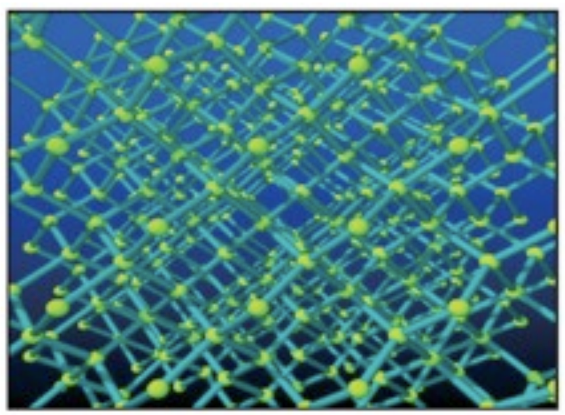
Transport: integrability and topology

Chicheley Hall, 22 April 2012



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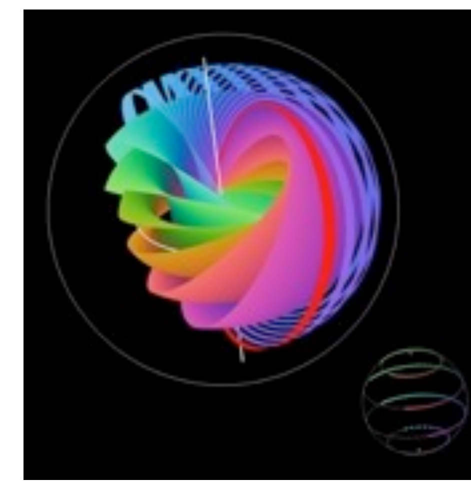


Motivation:

unconventional transport (linear conductivities)

1. Integrability in 1D systems and its consequences for transport
(Christoph Karrasch, Jens Bardarson, and JEM, PRL to appear)

2. Disorder and topological protection
(work with Balents, Essin, Turner, Vanderbilt, Ryu, Ludwig, et al.)

Basic idea: Many closed quantum systems are "localized" (have zero conductivity) with disorder, particularly in low dimensions.

Surfaces of many topological phases are a subtle way around this.

3. Can we use gravity to define topological phases beyond weak interactions?

Dissipationless transport

When is there a nonzero Drude weight *D*?

$$\sigma(\omega) = D\delta(\omega) + \dots$$

Two easy examples:

I. Superconductors (transport by condensate)

II. Part of the current is conserved: Mazur lower bound

$$D = \frac{1}{2LT} \lim_{t\to\infty} \langle J(t)J(0)\rangle \geq \frac{1}{2LT} \sum_k \frac{\langle JQ_k\rangle^2}{\langle {Q_k}^2\rangle}$$

Integrability

$$\sigma(\omega) = D\delta(\omega) + \dots$$

$$D = \frac{1}{2LT} \lim_{t\to\infty} \langle J(t)J(0)\rangle \geq \frac{1}{2LT} \sum_k \frac{\langle JQ_k\rangle^2}{\langle {Q_k}^2\rangle}$$

What about "integrable" models with an infinite number of conserved local quantities, none of which gives a lower bound?

Actually this happens quite often in 1D--simplest case is spinless interacting fermions (XXZ model in zero magnetic field).

$$H = \sum_i \left[J_{xx}(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + \Delta S_i^z S_{i+1}^z + h S_i^z\right]$$

The Drude weight is easy to calculate and nonzero at T=0.
20+ years of efforts to calculate it (or even show that it is nonzero) at T>0, by either analytical or numerical methods, until...

Some progress, 2011-present

$$\sigma(\omega) = D\delta(\omega) + \dots$$

$$D = \frac{1}{2LT} \lim_{t\to\infty} \langle J(t)J(0)\rangle \geq \frac{1}{2LT} \sum_k \frac{\langle JQ_k\rangle^2}{\langle {Q_k}^2\rangle}$$

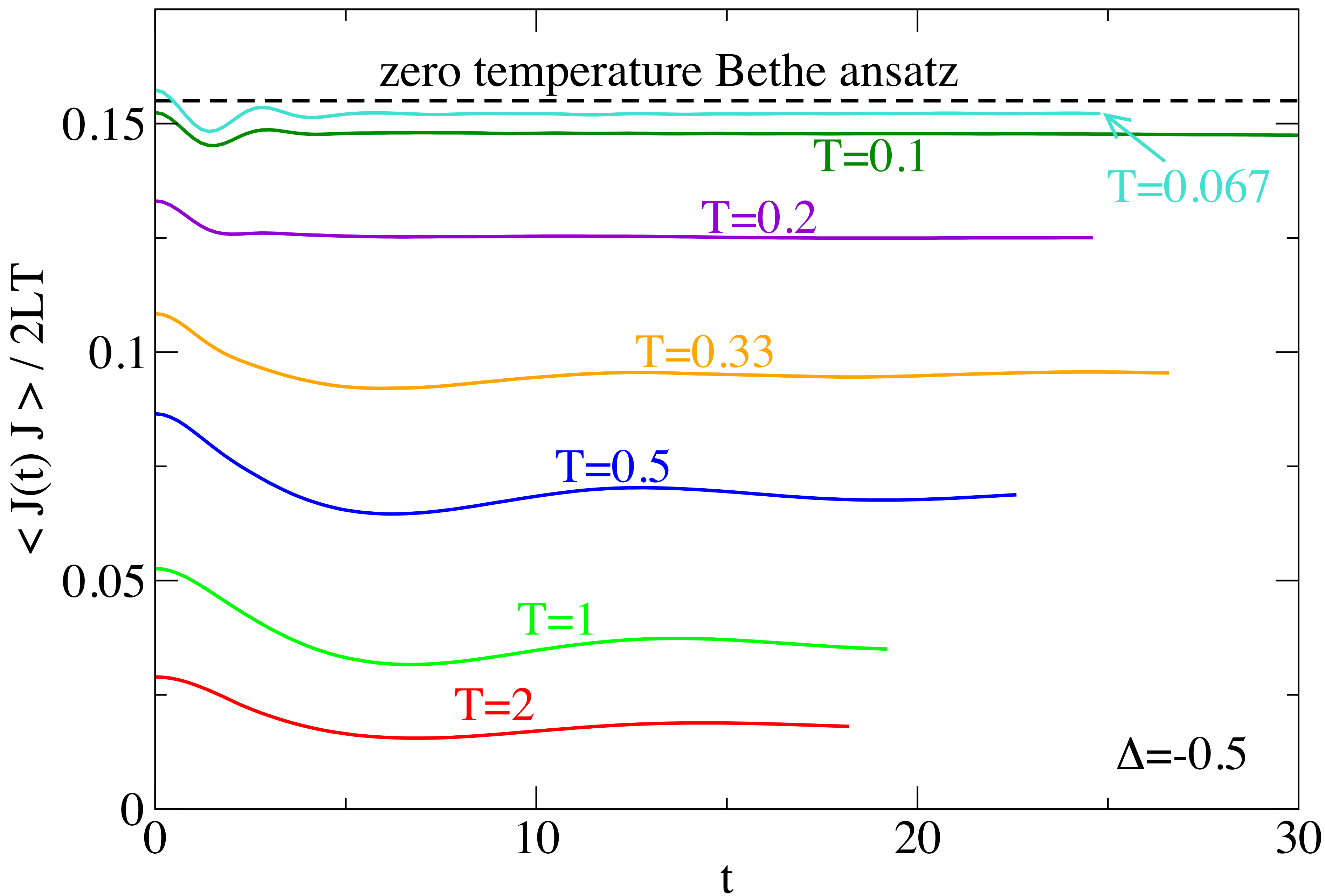
Prosen: there is an iterative process to construct a nonlocal quantity that gives a lower bound that depends "fractally" on anisotropy, with cusps at $\Delta=\cos(\pi/n)$.

KBM: The Drude weight can be calculated numerically for all but the lowest temperatures at positive Δ, and essentially all temperatures at negative Δ.

The lower bound appears to saturate the full value at the cusps.

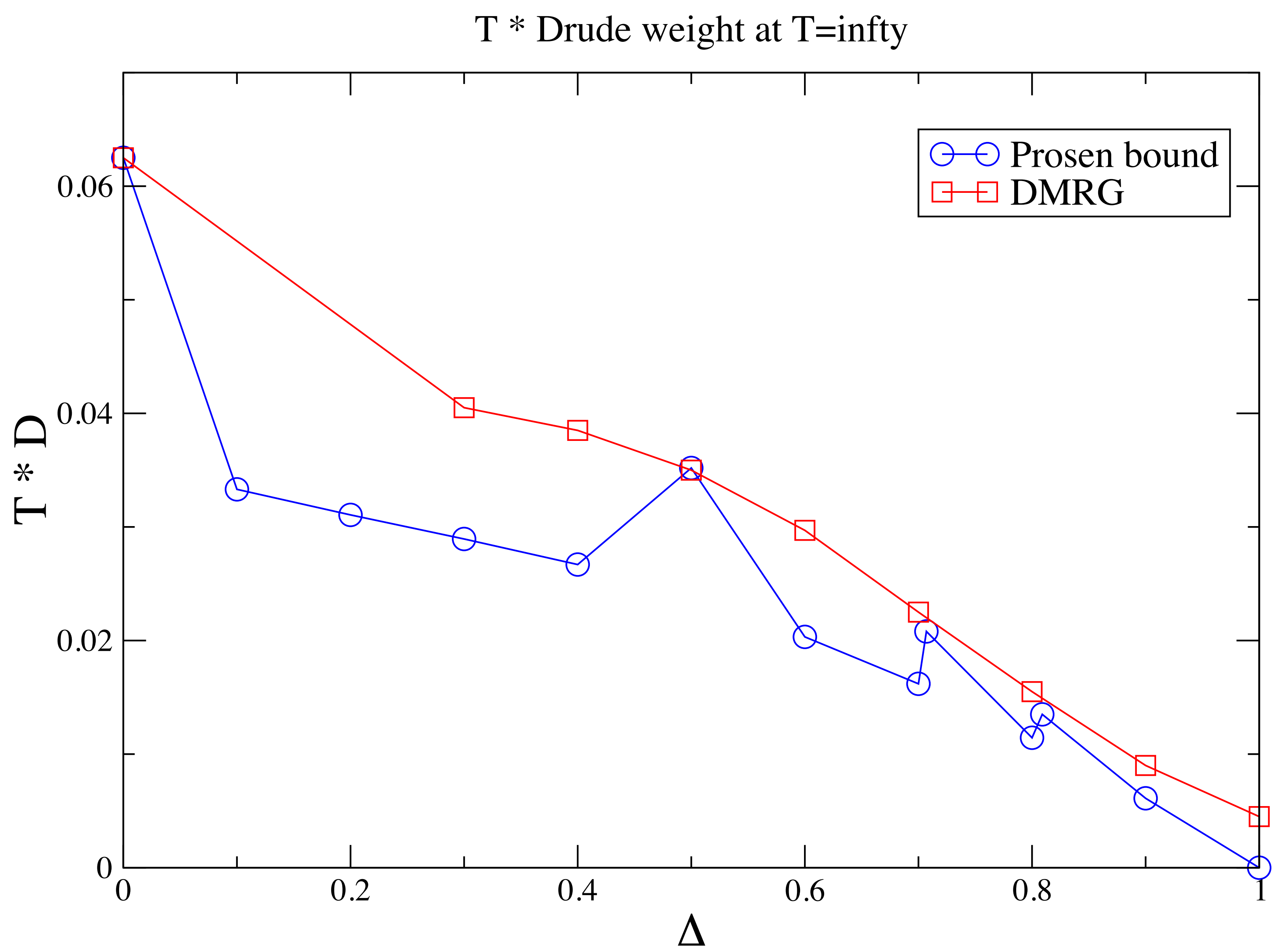
Drude weight of XXZ model

$$\sigma(\omega) = D\delta(\omega) + \ldots$$



Time-dependent matrix-product-state numerics using an "entanglement-based" trick

Agreement with lower bound:



Lessons

Without disorder, transport can be very sensitive to integrability--*gapless* integrable models seem to have nonzero Drude weight in general.

(What happens in supersymmetric models in d>1?)

Two approaches in AdS/CMT to compute transport:

1. add massive particles and look at some pre-equilibrium time scale

2. add a lattice (insufficient in XXZ example)

3. add disorder

How does transport in gapless systems respond to disorder (and interactions)?

Why does topology matter?

Intro to disordered electronic systems

For non-interacting systems, we understand essentially completely the effects of disorder.
For the simplest symmetries (orthogonal and unitary ensembles), disorder is localizing for essentially all states in 1D and 2D.

Real experimental systems typically have "dephasing" from interactions with phonons, which ultimately leads to a finite diffusion constant.

The combination of interactions and disorder in closed systems ("many-body localization") is not well understood even in 1D.
are the only two possibilities diffusive and localized? can there be subdiffusive scaling? ("glassy": $r \sim \log t$)

Gapped topological phases have "protected" surface states, i.e., no localization. In 1D these are dissipationless, but not in higher dimensions.

The "quantum spin Hall effect"

Spin-orbit coupling appears in nearly every atom and solid. Consider the standard atomic expression

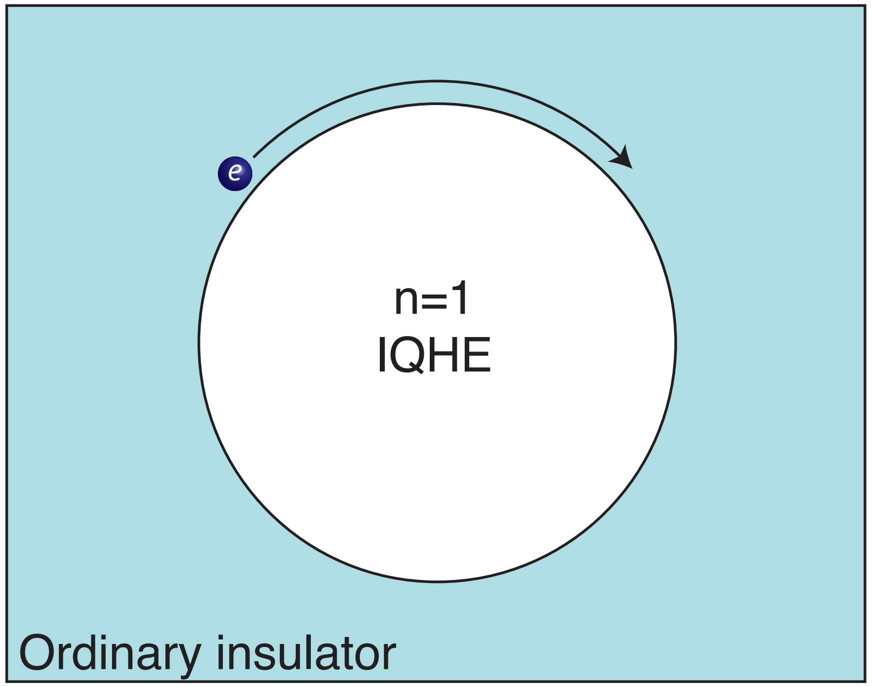
$$H_{SO} = \lambda \mathbf{L} \cdot \mathbf{S}$$

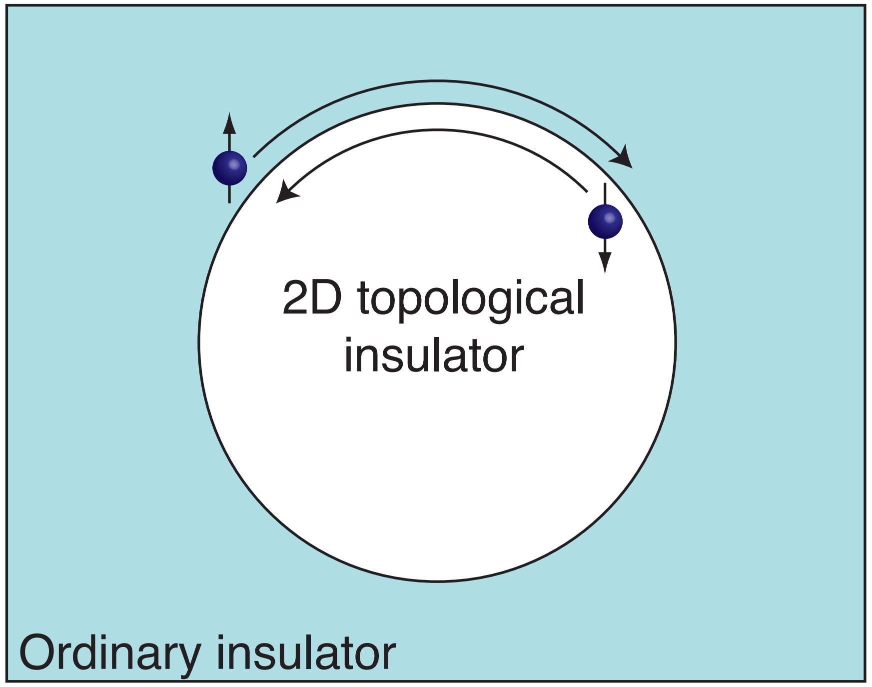
For a given spin, this term leads to a momentum-dependent force on the electron, somewhat like a magnetic field.

The spin-dependence means that the *time-reversal symmetry* of SO coupling (even) is different from a real magnetic field (odd).

It is possible to design lattice models where spin-orbit coupling has a remarkable effect: (Murakami, Nagaosa, Zhang 04; Kane, Mele 05)

spin-up and spin-down electrons are in IQHE states, with opposite "effective magnetic fields".





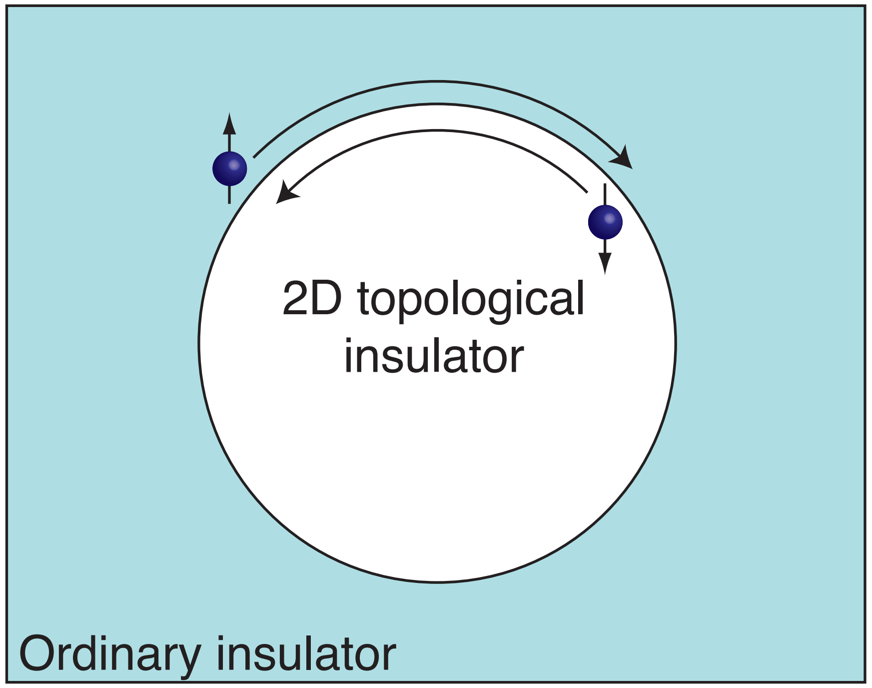
The 2D topological insulator

It was shown in 2005 (Kane and Mele) that, in real solids with all spins mixed and no "spin current", something of this physics does survive.

In a material with only spin-orbit, the "Chern number" mentioned before always vanishes.

Kane and Mele found a new topological invariant in time-reversal-invariant systems of fermions.

But it isn't an integer! It is a Chern *parity* ("odd" or "even"), or a "Z2 invariant".



Systems in the "odd" class are "2D topological insulators"

1. Where does this "odd-even" effect come from?
2. What is the Berry phase expression of the invariant?
3. How can this edge be seen?

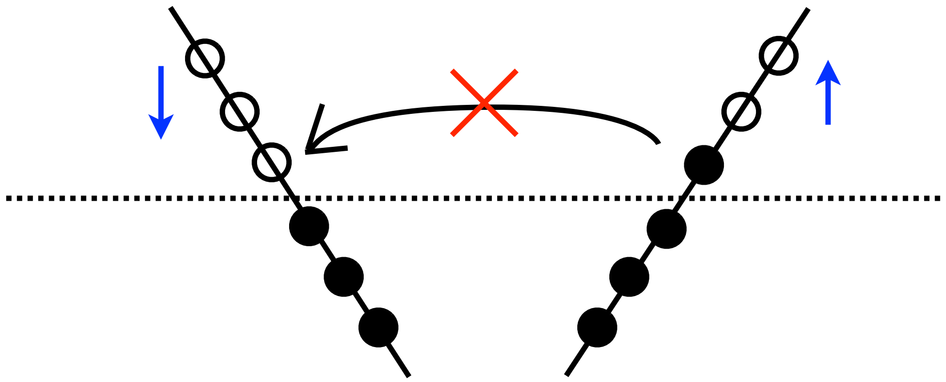
The 2D topological insulator

1. Where does this “odd-even” effect come from?

In a time-reversal-invariant system of electrons, all energy eigenstates come in degenerate pairs.

The two states in a pair cannot be mixed by any T-invariant perturbation. (disorder)

So an edge with a single Kramers pair of modes is perturbatively stable (C. Xu-JEM, C. Wu et al., 2006).



The 2D topological insulator

1. Where does this "odd-even" effect come from?

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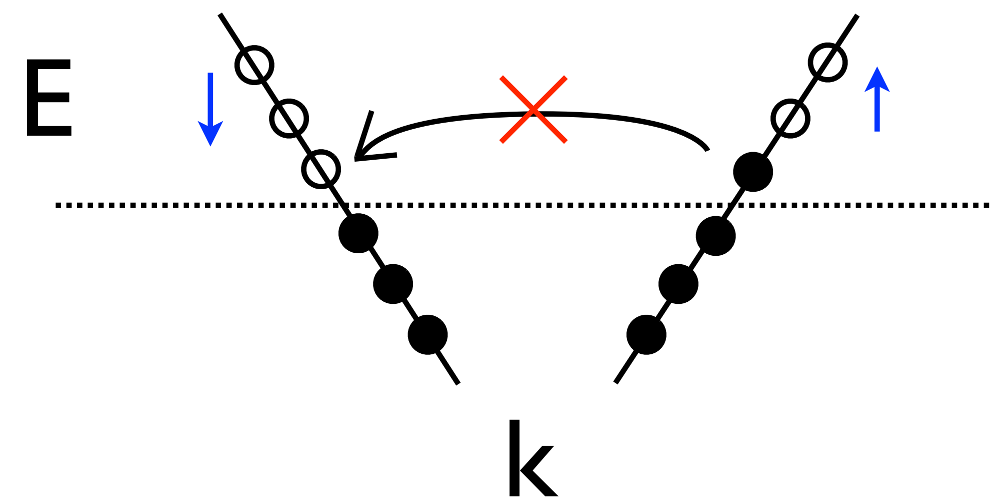
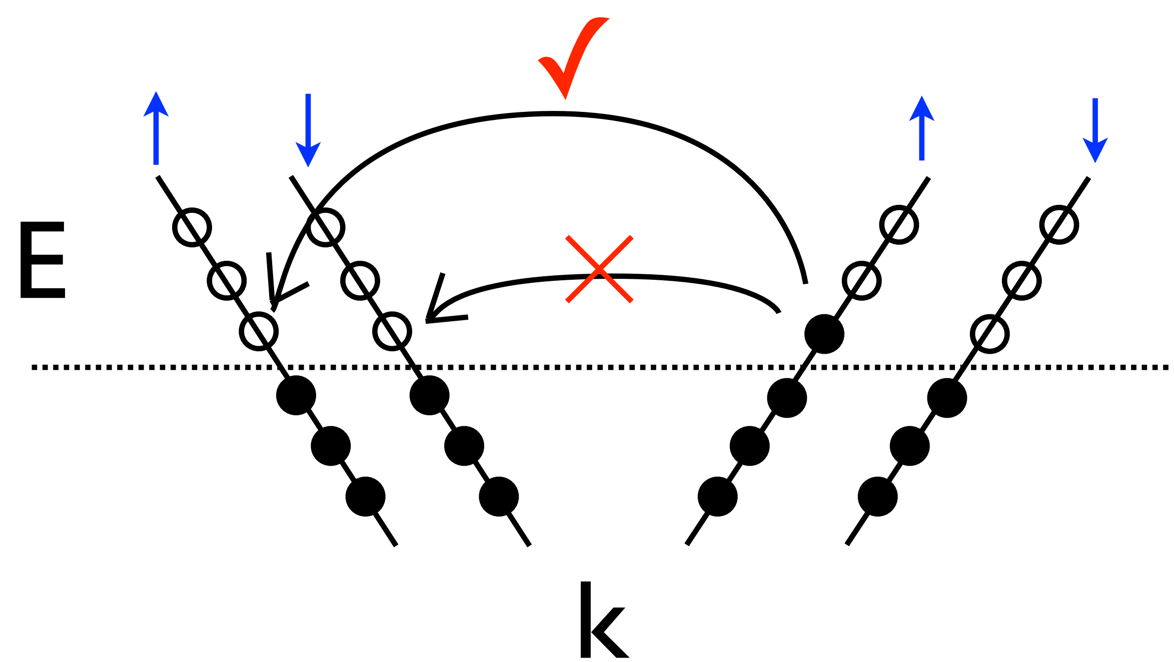


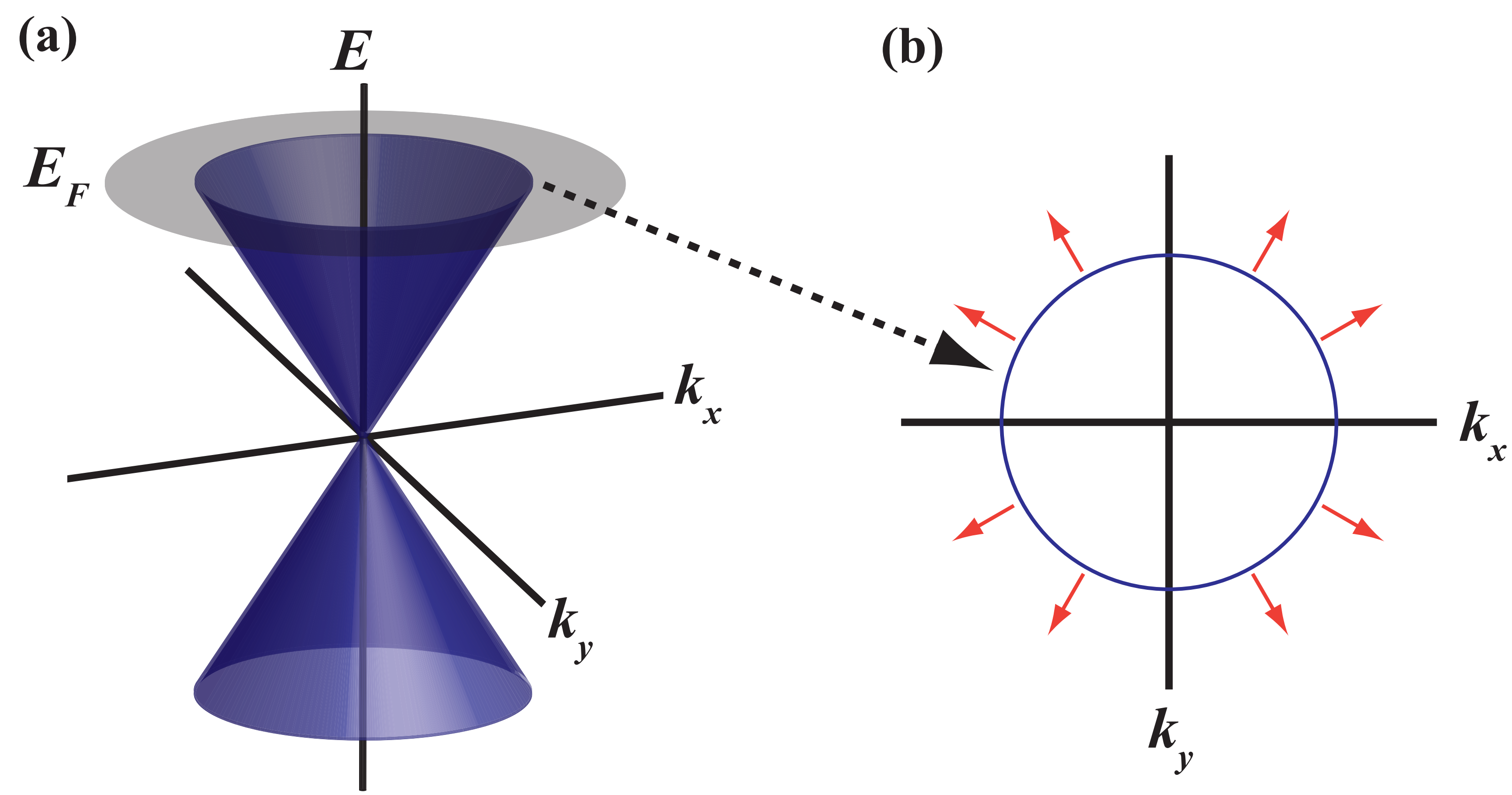
But this rule does not protect an ordinary quantum wire with 2 Kramers pairs:



The topological vs. ordinary distinction depends on time-reversal symmetry.

The 3D topological insulator surface is similarly protected by having an odd number of 2+1D Dirac fermions and time-reversal symmetry.

The robust 3D "strong topological insulator" has a metallic surface state, which in the simplest case is a single "Dirac fermion".



Some fairly common 3D materials are topological insulators.

Confirmed experimentally by ARPES measurements.

Electrodynamics in insulators

We know that the constants ε and μ in Maxwell's equations can be modified inside an ordinary insulator.

Particle physicists in the 1980s considered what happens if a 3D insulator creates a new term ("axion electrodynamics", Wilczek 1987)

$$\Delta\mathcal{L}_{EM} = \frac{\theta e^2}{2\pi h}\mathbf{E}\cdot\mathbf{B} = \frac{\theta e^2}{16\pi h}\epsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}.$$

This term is a total derivative, unlike other magnetoelectric couplings.
It is also "topological" by power-counting.

The angle θ is periodic and odd under T.

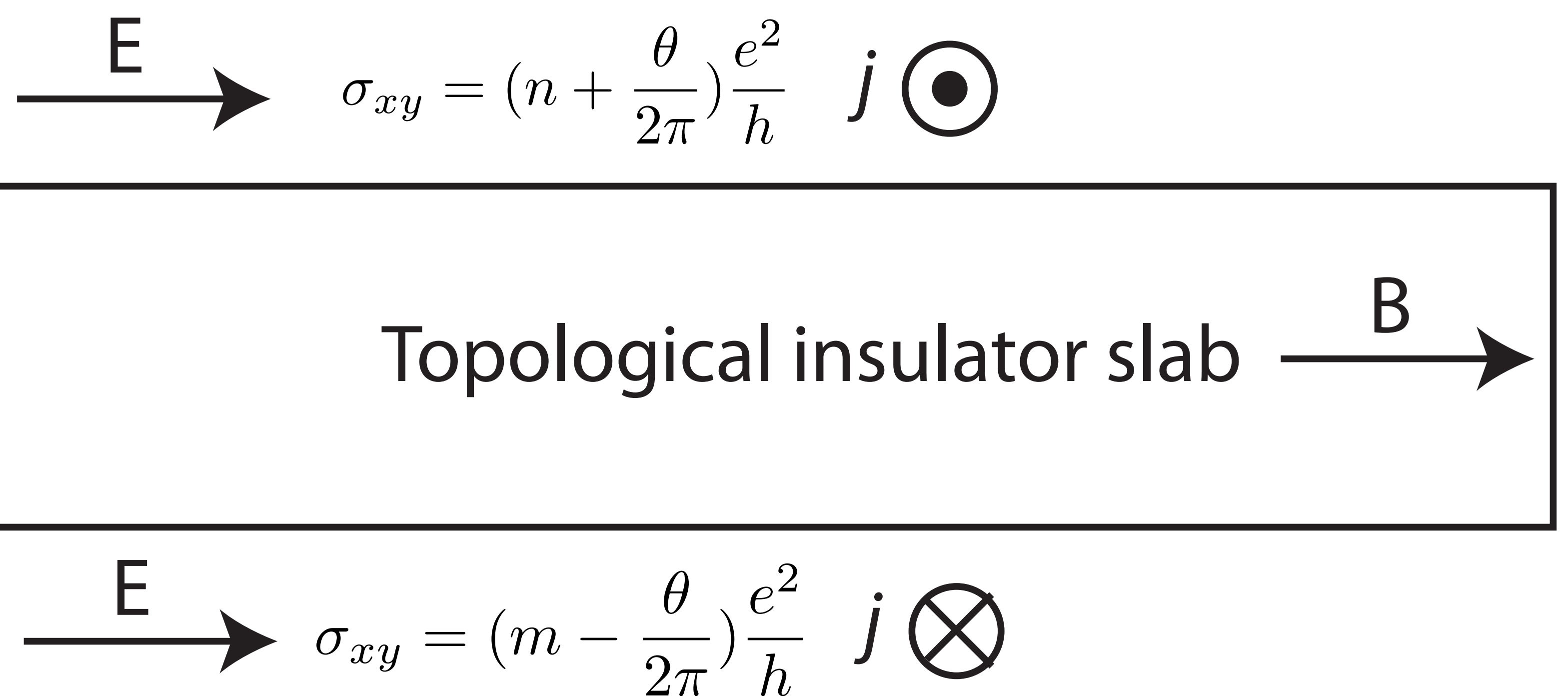
A T-invariant insulator can have *two* possible values: 0 or π.

Axion E&M, then and now

$$\Delta\mathcal{L}_{EM} = \frac{\theta e^2}{2\pi h}\mathbf{E}\cdot\mathbf{B} = \frac{\theta e^2}{16\pi h}\epsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}.$$

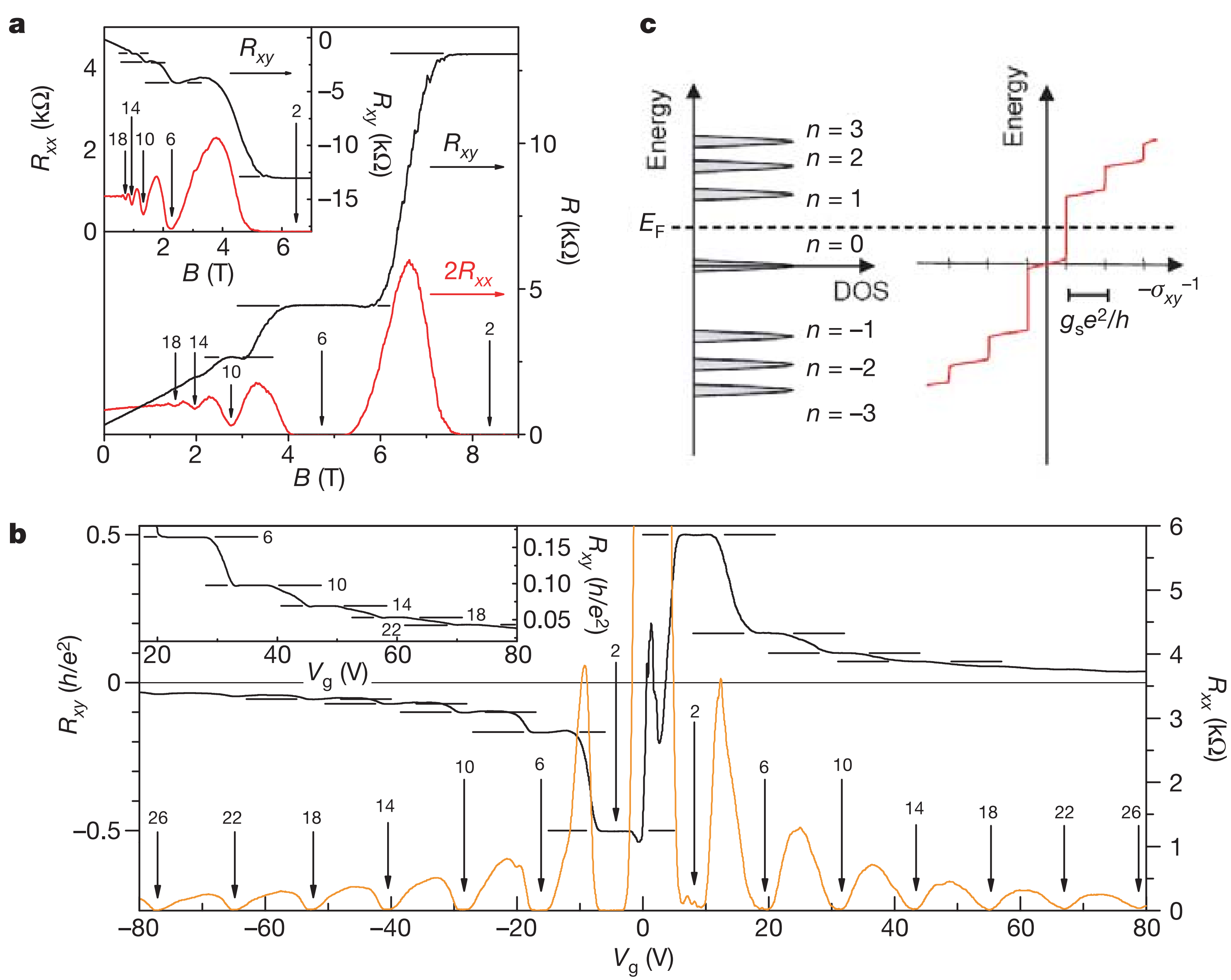
This explains a number of properties of the 3D topological insulator when its surfaces become gapped by breaking T-invariance:

Magnetoelectric effect:
applying B generates polarization P, applying E generates magnetization M)



Graphene QHE

The connection is that a single Dirac fermion contributes a *half-integer QHE*: this is seen directly in graphene if we recall the extra fourfold degeneracy. (Columbia data shown below)



Topological response

Idea of "axion electrodynamics in insulators"

there is a "topological" part of the magnetoelectric term

$$\Delta\mathcal{L}_{EM} = \frac{\theta e^2}{2\pi h}\mathbf{E}\cdot\mathbf{B} = \frac{\theta e^2}{16\pi h}\epsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}.$$

that is measured by the orbital magnetoelectric polarizability

$$\theta\frac{e^2}{2\pi h} = \frac{\partial M}{\partial E} = \frac{\partial}{\partial E}\frac{\partial}{\partial B}H = \frac{\partial P}{\partial B}$$

and computed by integrating the "Chern-Simons form" of the Berry phase

$$\theta = -\frac{1}{4\pi}\int_{\mathrm{BZ}} d^3k\ \epsilon_{ijk}\,\mathrm{Tr}[\mathcal{A}_i\partial_j\mathcal{A}_k - i\frac{2}{3}\mathcal{A}_i\mathcal{A}_j\mathcal{A}_k] \qquad (2)$$

(Qi, Hughes, Zhang, 2008; Essin, JEM, Vanderbilt 2009)

This integral is quantized only in T-invariant insulators, but contributes in all insulators.

The tenfold way

For "free-fermion" Hamiltonians, there are 10 symmetry classes robust to disorder.
There are 2 discrete symmetries (time-reversal and particle-hole), which can square to +1,-1, or be absent.

AZ class	space of projectors in momentum space	BL class	$N_f^{\min}$	fermionic replica NLσM target space	topological or WZW term
A	$\{\, Q(k) \in G_{m,m+n}(\mathbb{C}) \,\}$	**0**	1	$\mathrm{U}(2N)/\mathrm{U}(N) \times \mathrm{U}(N)$	Pruisken
AI	$\{\, Q(k) \in G_{m,m+n}(\mathbb{C}) \,\vert\, Q(k)^* = Q(-k) \,\}$	$\mathbf{4}_+$	2	$\mathrm{Sp}(2N)/\mathrm{Sp}(N) \times \mathrm{Sp}(N)$	N/A
AII	$\{\, Q(k) \in G_{2m,2(m+n)}(\mathbb{C}) \,\vert\, (i\sigma_y)Q(k)^*(-i\sigma_y) = Q(-k) \,\}$	$\mathbf{3}_+$	1	$\mathrm{O}(2N)/\mathrm{O}(N) \times \mathrm{O}(N)$	$\mathbb{Z}_2$
AIII	$\{\, q(k) \in \mathrm{U}(m) \,\}$	**1** or **2**	1 or 2	$\mathrm{U}(N) \times \mathrm{U}(N)/\mathrm{U}(N)$	WZW
BDI	$\{\, q(k) \in \mathrm{U}(m) \vert q(k)^* = q(-k) \,\}$	$\mathbf{9}_+$	2	$\mathrm{U}(2N)/\mathrm{Sp}(N)$	N/A
CII	$\{\, q(k) \in \mathrm{U}(2m) \,\vert\, (i\sigma_y)q(k)^*(-i\sigma_y) = q(-k) \,\}$	$\mathbf{9}_-$	2	$\mathrm{U}(N)/\mathrm{O}(N)$	$\mathbb{Z}_2$
D	$\{\, Q(k) \in G_{m,2m}(\mathbb{C}) \,\vert\, \tau_x Q(k)^* \tau_x = -Q(-k) \,\}$	$\mathbf{3}_-$	1	$\mathrm{O}(2N)/\mathrm{U}(N)$	Pruisken
C	$\{\, Q(k) \in G_{m,2m}(\mathbb{C}) \,\vert\, \tau_y Q(k)^* \tau_y = -Q(-k) \,\}$	$\mathbf{4}_-$	2	$\mathrm{Sp}(N)/\mathrm{U}(N)$	Pruisken
DIII	$\{\, q(k) \in \mathrm{U}(2m) \,\vert\, \sigma_z q(k)^T \sigma_z = -q(-k) \,\}$	**5** or **7**	1 or 2	$\mathrm{O}(N) \times \mathrm{O}(N)/\mathrm{O}(N)$	WZW
CI	$\{\, q(k) \in \mathrm{U}(m) \,\vert\, q(k)^T = q(-k) \,\}$	**6** or **8**	2 or 4	$\mathrm{Sp}(N) \times \mathrm{Sp}(N)/\mathrm{Sp}(N)$	WZW

TABLE II: The space of projectors in momentum space for each Altland-Zirnbauer (AZ) class. The BL classes represent the classification of 2D *Dirac Hamiltonians* obtained by Bernard and LeClair,[56] and $N_f^{\min}$ is the smallest possible number of flavors of 2D two-component Dirac fermions. The fermionic replica NLσM target spaces, with possible 2D critical behavior [in terms of whether it is possible for a given NLσM to have a topological term, either of Pruisken (IQHE) type or $\mathbb{Z}_2$ type, or a Wess-Zumino-Witten (WZW) term] are also listed according to Refs. 82 and 83.

from Schnyder, Ryu, Ludwig

Other responses & symmetries

Can we understand this response in a more fundamental way, and use similar arguments to show that other topological insulators and superconductors in the "periodic table" (Schynder et al.; Kitaev) are stable?

One simple option: there can be a conserved SU(2) spin current (gives electric field). In superconducting classes, charge may not be conserved; *energy* conserved in all classes.

TABLE I. Electromagnetic and gravitational (thermal) responses for five out of ten Altland-Zirnbauer symmetry classes (AII, CI, CII, DIII, and AIII). The assumptions made in the first four classes are that U(1) conserved currents arise from electrical charge and that SU(2) conserved currents arise from spin. In class AIII (as indicated by asterisks), the U(1) conservation law may arise either from charge or one component of spin.

Symmetry	Charge	Gravitational	Dipole
AII	√	√	
CI		√	√
CII		√	√
DIII		√	
AIII	*	√	*

Periodic table from anomalies?

Result: we can understand how some integer-valued classes are related to *anomalies* of different types, indicated by different colors in the table below. (Ryu, JEM, Ludwig)

The other classes (incl. Z2) may be related to "mixed" or "global" anomalies?

TABLE II. Topological insulators (superconductors) with an integer ($\mathbb{Z}$) classification, (a) in the complex symmetry classes, predicted from the chiral U(1) anomaly, and (b) in the real symmetry classes, predicted from the gravitational anomaly (red), the chiral anomaly in the presence of background gravity (blue), and the chiral anomaly in the presence of both background gravity and U(1) gauge field (green).

Cartan\\d	0	1	2	3	4	5	6	7	8	9	10	11	$\cdots$
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\cdots$
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	$\cdots$
AI	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\cdots$
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$\cdots$
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	$\cdots$
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$\cdots$
AII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\cdots$
CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\cdots$
C	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\cdots$
CI	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	$\cdots$

Physical meaning of gravitational response

We can understand the meaning of the gravitational term by defining a rest frame and applying a small thermal gradient to a solid cylinder of topological superconductor.

"Gravitoelectromagnetism":
the first non-Newtonian terms can be described as "electric" and "magnetic" components.

The electric component includes the usual Newtonian force, while B includes effects such as rotational frame-dragging.

A temperature gradient induces a rotational energy flow, which would cause a gyroscope placed along the axis of the cylinder to co-rotate.

$$\approx \theta \mathbf{E}_g \cdot \mathbf{B}_g$$

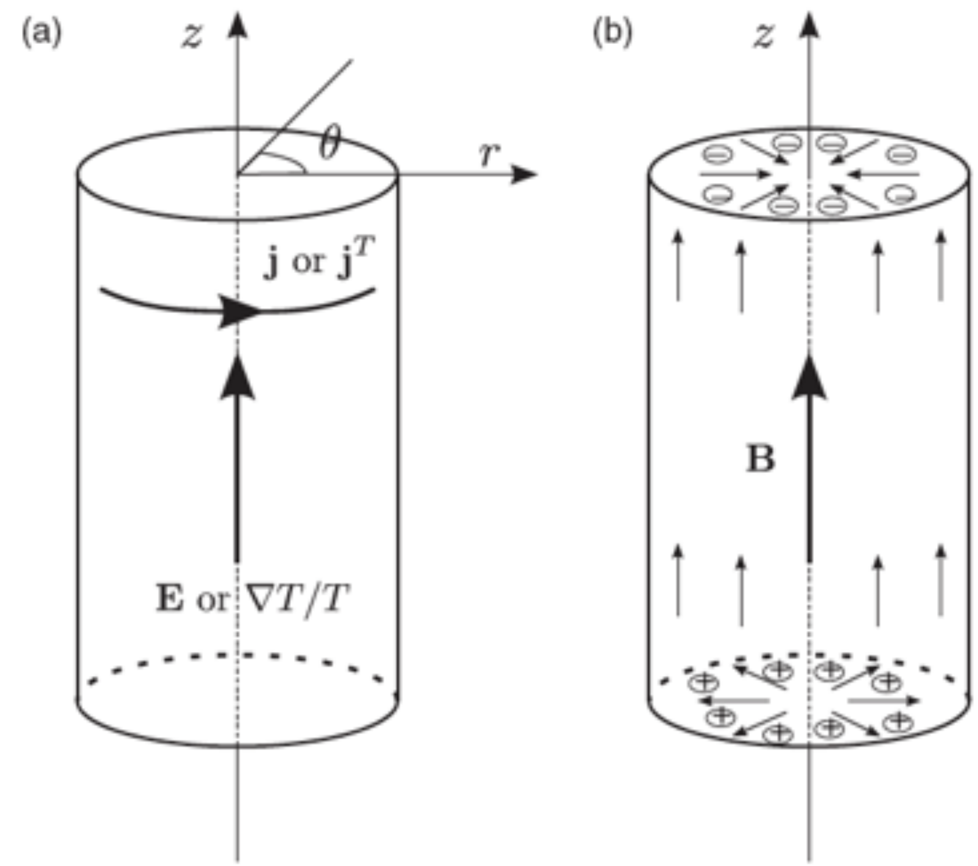


FIG. 1. Electric and thermal response of topological insulators, and thermal response of topological triplet superconductors, in a cylindrical geometry. (a) Electric (**j**) or thermal ($\mathbf{j}^T$) current driven by applied electric field (**E**) or thermal gradient ($\nabla T/T$). (b) A response dual to (a) where an applied magnetic field in the z direction induces charge polarization.

Gravitational anomaly derivation

Luttinger pointed out that thermal conductivity is connected to gravitational response: the required current can be obtained by variation with respect to the metric.

This variation is not uniquely specified for a lattice model, so we focus on the "Dirac representative". Will focus here on class DIII in 3 dimensions (superfluid 3He).
Sign of Dirac mass determines "ordinary" or "topological".

$$S[m,\bar{\psi},\psi,e] = \int d^4x\,\sqrt{g}\mathcal{L},$$

$$\mathcal{L} = \bar{\psi} e_a{}^{\mu} i\gamma^a \left(\partial_\mu - \frac{i}{2}\omega_\mu{}^{ab}\Sigma_{ab}\right)\psi - m\bar{\psi}\psi,$$

Chiral symmetry of H: $\psi \to \psi = e^{i\phi\gamma_5/2}\psi', \quad \psi^\dagger \to \psi^\dagger = \psi^{\dagger\prime} e^{-i\phi\gamma_5/2},$

Anomaly from Jacobian: $W_{\text{eff}}[m,e] \neq W_{\text{eff}}[-m,e]$

Difference is integral of Dirac genus:
(However, see preprint of Michael Stone for possible problems with GR formulation)

$$\begin{aligned} W^\theta_{\text{eff}} &:= -\ln\mathcal{J} \\ &= \theta\,\frac{1}{2}\left[\frac{1}{2\times 384\pi^2}\int d^4x\sqrt{g}\epsilon^{cdef}R^a{}_{bcd}R^b{}_{aef}\right] \end{aligned}$$

Open questions:
“fractional” topological phases
gapless “topological” phases

Fractional quantum Hall phases in 2+1D can have Abelian or non-Abelian “anyons” (particles with statistics neither bosonic nor fermionic).

These are connected to 1+1D CFTs in a beautiful way: quasiparticle braiding is related to CFT fusion rules; the gapless edge realizes a CFT.

We now have an “integer” topological phase in 3D. Are there fractional gapped topological phases in 3D? What are their surfaces like?

What are the transport signatures of fractionalization?

Experimental evidence for 2D *gapless* spin liquid

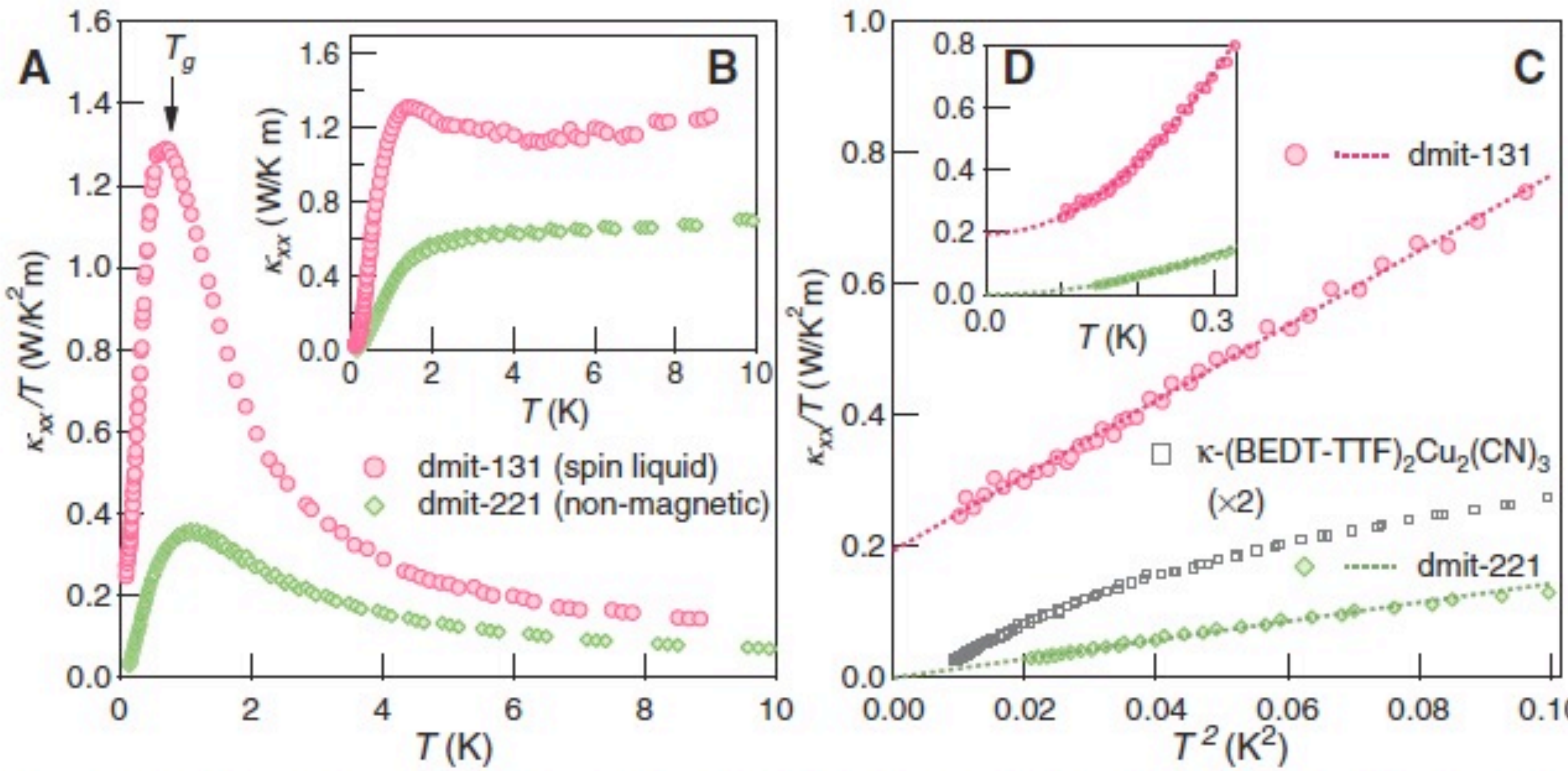


Fig. 2. The temperature dependence of $\kappa_{xx}(T)/T$ (**A**) and $\kappa_{xx}(T)$ (**B**) of dmit-131 (pink) and dmit-221 (green) below 10 K in zero field [$\kappa_{xx}(T)$ is the thermal conductivity]. A clear peak in κ_{xx}/T is observed in dmit-131 at $T_g \sim 1$ K, which is also seen as a hump in κ_{xx}. Lower temperature plot of $\kappa_{xx}(T)/T$ as a function of T^2 (**C**) and T (**D**) of dmit-131, dmit-221, and κ-(BEDT-TTF)$_2$Cu$_2$(CN)$_3$ (black) (*18*). A clear residual of $\kappa_{xx}(T)/T$ is resolved in dmit-131 in the zero-temperature limit.

M. Yamashita et al., Science 2009

Spinon Fermi surface?

Three questions about AdS/CMT for transport

How much "uniqueness of solutions" can we expect?

Is AdS a very general framework that describes more than the "natural physical space" of field theories?
At least in 1+1d, that space is pretty tightly constrained.

How are the levels of transport approximations reflected in AdS/CMT?

At low order in scattering, one gets a cross section -> diffusion.
From resumming crossed diagrams, one gets localization.
Adding a bath restores diffusion.
Adding interactions but no bath gives ???.

How do anyons (simple fractionalization) show up in AdS?