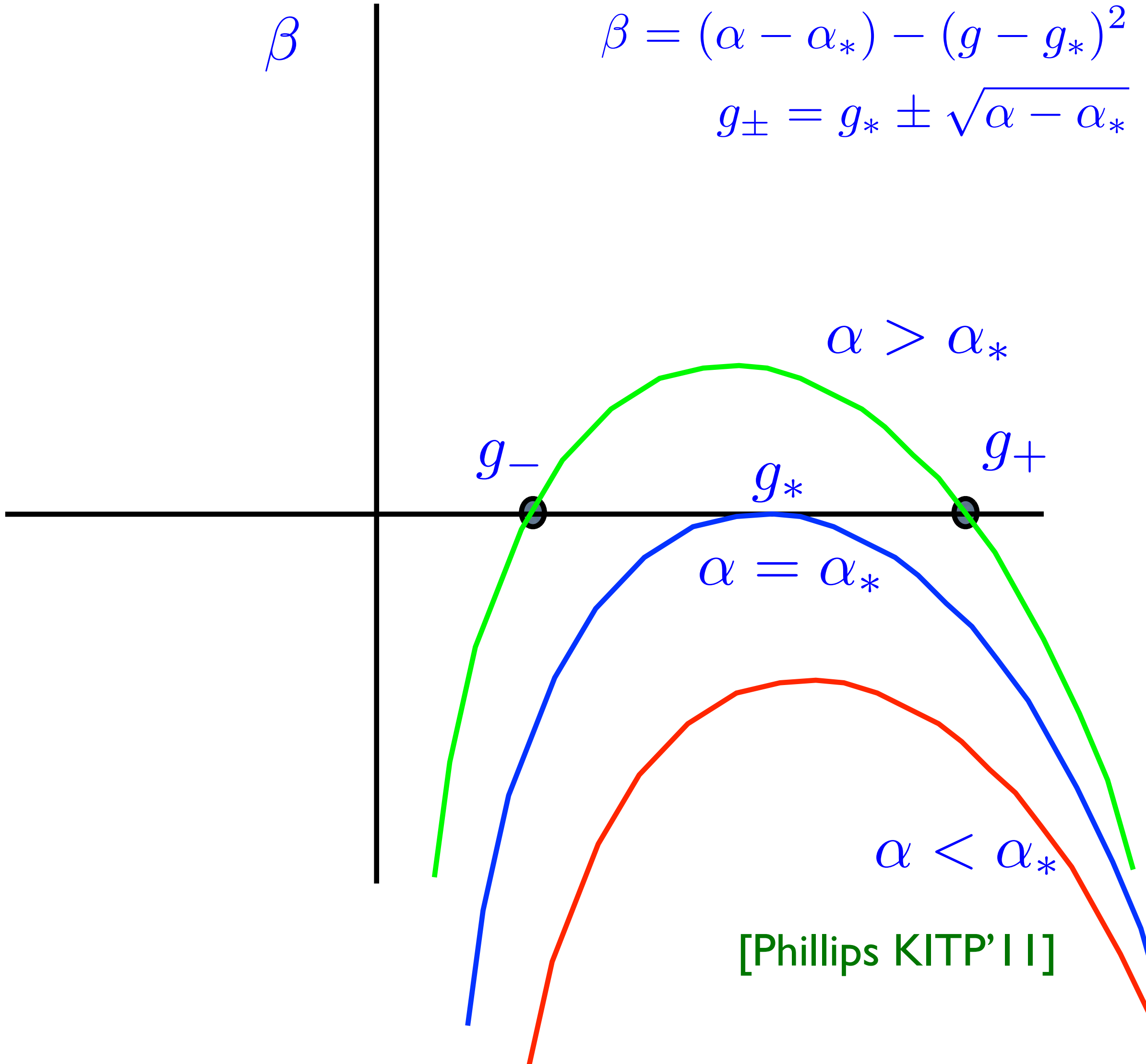
[Iqbal, Liu, Mezei]

"Marginal" Susceptibilities @ marginal QCP

$$\mathrm{Im}\chi \sim \begin{cases} \frac{\omega}{T} & \omega \ll T \\ \mathrm{sgn}(\omega) & \omega \gg T \end{cases}$$

- Semi-local Quantum Liquid: Fermions
  - Log-oscillatory = Discrete Scale Invariance



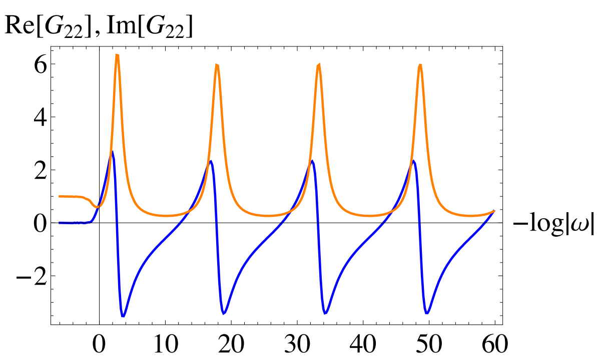


[Liu, McGreevy, Vegh]

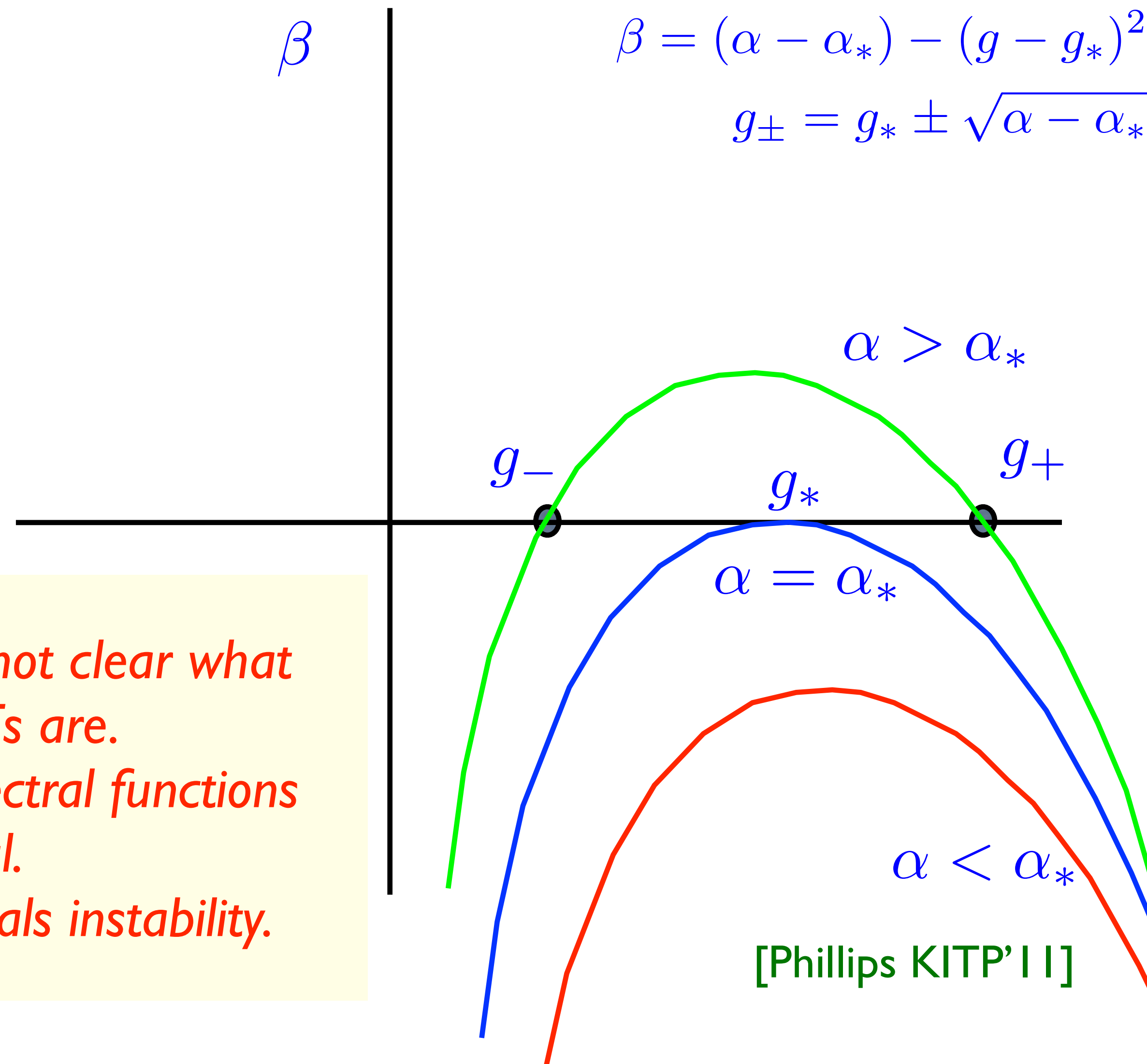
[Phillips KITP'11]

The LQCP of extremal AdS-RN?

- Semi-local Quantum Liquid: Fermions
  - Log-oscillatory = Discrete Scale Invariance



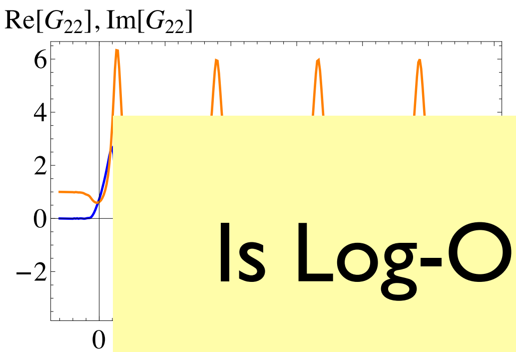
[Liu, McGreevy, Vegh]

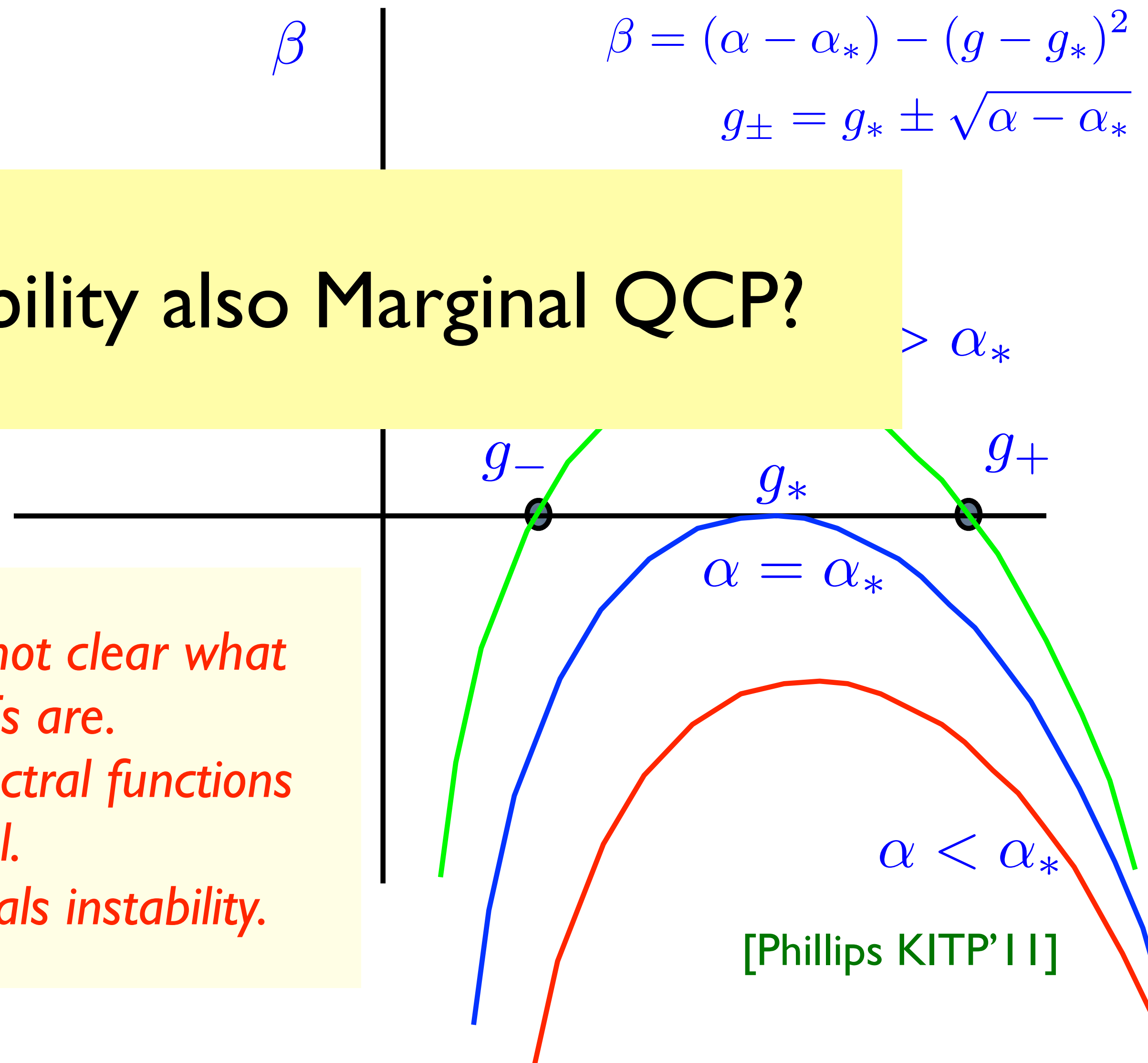


[Phillips KITP'11]

- *If true, it is not clear what the two CFTs are.*
- *Fermion spectral functions not marginal.*
- *Log-osc signals instability.*

- Semi-local Quantum Liquid: Fermions
  - Log-oscillatory = Discrete Scale Invariance





Is Log-Osc instability also Marginal QCP?

- *If true, it is not clear what the two CFTs are.*
- *Fermion spectral functions not marginal.*
- *Log-osc signals instability.*

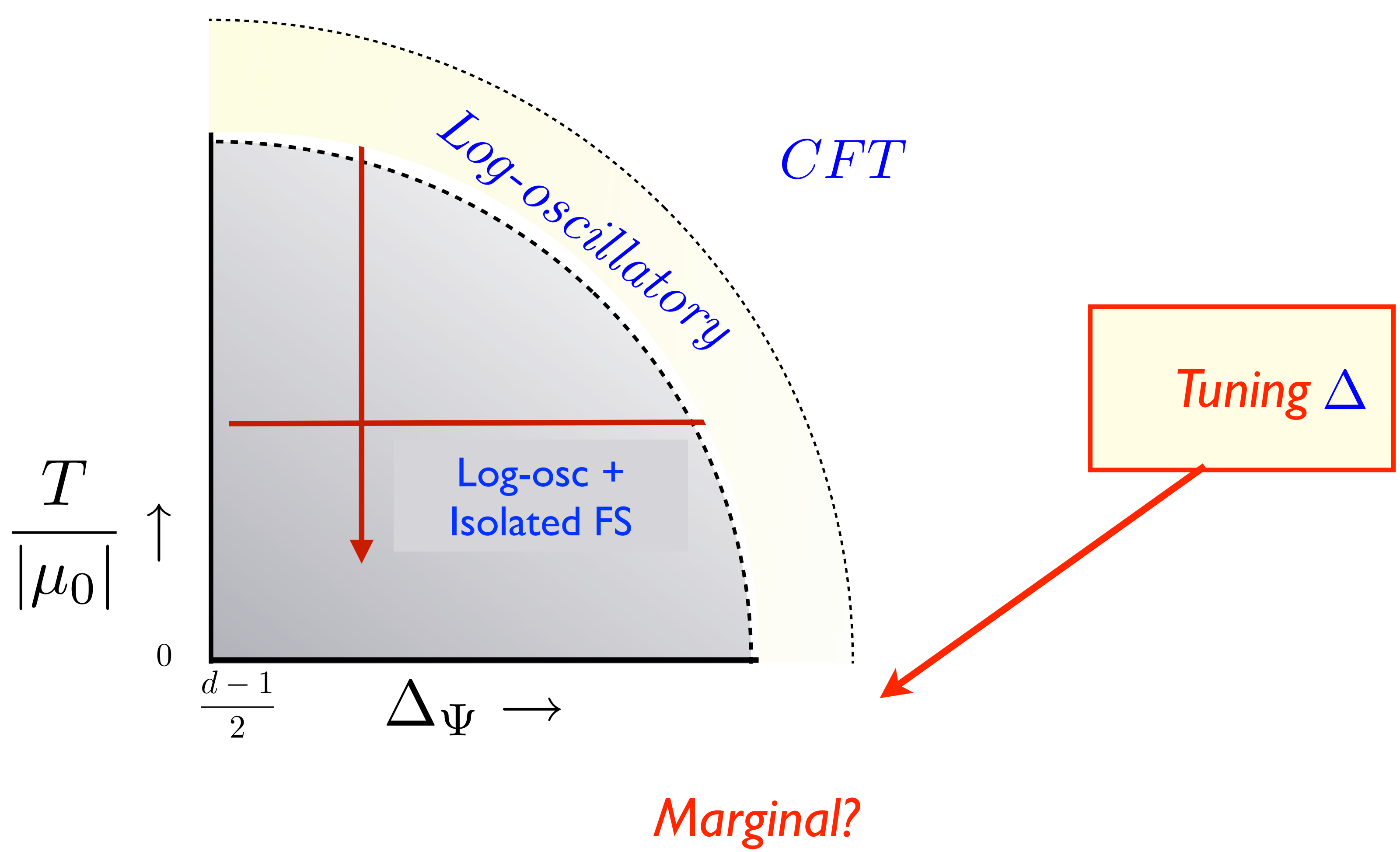
[Liu, McGreevy, Vegh]

[Phillips KITP'11]

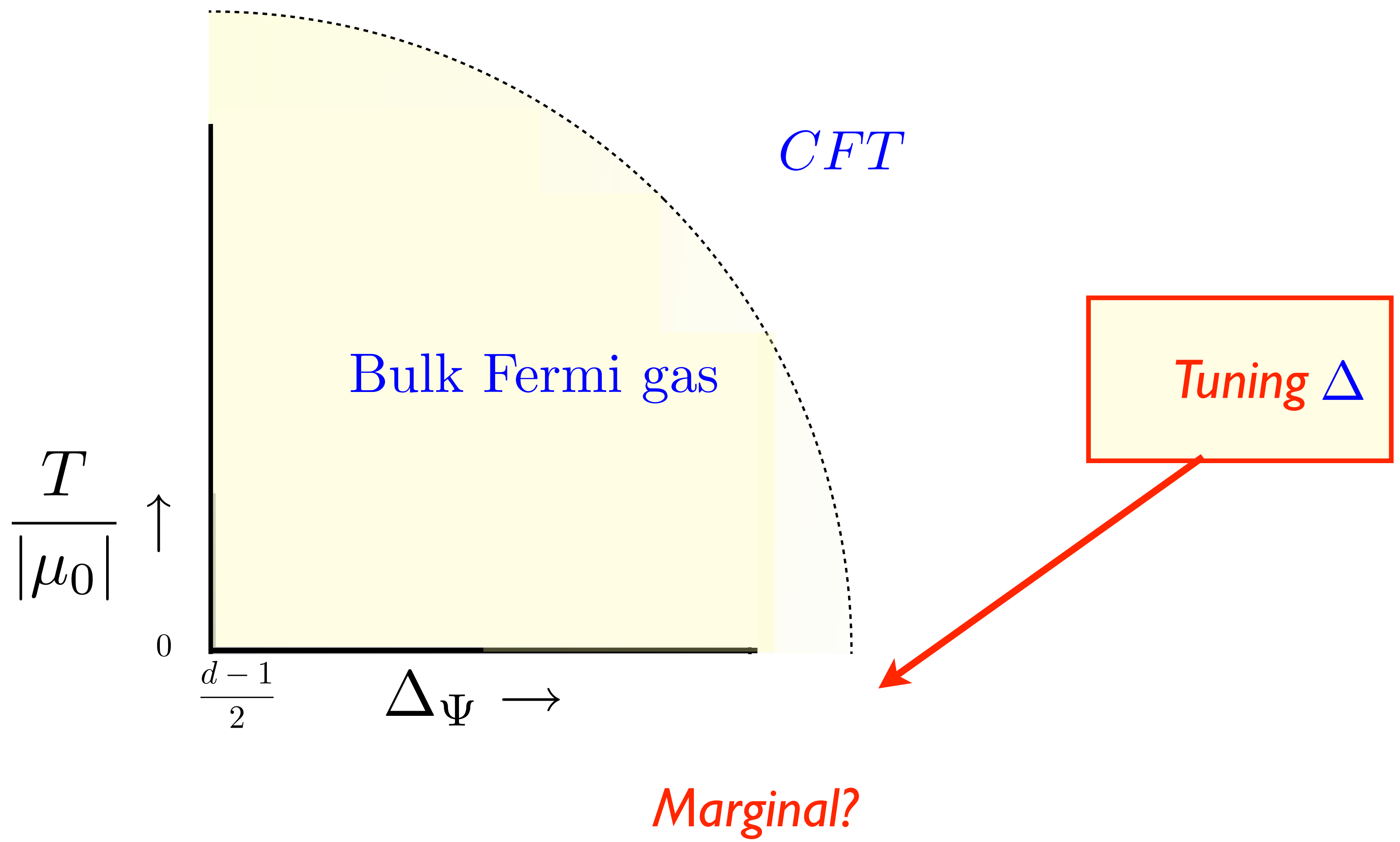
- A Fermi-gas in AdS: various approaches

Fluid approx	Dirac Hair	Hartree-Fock
- # occ.wave func. large - # radial harm. large	- # occ.wave func: one - # radial harm: one	- # occ.wave func: arb. - # radial harm: arb.
- # FS large - # species large	- # FS: one	- # FS: arb.
Neutron/Electron star:	By construction	Exact, iterative procedure...
de Boer, Papadodimas, Verlinde; Hartnoll, Tavanfar, Hofman;	Cubrovic, Zaanen, KS.	Sachdev Allais, McGreevy, Josephine Suh

- Semi-local Quantum Liquid: Fermions
  - Log-oscillatory = Instability

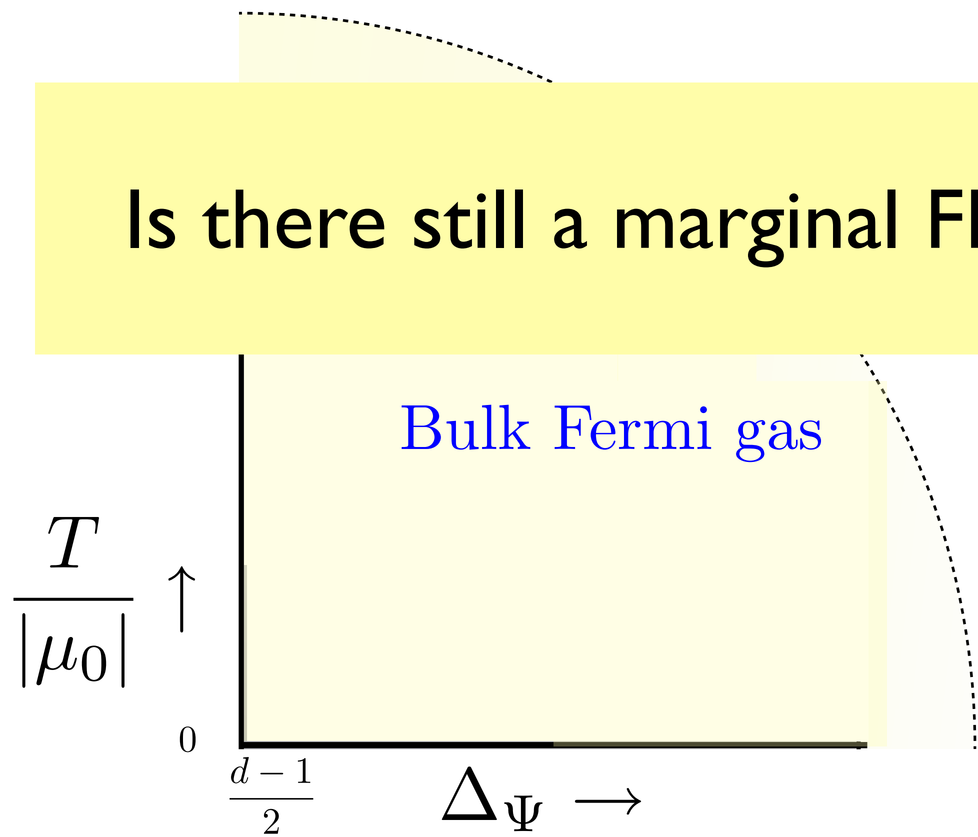


- Semi-local Quantum Liquid: Fermions
  - Log-oscillatory = Instability



- Semi-local Quantum Liquid: Fermions
  - Log-oscillatory = Instability

Is there still a marginal FL?

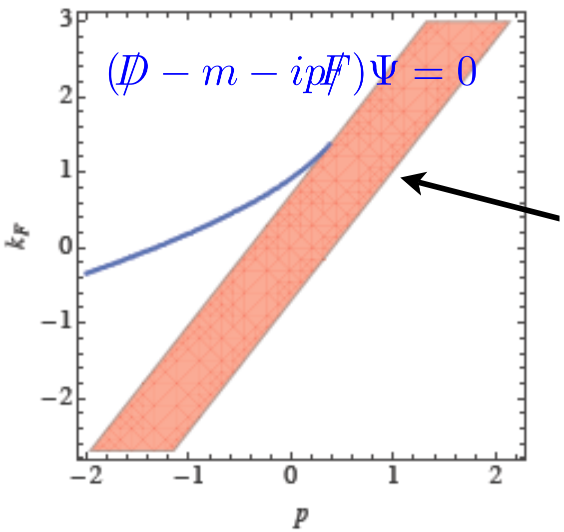


Tuning Δ

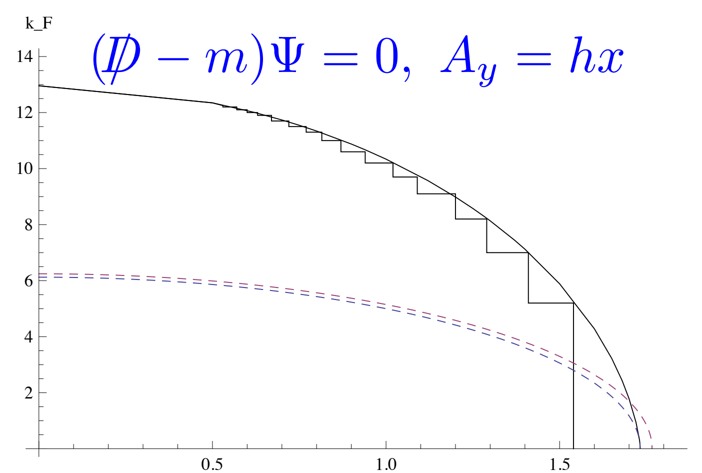
Marginal?

- Other instabilities of fermions in AdS-RN?
  - Charged susceptibilities (Fermi pairing) [Hartman, Hartnoll]

- Other instabilities of fermions in AdS-RN?
  - Charged susceptibilities (Fermi pairing) [Hartman, Hartnoll]
  - *Tuning* Δ FL to NFL; log-oscillatory vs isolated (order N))



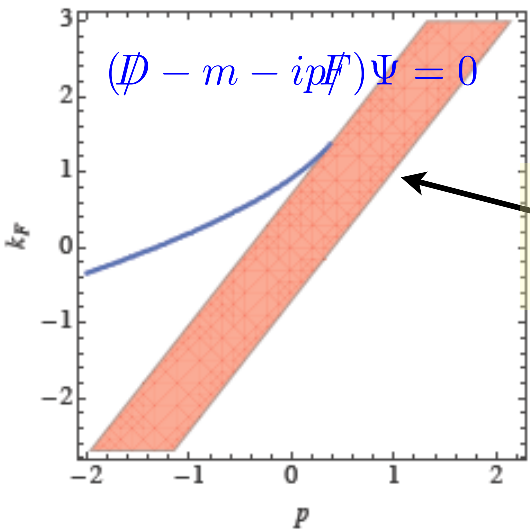
[Edalati, Leigh, Phillips]



[Gubankova, Brill, Cubrovic, Schijven, KS, Zaanen; Gubankova et al]

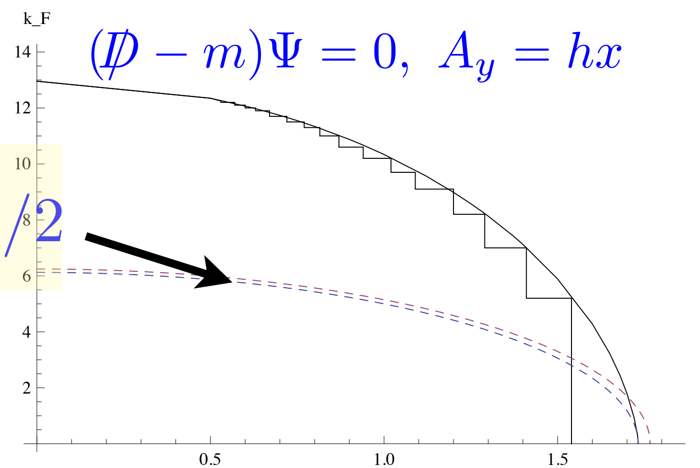
[possibly related to Bolognesi, Tong]

- Other instabilities of fermions in AdS-RN?
  - Charged susceptibilities (Fermi pairing) [Hartman, Hartnoll]
  - $\boxed{\textit{Tuning}\,\Delta}$ FL to NFL; log-oscillatory vs isolated (order N))



[Edalati, Leigh, Phillips]

$\nu_{k_F} < 1/2$



[Gubankova, Brill, Cubrovic, Schijven, KS, Zaanen; Gubankova et al]

[possibly related to Bolognesi, Tong]

- If FL to NFL is related to a macroscopic instability, then it selects marginal FL.

When is large N good? When is it bad?

- Holographic Superconductor
  - 1/N corrections restore ALRO [Anninos, Hartnoll, Iqbal]
  - 1/N corrections ought to capture any non-mean field [e.g. She, Overbosch, Liu, KS, Sun, Zaanen]

- Fermions
  - Single Fermi surface (Dirac Hair/Hartree Fock) is 1/N effect.

When is large N good? When is it bad?

- Holographic Superconductor
  - 1/N corrections restore ALRO [Anninos, Hartnoll, Iqbal]
  - 1/N corrections ought to capture any non-mean field [e.g. She, Overbosch, Liu, KS, Sun, Zaanen]

- Fermions
  - Single Fermi surface (Dirac Hair/Hartree Fock) is 1/N effect.
  - Isolated FL to NFL Fermions vs Confin-Deconfined

- O(1): marginal FL

- O(N): stability of LFL

When is large N good? When is it bad?

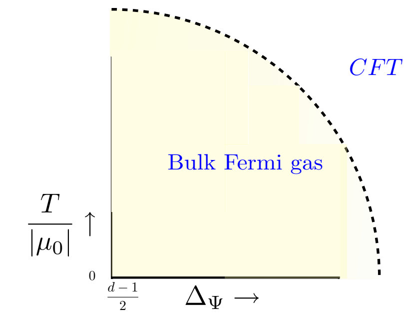
- Holographic Superconductor
  - 1/N corrections restore ALRO [Anninos, Hartnoll, Iqbal]
  - 1/N corrections ought to capture any non-mean field
    ...sch, Liu, KS, Sun, Zaanen]

Is there still a marginal FL?
(simply 1/N?)

- Ferm...
  - Sin... ...x) is 1/N effect.
  - Isolated FL to NFL Fermions vs Confin-Deconfined

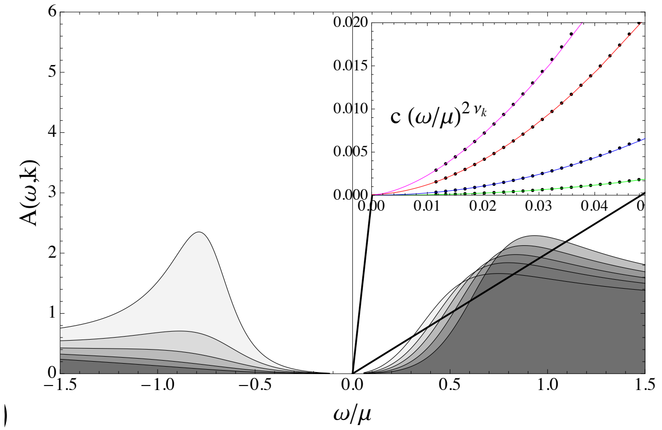
- O(1): marginal FL

- O(N): stability of LFL

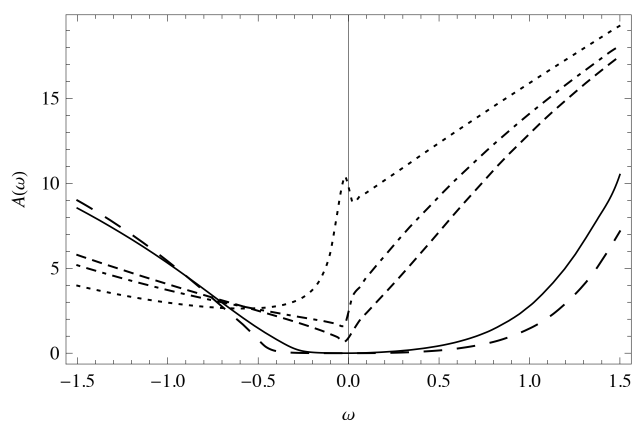


- What if there is no instability?
  - Tuning Δ all the way?

$$\mathrm{Im} G_R(\omega, k) \sim \mathrm{Im}\mathcal{G}(\omega, k) \qquad \mathcal{G}(\omega, k) \sim \omega^{2\nu_k}$$



[Gauntlett, Sonner, Waldram]

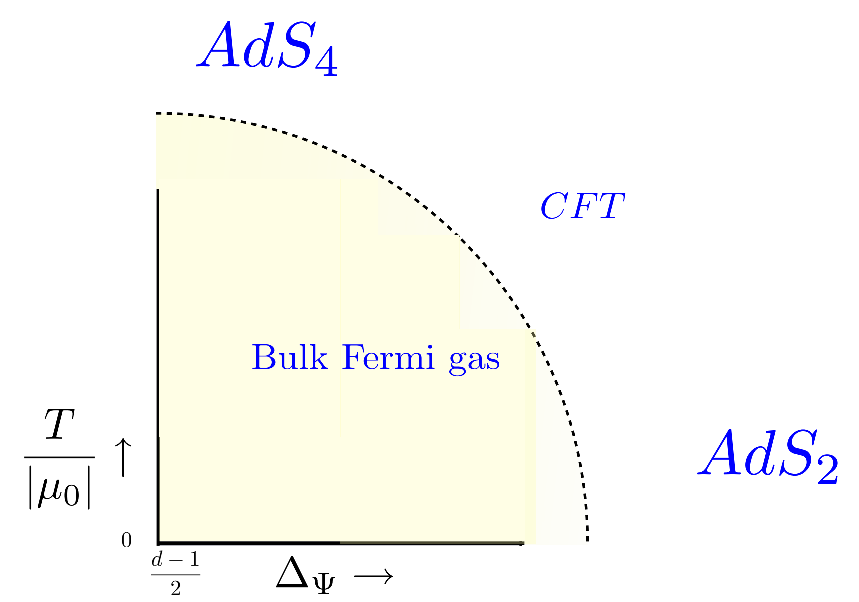


[Edalati, Leigh, Phillips]

- Connection to Strange Metal? Local Quantum Critical?

- What if there is no instability?
  - Tuning Δ all the way?

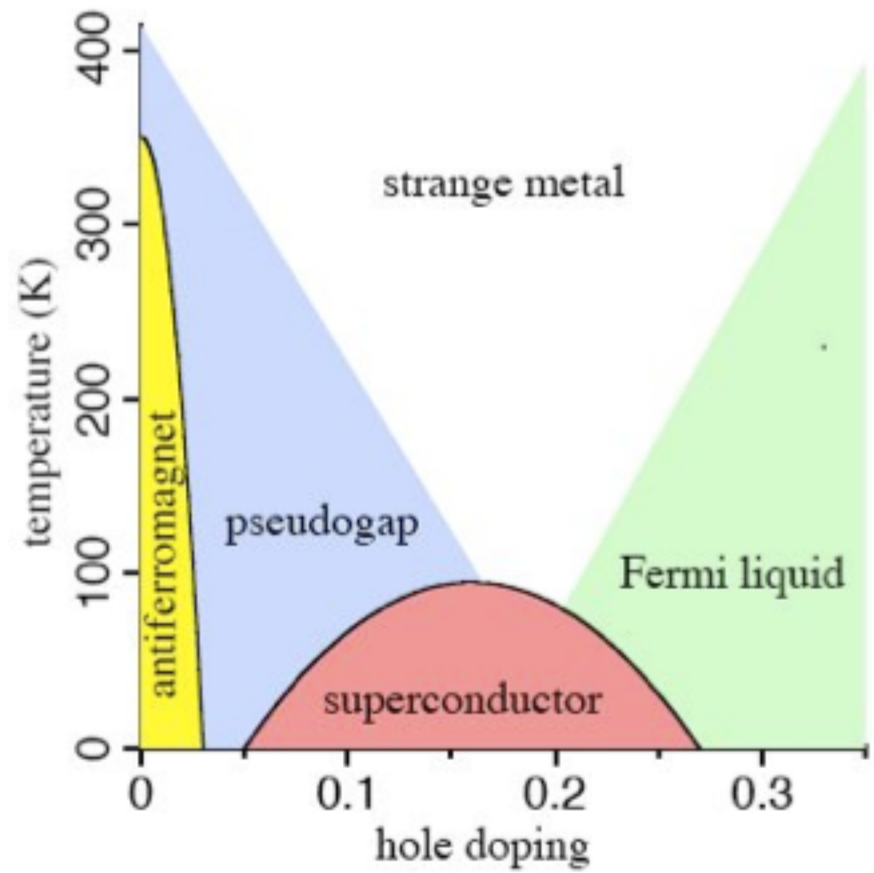
$$\mathrm{Im}G_R(\omega,k) \sim \mathrm{Im}\mathcal{G}(\omega,k) \qquad \mathcal{G}(\omega,k) \sim \omega^{2\nu_k}$$



- Connection to Strange Metal? Local Quantum Critical?

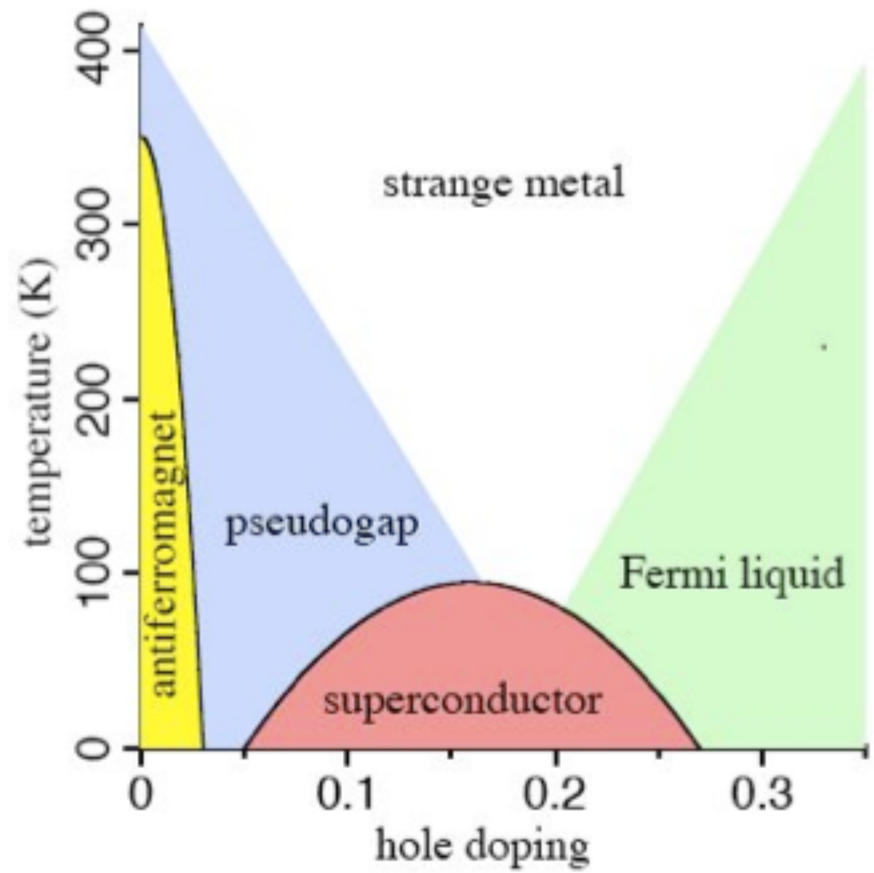
What a theory of High Tc superconductivity should explain...

- Strange Metal State
  - Local Quantum Criticality
  - Linear Resistivity
- High T superconductivity
  - What is the Glue?
- Stability of the FL
- The many orders of the Pseudogap/AFM phase
  - Nature of the QCP/Lattice effects
- ...

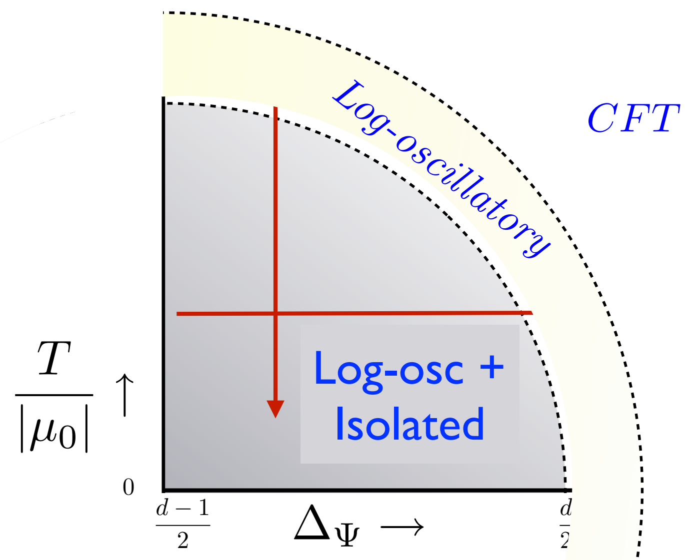


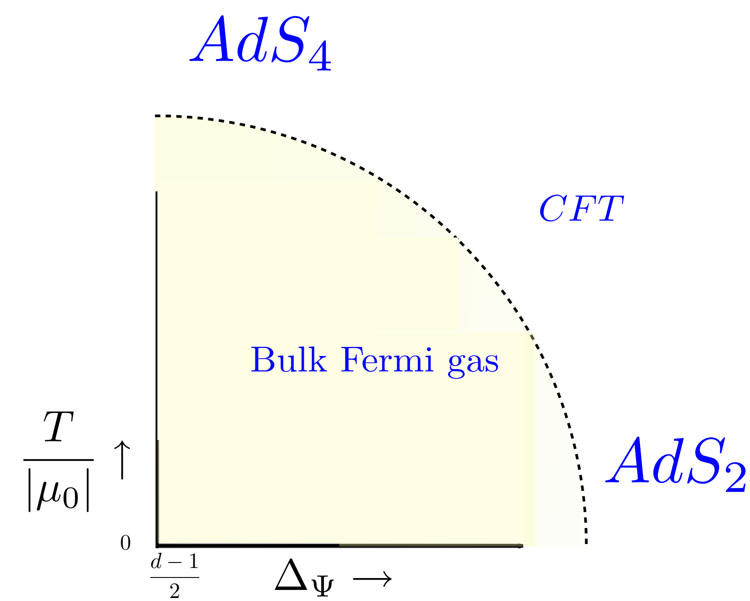
What a theory of High Tc superconductivity should explain...

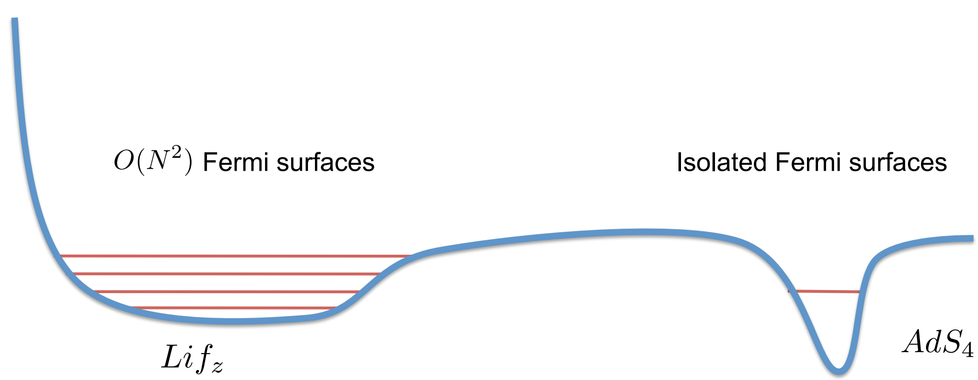
- Strange Metal State
  - Local Quantum Criticality
  - Linear Resistivity
- High T superconductivity
  - What is the Glue?
- **Stability of the FL**
- The many orders of the Pseudogap/AFM phase
  - Nature of the QCP/Lattice effects
- ...



- Dominant Instability of AdS-RN w. fermions?
  - Holographic Fermions ...Requires understanding of the other instabilities



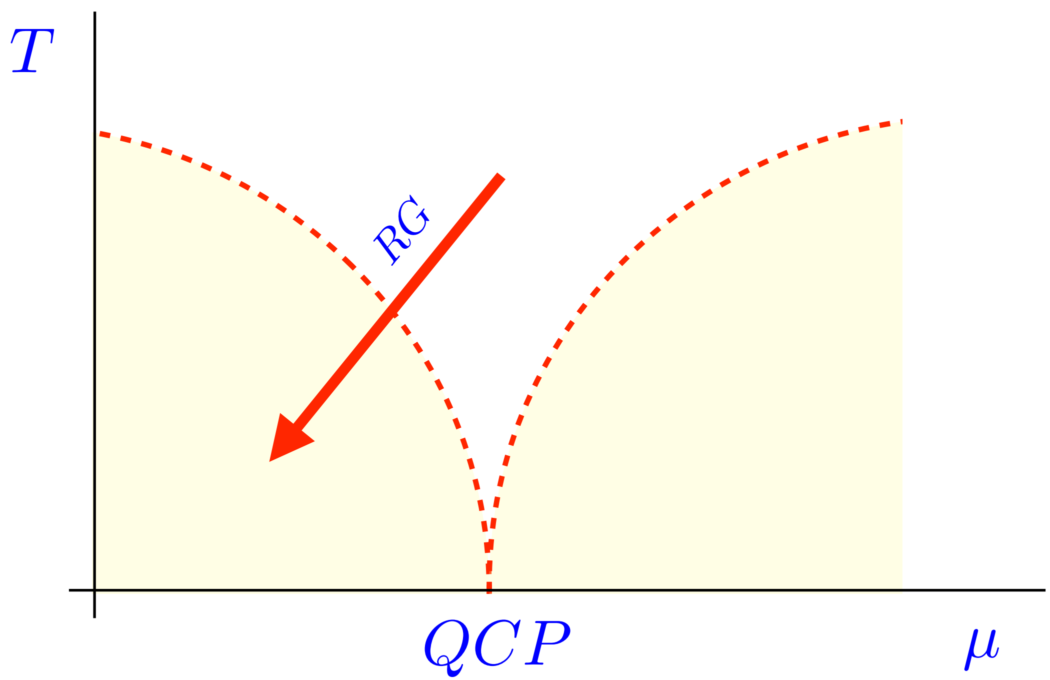




Why is only the marginal Fermi liquid observed?

- Holographic Strange Metal State = Near Horizon AdS-RN
  - Local Quantum Criticality (yes)
  - Linear Resistivity (no)

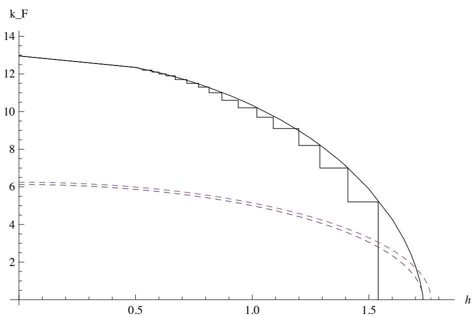
-Need another scale/parameter

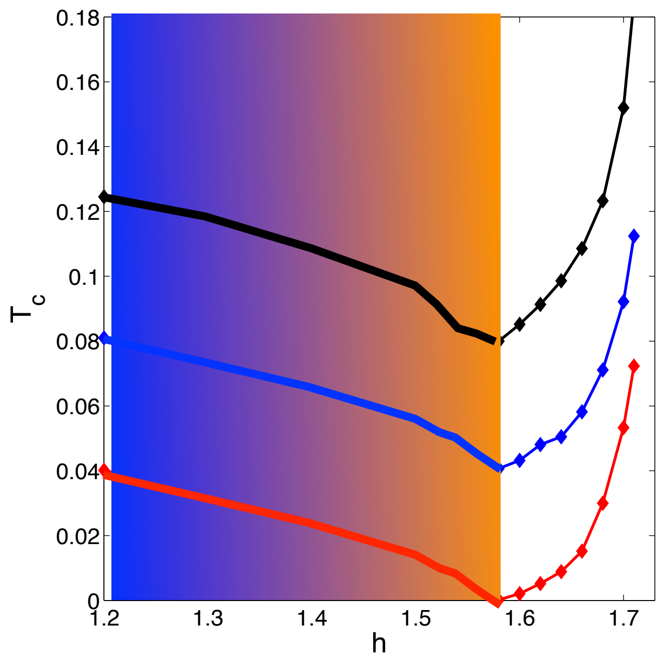


Why is only the marginal Fermi liquid observed?

- Holographic Strange Metal State = Near Horizon AdS-RN
  - Local Quantum Criticality (yes)
  - Linear Resistivity (no)

-Need another scale/parameter





$$(\not{D} - m)\Psi = 0, \;\; A_y = hx$$

$$\nu_{k_F} = 1/2$$

[Gubankova et al, Bolognesi, Tong]

Can we explain high Tc?

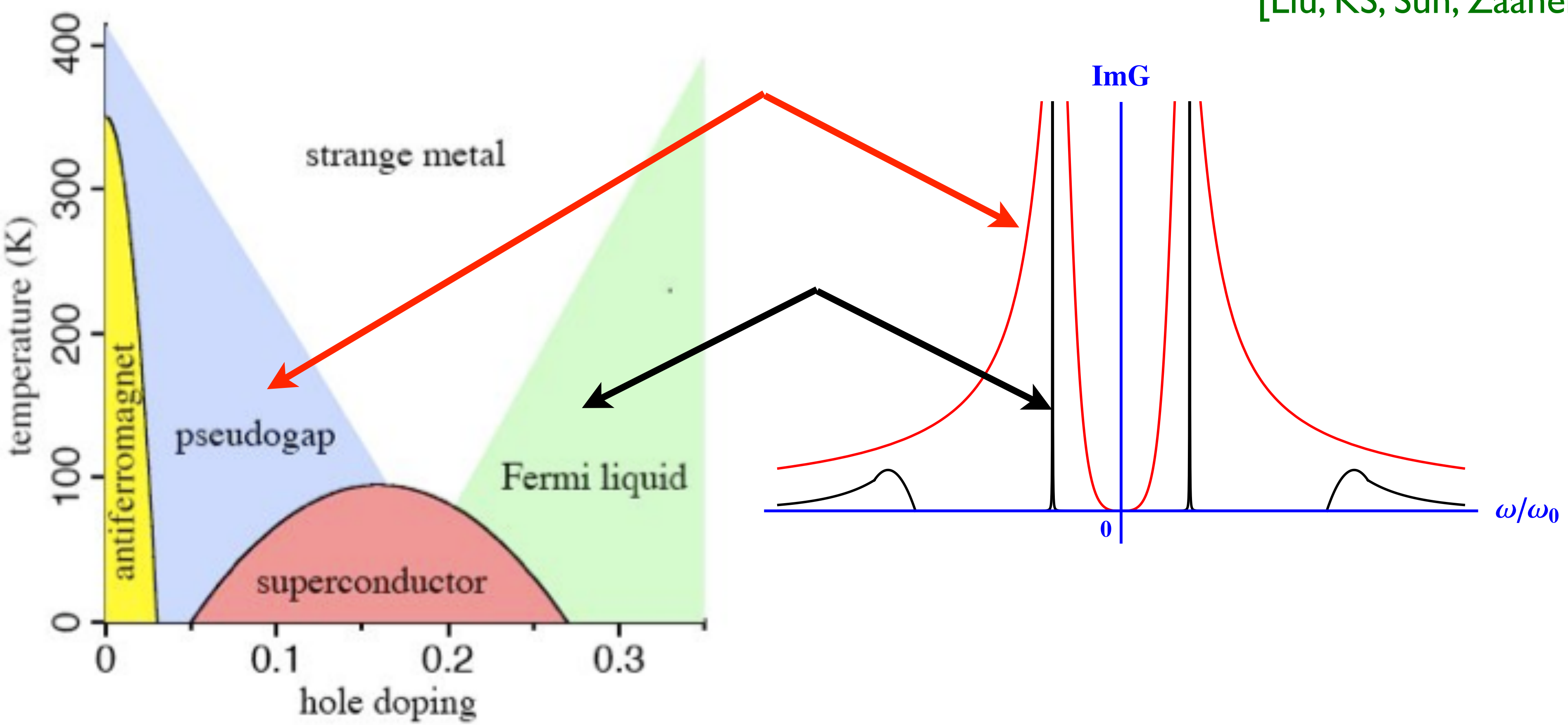
- Dominant Instability of AdS-RN w. fermions?
  - Holographic Fermions ...Requires understanding of the other instabilities

[Hartman, Hartnoll]

- The pseudogap mystery...

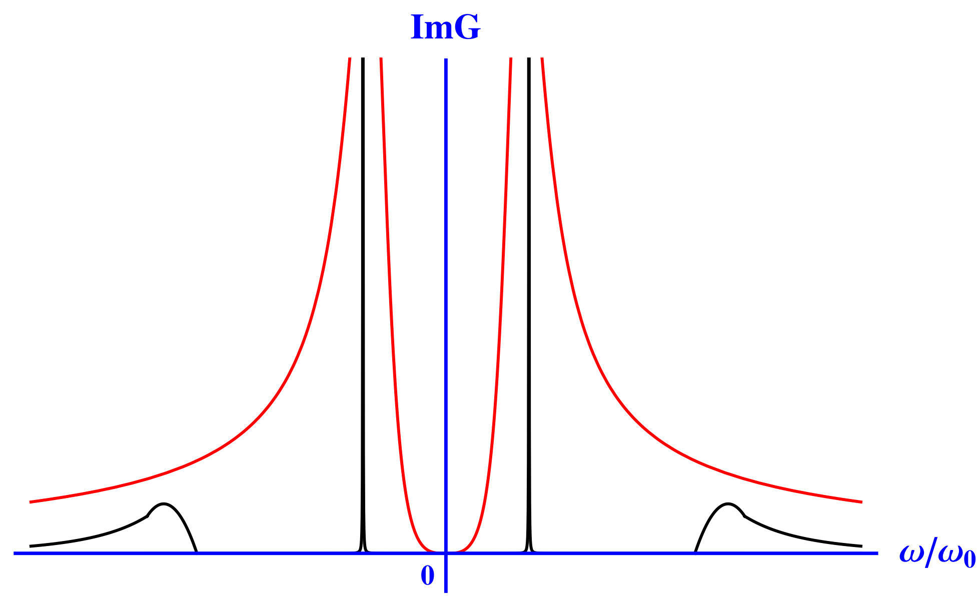
[Pseudogap in AdS/CMT:
Iqbal, Liu, Mezei,Si;
Aperis et al;
Edalati et al;
....]

[Liu, KS, Sun, Zaanen]



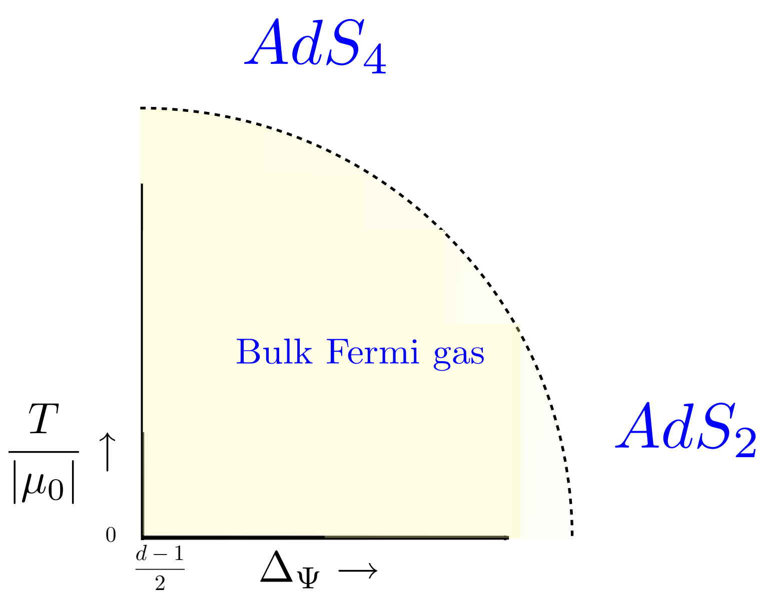
- The pseudogap mystery...
    - AdS2 metal is pseudogap for free

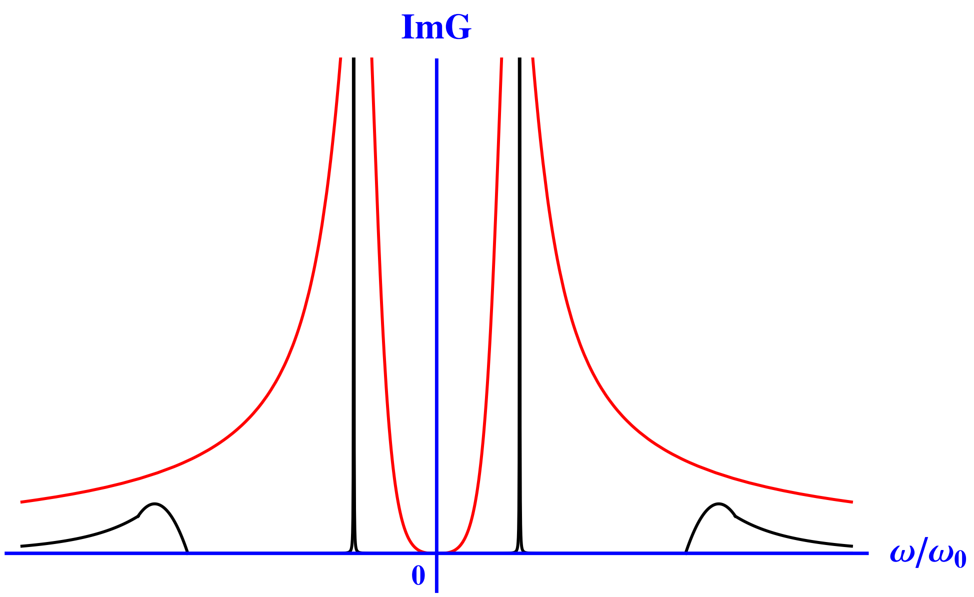
$$\mathrm{Im}G_R(\omega,k) \sim \mathrm{Im}\mathcal{G}(\omega,k) \qquad\qquad \mathcal{G}(\omega,k) \sim \omega^{2\nu_k}$$



- The pseudogap mystery...
  - AdS2 metal is pseudogap for free

$$\mathrm{Im}G_R(\omega,k) \sim \mathrm{Im}\mathcal{G}(\omega,k) \qquad \mathcal{G}(\omega,k) \sim \omega^{2\nu_k}$$

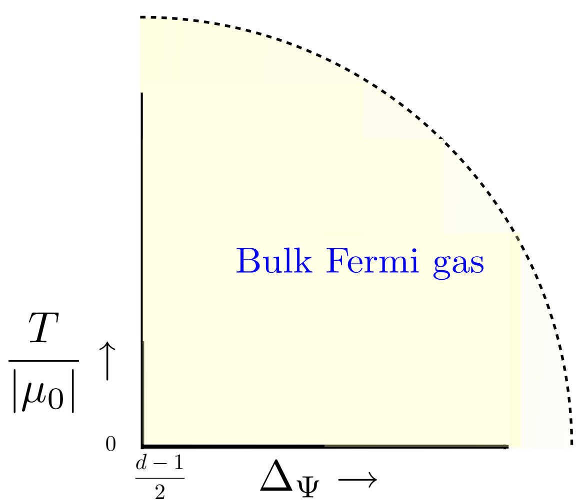




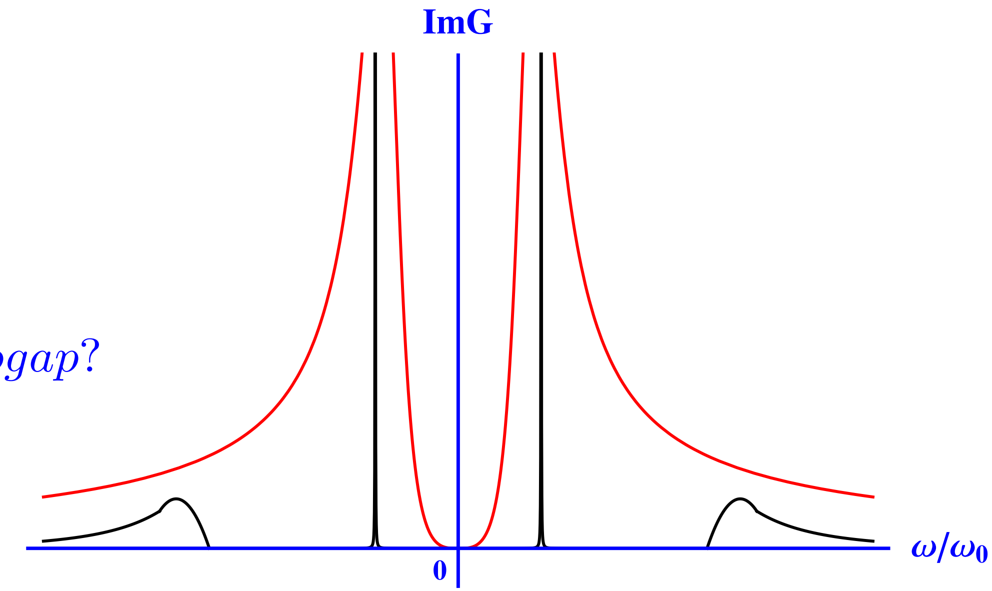
- The pseudogap mystery...
  - AdS2 metal is pseudogap for free

$$\mathrm{Im}G_R(\omega,k) \sim \mathrm{Im}\mathcal{G}(\omega,k) \qquad \mathcal{G}(\omega,k) \sim \omega^{2\nu_k}$$

AdS_4



$Pseudogap?$

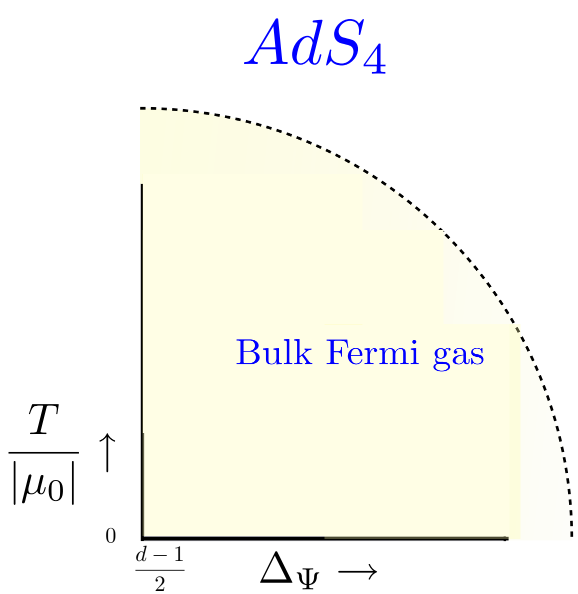


- The pseudogap mystery...

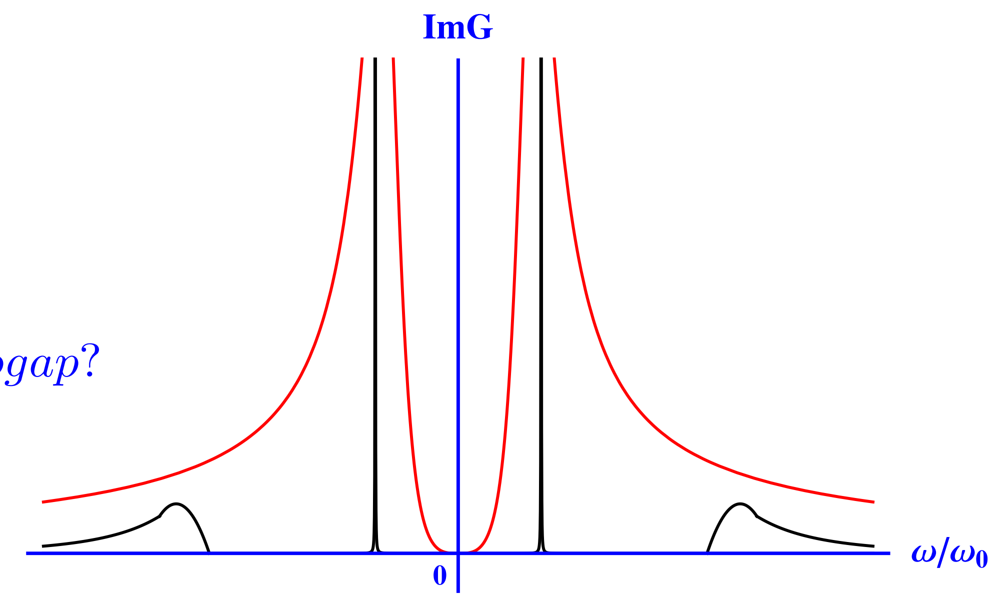
- Two holographic mysteries...

  - AdS2 metal is pseudogap for free

$$\mathrm{Im}G_R(\omega,k) \sim \mathrm{Im}\mathcal{G}(\omega,k) \qquad \mathcal{G}(\omega,k) \sim \omega^{2\nu_k}$$



$Pseudogap?$

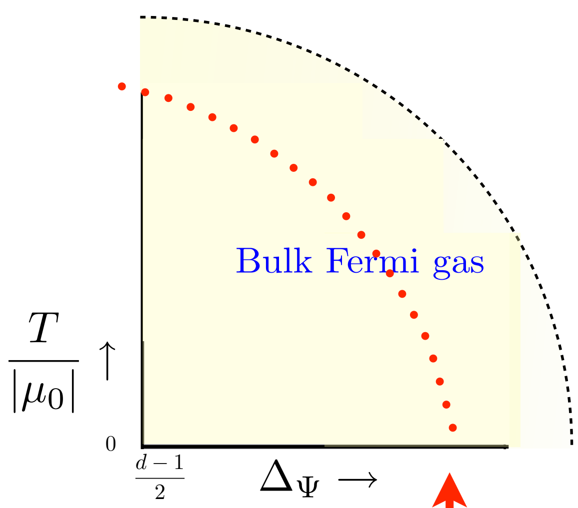


- The pseudogap mystery…
  - AdS2 metal is pseudogap for free

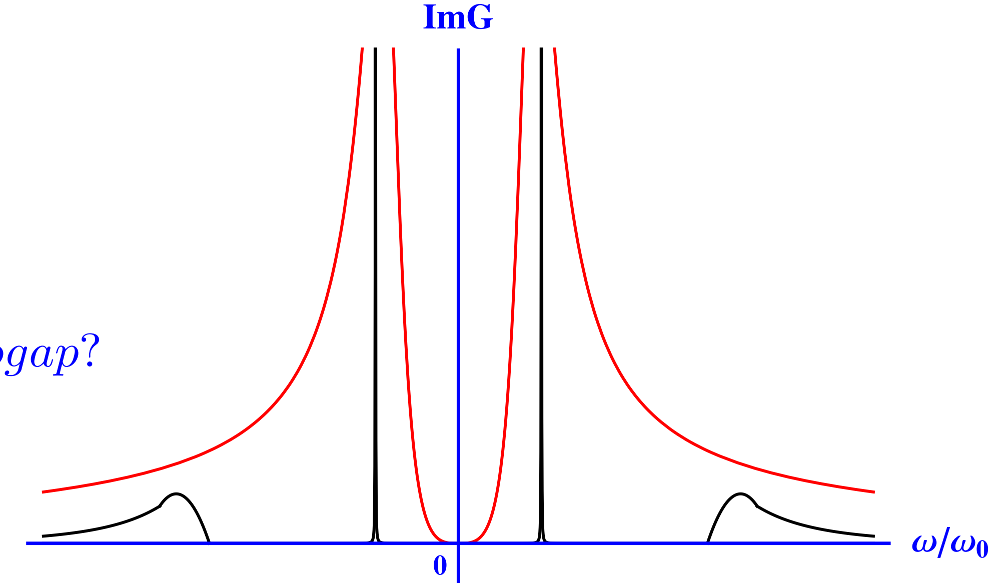
- Two holographic mysteries…

$$\mathrm{Im}G_R(\omega, k) \sim \mathrm{Im}\mathcal{G}(\omega, k) \qquad \mathcal{G}(\omega, k) \sim \omega^{2\nu_k}$$

AdS_4



$Pseudogap?$



- Marginal FL to NFL transition

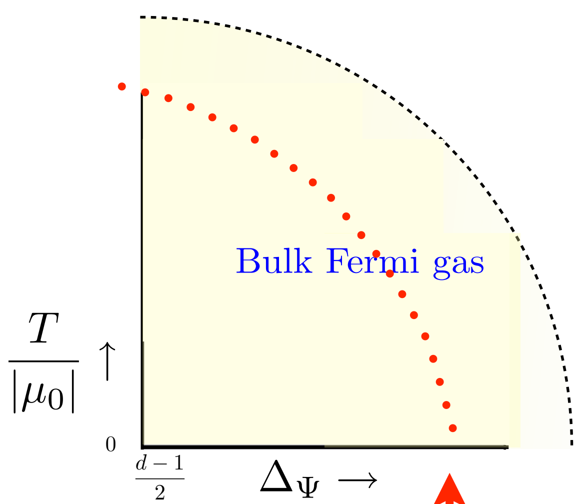
- The pseudogap mystery...

- Two holographic mysteries...

  - AdS2 metal is pseudogap for free

$$\mathrm{Im}G_R(\omega, k) \sim \mathrm{Im}\mathcal{G}(\omega, k) \qquad \mathcal{G}(\omega, k) \sim \omega^{2\nu_k}$$

AdS_4

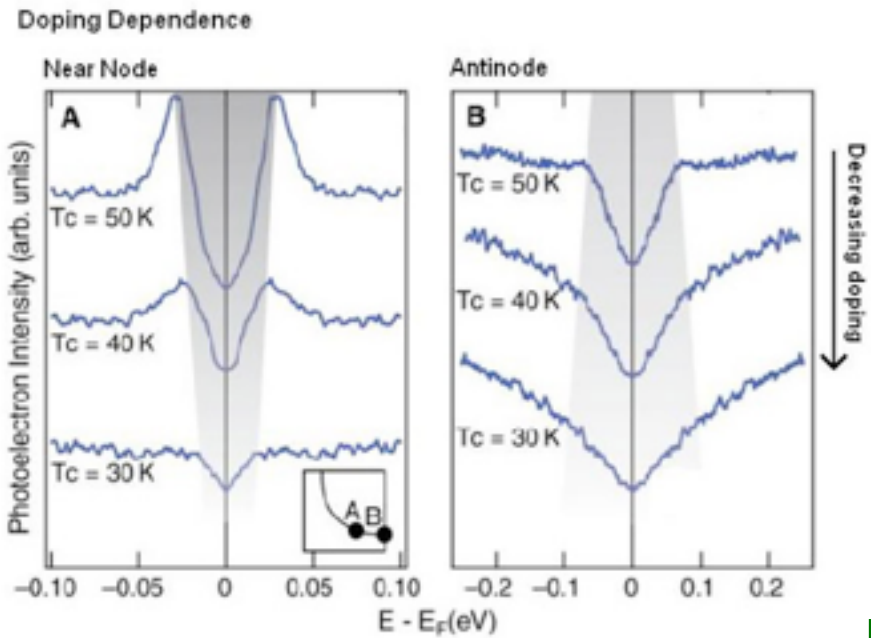


$Pseudogap?$

Pseudogap is thought to be connected to lattice features...

- Marginal FL to NFL transition

Experimental Pseudogap features



[Shen et al]

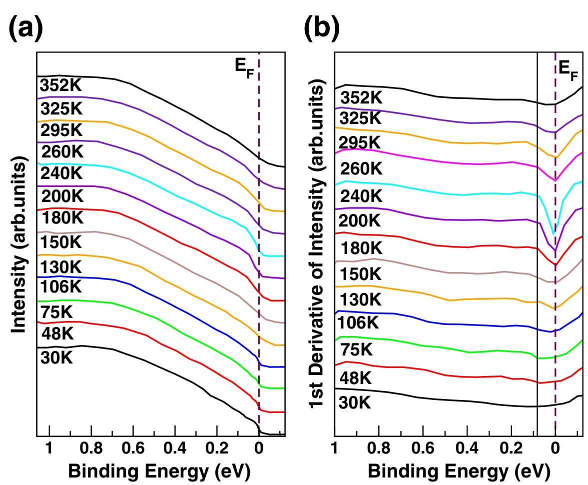
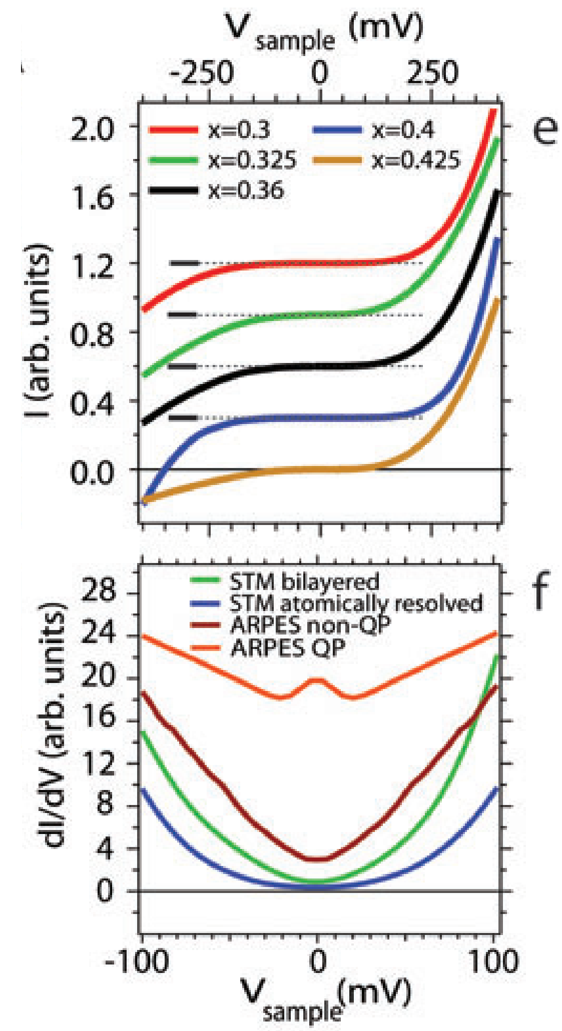


Fig. 2. (a) High-resolution valence band photoemission spectra of the near E_F region obtained by using 50 eV photons. (b) First derivative of the spectra shown the panel (a) to highlight the temperature dependent dip near E_F resulting the formation of a pseudogap. (The temperature dependent dip near E_F corresponds to the energy scale of 80 meV marked by the solid line, measures the pseudogap.)

[Ulfat et al]

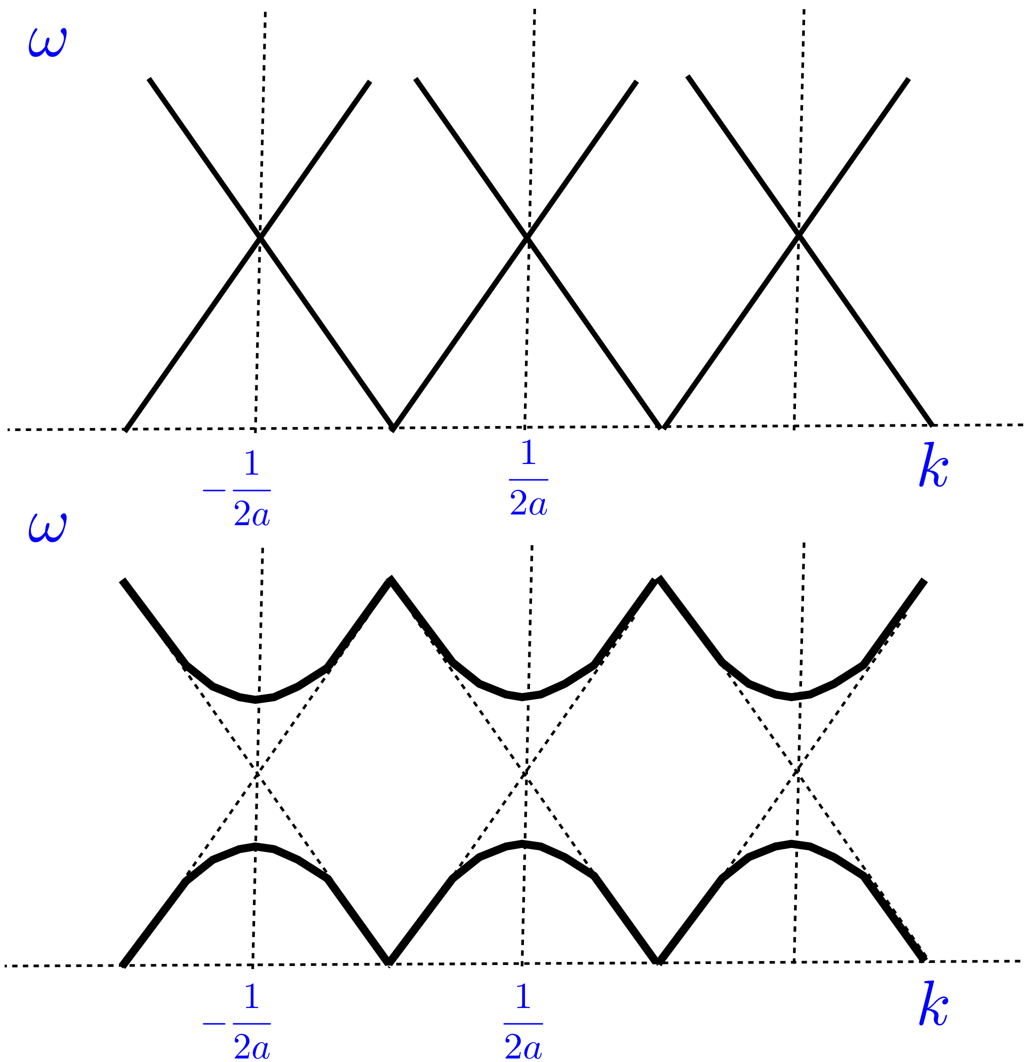


[Golden et al]

Holographic Fermions on the lattice

Lattice aspects of holographic fermions

- Solid State 101:
  - Bands from eigenvalue repulsion



$$\Psi(x) = \int_{-1/2a}^{1/2a} \frac{dk}{2\pi} \sum_{\ell \in Z} \Phi^{(\ell)}(k) e^{i(k+\ell K)x}$$

"Umklapp"

[Other aspects of Holographic Umklapp: Hartnoll,Hofman]

- Computing the spectral function from AdS/CFT

  - near the AdS boundary

$$\Psi(r) = \mathcal{A}\begin{pmatrix}0\\1\end{pmatrix} r^{m} + \mathcal{B}\begin{pmatrix}1\\0\end{pmatrix} r^{-m} + \ldots$$

  - The spectral function

$$G_R = \mathcal{B}\mathcal{A}^{-1}$$

  - Matching the near-horizon AdS2 behavior to the boundary behavior

$$G_R = \frac{b^{(0)} + \omega b^{(1)} + \ldots + \mathcal{G}(\omega,k)(\bar{b}^{(0)} + \omega \bar{b}^{(1)} + \ldots)}{a^{(0)} + \omega a^{(1)} + \ldots + \mathcal{G}(\omega,k)(\bar{a}^{(0)} + \omega \bar{a}^{(1)} + \ldots)}$$

with the AdS2 Green's function

$$\mathcal{G}(\omega,k) \sim \omega^{2\nu_k}$$

[Faulkner, Liu, McGreevy, Vegh]

- The novelty is the exponential IR local quantum criticality

$$\mathcal{G}(\omega, k) \sim \omega^{2\nu_k}$$

  - Large effect for the AdS2 metal

$$\mathrm{Im} G_R(\omega, k) \sim \mathrm{Im}\mathcal{G}(\omega, k)$$

  - Lattice induced dispersive effect in the FL regime

$$G_R \sim \frac{1}{\omega - v_F k + \mathcal{G}}$$

- Lattice periodicity by a modulated chemical potential

$$
\begin{aligned}
ds^2 =& -r^2 f(r)dt^2 + \frac{dr^2}{r^2 f(r)} + r^2(dx^2 + dy^2) \\
A_t =& [\mu_0 + \mu_1(x)](1 - \frac{1}{r}) \\
f(r) =& 1 - \frac{1+Q^2}{r^3} + \frac{Q^2}{r^4} \\
\mu_0 =& 2Q
\end{aligned}
$$

ignore gravitational backreaction

$$
\mu_1 = 2\epsilon \cos(\frac{x}{a}) \ , \quad \epsilon \ll Q
$$

[Flauger et al; Aperis et al; Horowitz]

- Different from explicit "brane intersection" lattices:

[Kachru, Karch, Yaida]

Standard exercise but much more intricate!

- Solve Dirac equation .... in perturbation in $\delta\mu \sim \epsilon$

$$(e^{\mu}_{A}\mathcal{D}_{\mu}\Gamma^{A} + m)\Psi = 0$$

$$\Psi(x) = \int_{-1/2a}^{1/2a} \frac{dk}{2\pi} \sum_{\ell \in Z} \Phi^{(\ell)}(k) e^{i(k+\ell K)x}$$

  - single particle Green's function

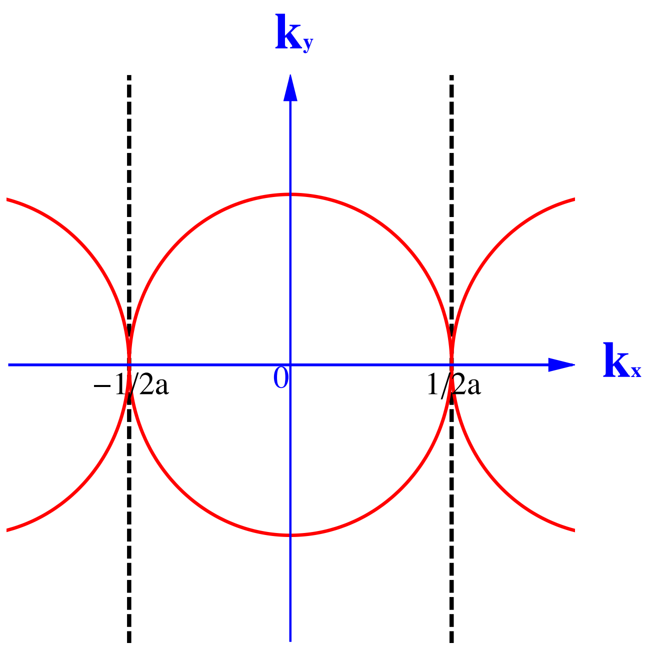
$$G_R = \mathcal{B}\mathcal{A}^{-1}$$

$$\mathcal{A}_{\alpha\ell,\beta\ell'} = \begin{pmatrix} \ddots & & & \\ & C\delta_{\alpha\beta} + \epsilon^2 S_{\alpha\beta} + \dots & \epsilon F_{\alpha\beta} + \dots & \\ & \epsilon F^{\dagger}_{\alpha\beta} + \dots & C\delta_{\alpha\beta} + \epsilon^2 S_{\alpha\beta} + \dots & \epsilon F_{\alpha\beta} + \dots \\ & & \epsilon F^{\dagger}_{\alpha\beta} + \dots & C\delta_{\alpha\beta} + \epsilon^2 S_{\alpha\beta} + \dots \\ & & & & \ddots \end{pmatrix}$$

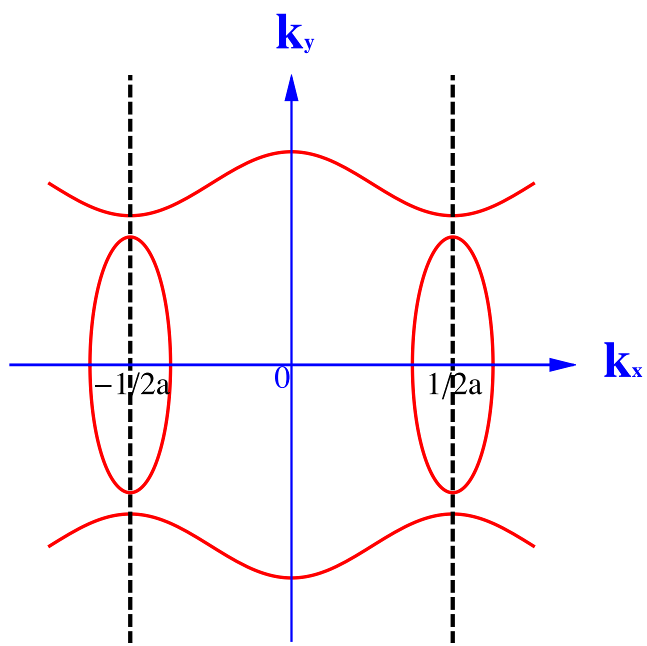
  - and similar for $\mathcal{B}_{\alpha\ell,\beta\ell'}$

- In the Fermi-liquid regime ($q \gg m$): a bandgap
  - Predictable from semi-holography

$$G_R \sim \frac{1}{\omega - v_F k + \mathcal{G}}$$



$$k_F \sim \mu_0 = K \equiv \frac{1}{a}$$



$$k_F \sim \mu_0 \gg K \equiv \frac{1}{a}$$

[Liu, Sun, KS, Zaanen]

- What happens to $\mathcal{G}(\omega, k)$

$$\mathcal{G}(\omega, k) = \alpha_k \omega^{2\nu_k} + \beta_k^{(-)} \omega^{2\nu_{k-K}} + \beta_k^{(0)} \omega^{2\nu_k} \ln \omega + \beta_k^{(+)} \omega^{2\nu_{k+K}} + \ldots$$

  - Term with $\beta_k^{(0)}$ is basically first term in $\nu_k + \delta\mu \dfrac{\partial \nu_k}{\partial \mu} + \ldots$

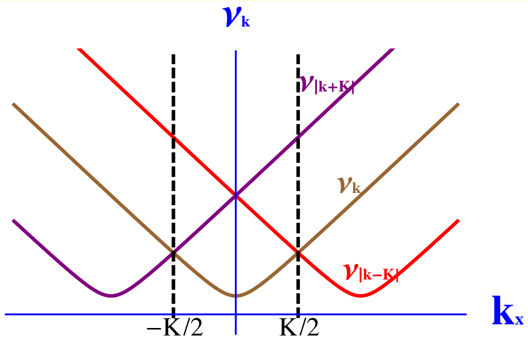
Novel Physics: Periodic Dispersion

- What happens to $\mathcal{G}(\omega, k)$

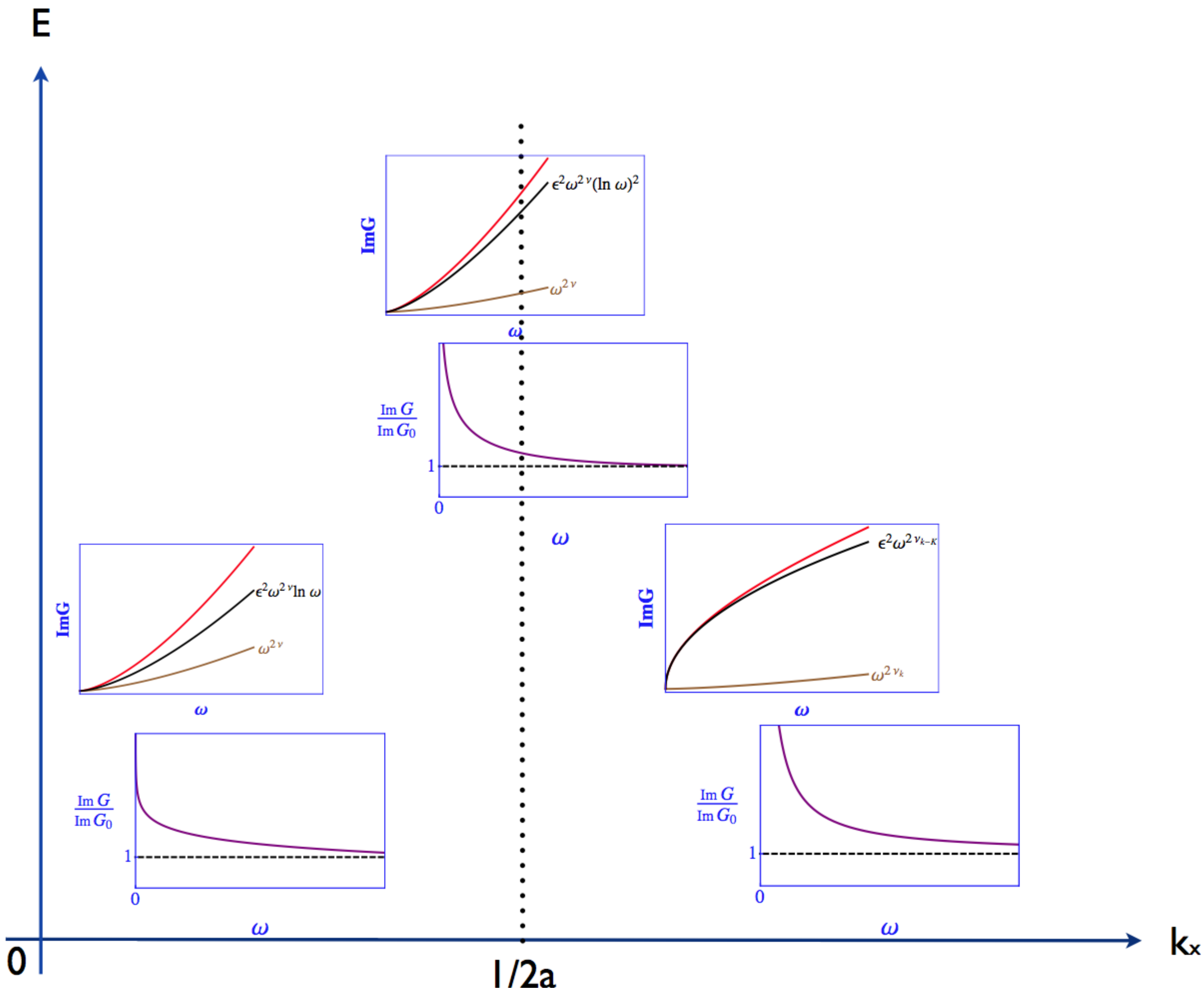
$$\mathcal{G}(\omega, k) = \alpha_k \omega^{2\nu_k} + \beta_k^{(-)} \omega^{2\nu_{k-K}} + \beta_k^{(0)} \omega^{2\nu_k} \ln \omega + \beta_k^{(+)} \omega^{2\nu_{k+K}} + \dots$$

- Term with $\beta_k^{(0)}$ is basically first term in $\nu_k + \delta\mu \dfrac{\partial \nu_k}{\partial \mu} + \dots$
- Other term are true lattice effects:

(I) Qualitative change the IR behavior
(II) Quantitative effect depends on the Brillioun zone

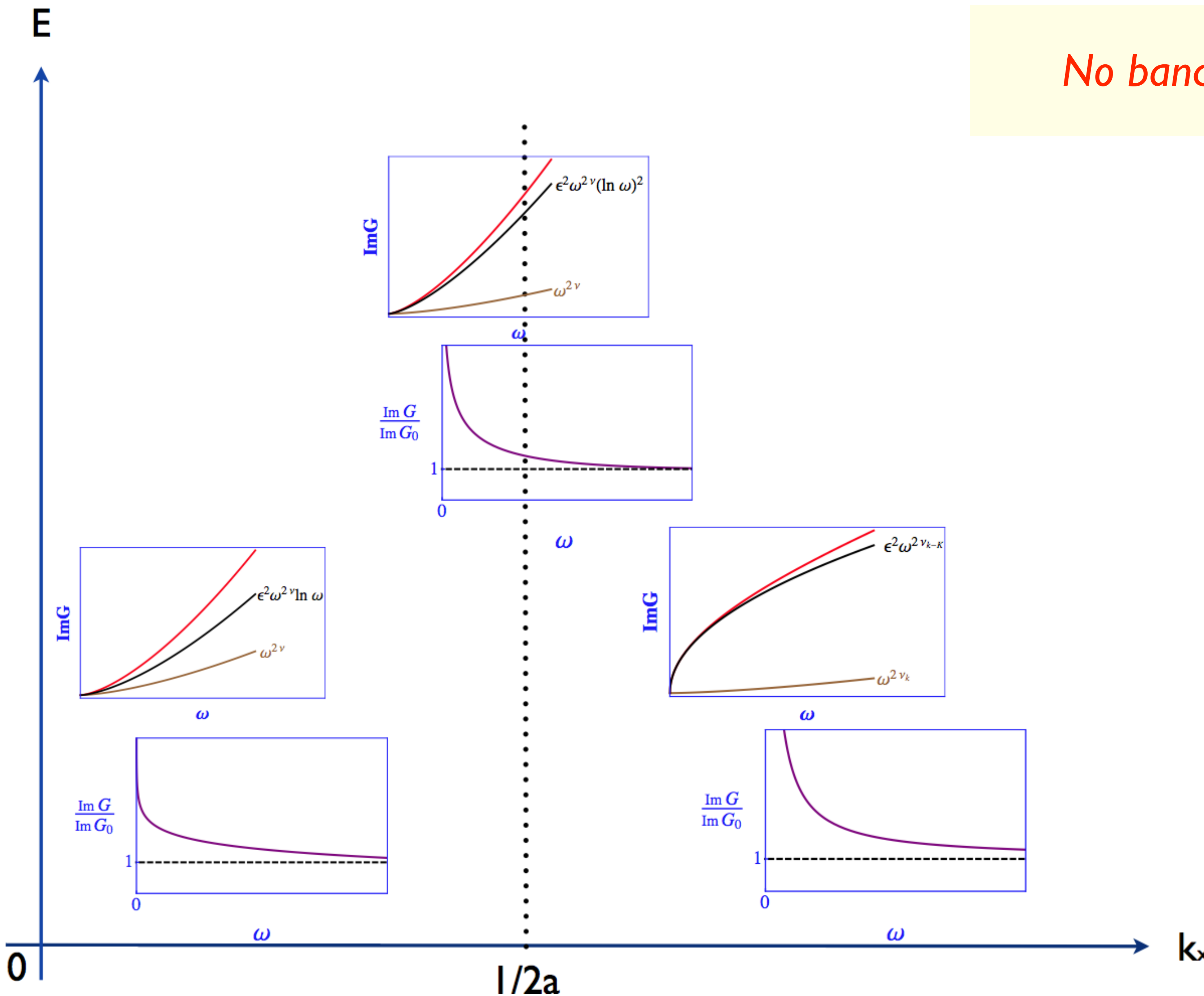


The AdS2 metal latticized



The AdS2 metal latticized

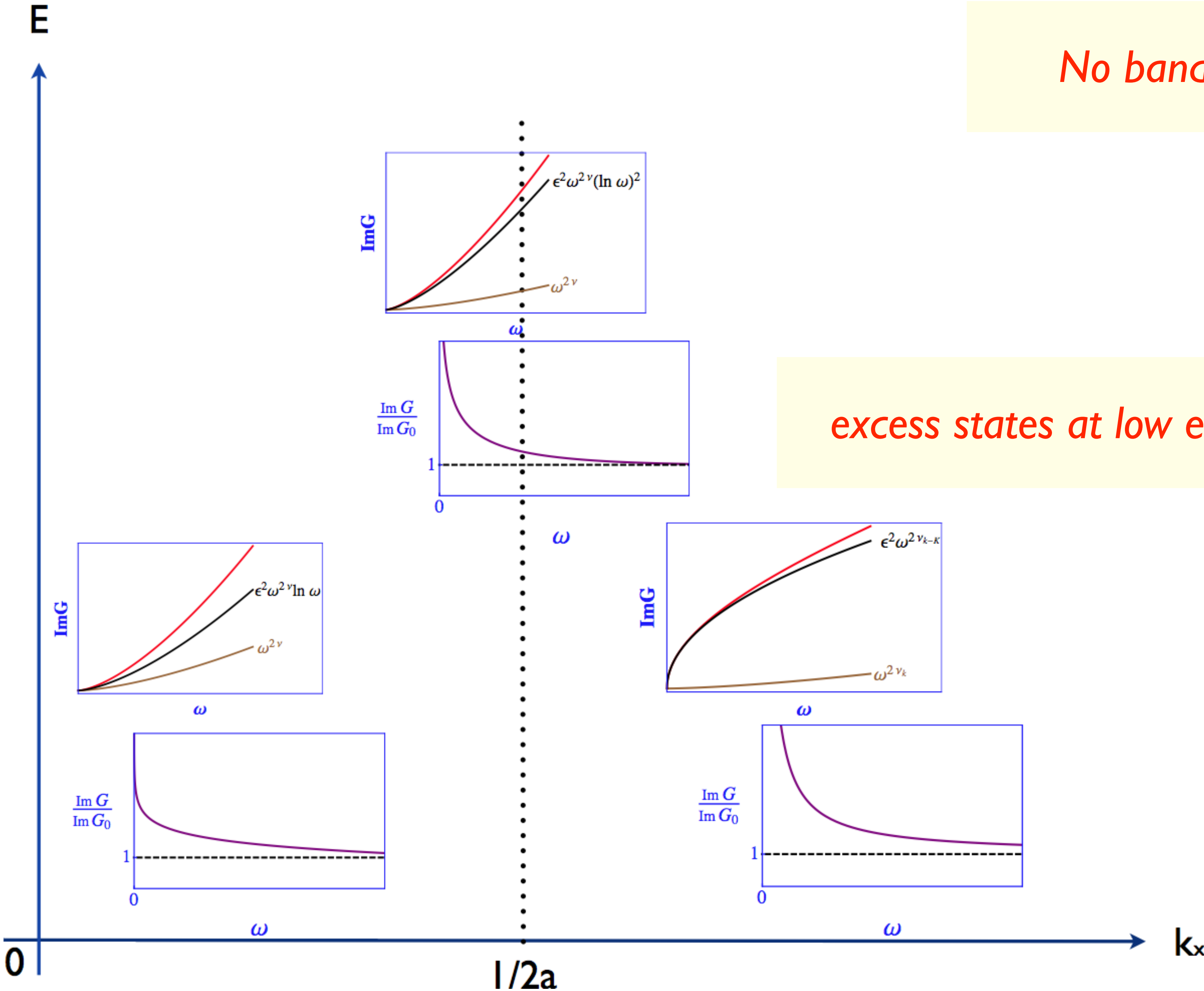
No bandgap



The AdS2 metal latticized

No bandgap

excess states at low energies



Summary

- In FL regime higher BZs are softened
  - width of QP peak $\mathrm{Im}\Sigma = \mathcal{G}$

1st Brillioun Zone $\mathcal{G}(\omega, k) = \alpha_k \omega^{2\nu_k} + \beta_k^{(0)} \omega^{2\nu_k} \ln \omega + \ldots$

2nd Brillioun Zone $\mathcal{G}(\omega, k) = \beta_k^{(-)} \omega^{2\nu_{k-K}} + \ldots$

Summary

- In FL regime higher BZs are softened
  - width of QP peak $\mathrm{Im}\Sigma = \mathcal{G}$

1st Brillioun Zone $\mathcal{G}(\omega, k) = \alpha_k \omega^{2\nu_k} + \beta_k^{(0)} \omega^{2\nu_k} \ln\omega + \dots$

2nd Brillioun Zone $\mathcal{G}(\omega, k) = \beta_k^{(-)} \omega^{2\nu_{k-K}} + \dots$

- The AdS2 metal *(natural holographic pseudogap)*

(I) Qualitative change the IR behavior *Thermodynamics?*

(II) Quantitative effect depends on the Brillioun zone

Summary

- In FL regime higher BZs are softened
  - width of QP peak $\mathrm{Im}\Sigma = \mathcal{G}$

1st Brillioun Zone $\mathcal{G}(\omega, k) = \alpha_k \omega^{2\nu_k} + \beta_k^{(0)} \omega^{2\nu_k} \ln \omega + \dots$

2nd Brillioun Zone $\mathcal{G}(\omega, k) = \beta_k^{(-)} \omega^{2\nu_{k-K}} + \dots$

- The AdS2 metal *(natural holographic pseudogap)*

(I) Qualitative change the IR behavior *Thermodynamics?*
(II) Quantitative effect depends on the Brillioun zone

If so, could this be seen in experiment....?

Conclusion

- Holographic Fermions
  - Does its success, NFL and MFL, survive the stability?
  - What is the physics of marginality?

    the confluence of two QCPs?

    a FL to NFL transition?

    both?
  - What are the other instabilities?

    Onset of superconductivity; pseudogap, other orders?

    Lattice effects?

Conclusion

- Holographic Fermions
  - Does its success, NFL and MFL, survive the stability?
  - What is the physics of marginality?

    the confluence of two QCPs?

    a FL to NFL transition?

    both?
  - What are the other instabilities?

    Onset of superconductivity; pseudogap, other orders?

    Lattice effects?

Conclusion

- Holographic Fermions
  - Does its success, NFL and MFL, survive the stability?
  - What is the physics of marginality?

    the confluence of two QCPs?

    a FL to NFL transition?

    both?
  - What are the other instabilities?

    Onset of superconductivity; pseudogap, other orders?

    Lattice effects?

Thank you.