Holography from probe branes:

Phase transitions in QCD and beyond

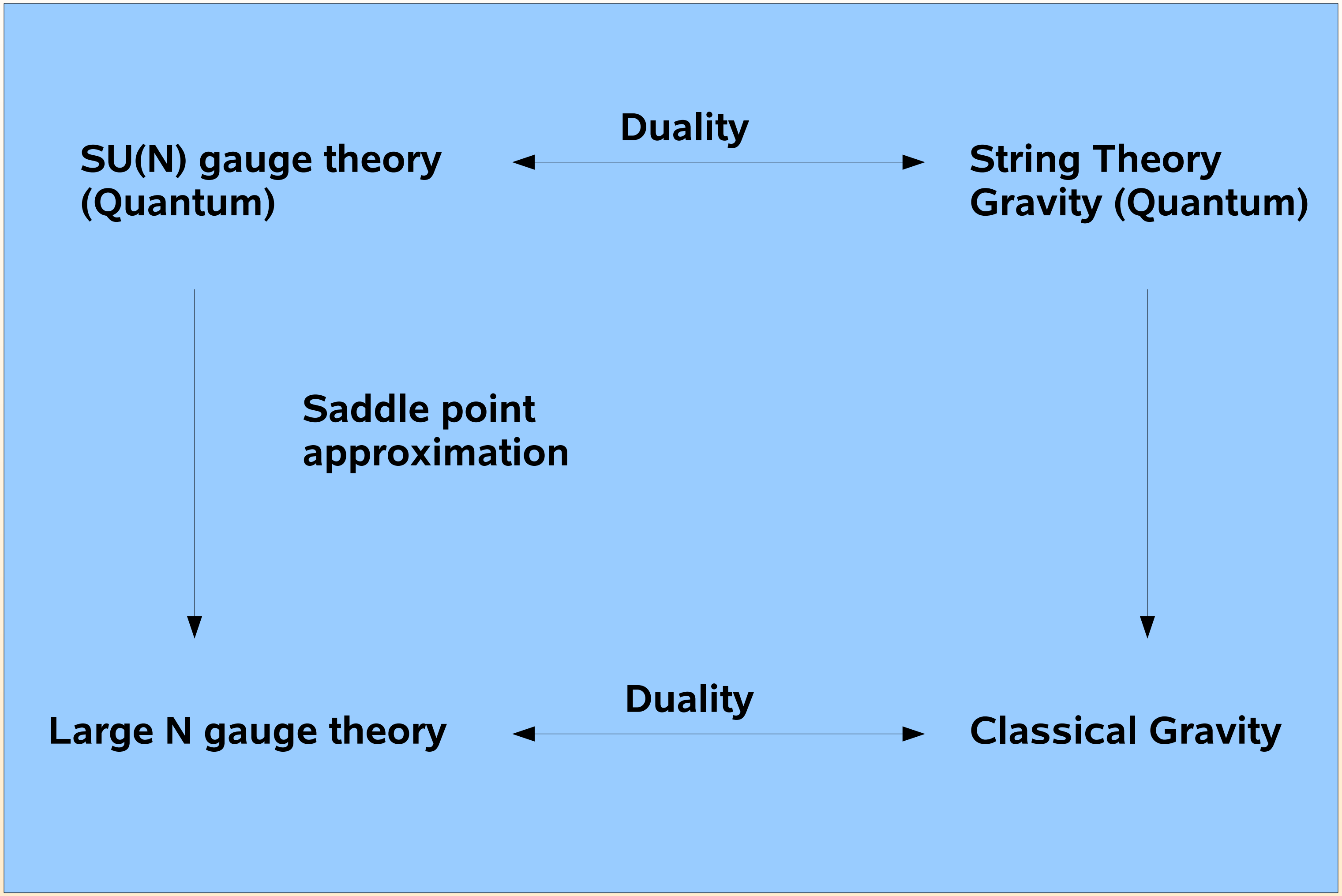
Johanna Erdmenger

Max–Planck–Institut für Physik, München

Outline

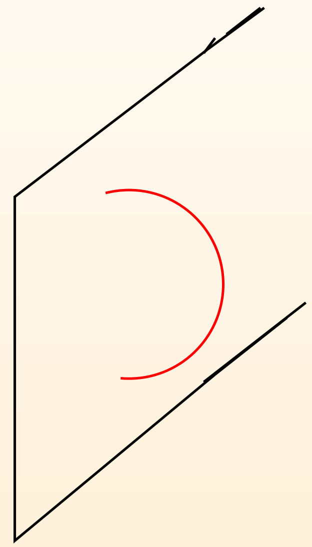
1. String theory origin of gauge/gravity duality
2. Probe brane holography
3. Phase transitions of relevance for QCD
4. Phase transitions of relevance for condensed matter

AdS/CFT correspondence (Maldacena 1997)



D-Branes

D-branes are hypersurfaces embedded into 9+1 dimensional space



D3-Branes: (3+1)-dimensional hypersurfaces

Open Strings may end on these hypersurfaces ⇔ Dynamics

D-Branes

Low-energy limit (Strings point-like) $\Rightarrow$

Open Strings $\Leftrightarrow$ Dynamics of gauge fields on the brane

Low-energy limit (Strings point-like) $\Rightarrow$

Open Strings $\Leftrightarrow$ Dynamics of gauge fields on the brane

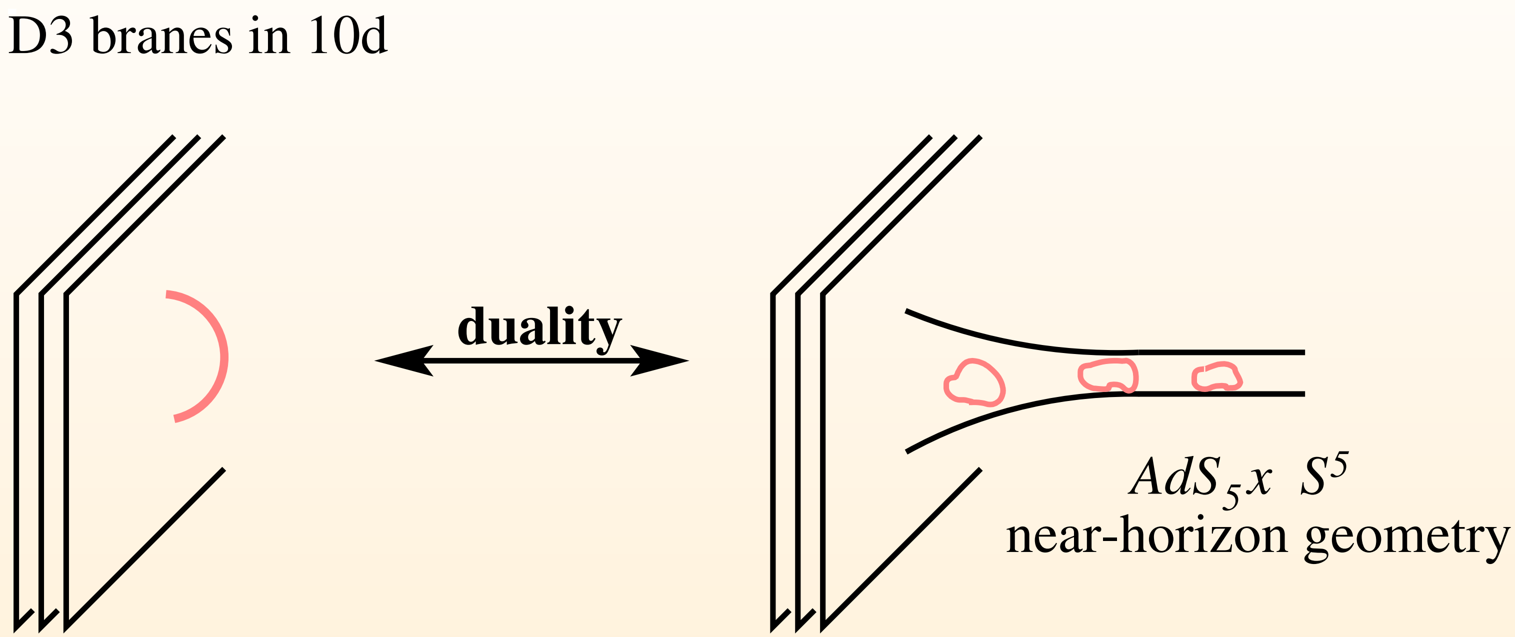
Second interpretation of the D-branes:

Solitonic solutions of ten-dimensional supergravity

Heavy objects which curve the space around them

Excitations: Closed strings

String theory origin of the AdS/CFT correspondence



$\Downarrow$ Low energy limit

Supersymmetric $SU(N)$ gauge theory in four dimensions ($N \to \infty$)

Supergravity on the space $AdS_5 \times S^5$

Generalized AdS/CFT Correspondence: Gauge/gravity duality

Generalizations:

1. Symmetry requirements are relaxed in a controlled way

   $\Rightarrow$ RG flows

   $\Rightarrow$ Theories with confinement similar to QCD

2. More degrees of freedom are added (Examples: quarks, electrons)

Quarks in the AdS/CFT correspondence

Add D7-Branes (Hypersurfaces)

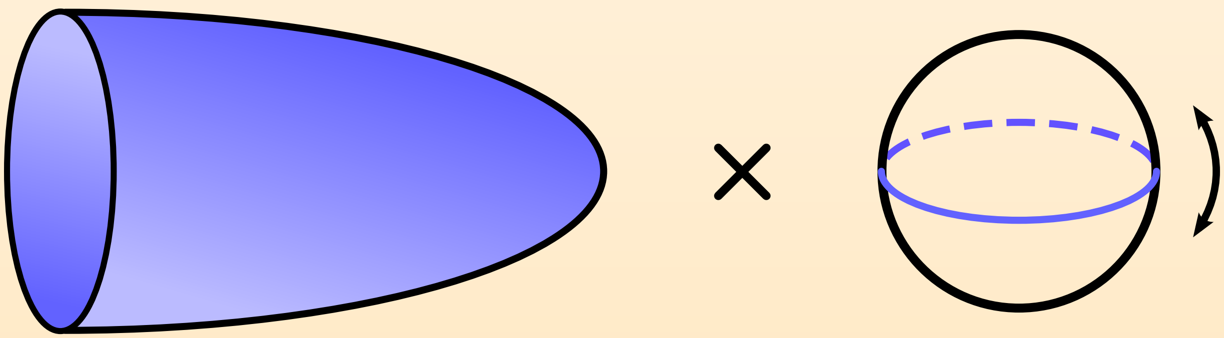
Karch, Katz '02; Mateos, Myers et al '03

	0	1	2	3	4	5	6	7	8	9
N D3	X	X	X	X						
1,2 D7	X	X	X	X	X	X	X	X		

Quarks: Low energy limit of open strings between D3- and D7-Branes

Lagrangian of dual field theory known: $\mathcal{L} = \mathcal{L}_{\mathcal{N}=4} + \mathcal{L}(\psi^i, \phi^i)$

Meson ($\bar{\psi}\psi$) masses from fluctuations of the D7-brane:



DBI (Dirac-Born-Infeld) action:

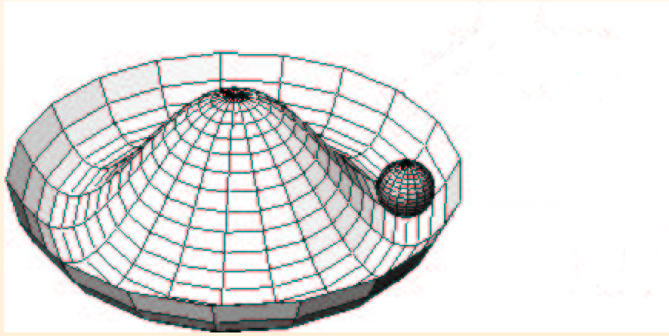
$$S_{DBI} = -T_7 \int d^8\xi \, \mathrm{tr}\sqrt{|det(P[G] + 2\pi\alpha' F)|}$$

Contributions of order N_f/N_c

Babington, J.E., Evans, Guralnik, Kirsch PRD 2004

Gravitational realization of

Spontaneous chiral symmetry breaking

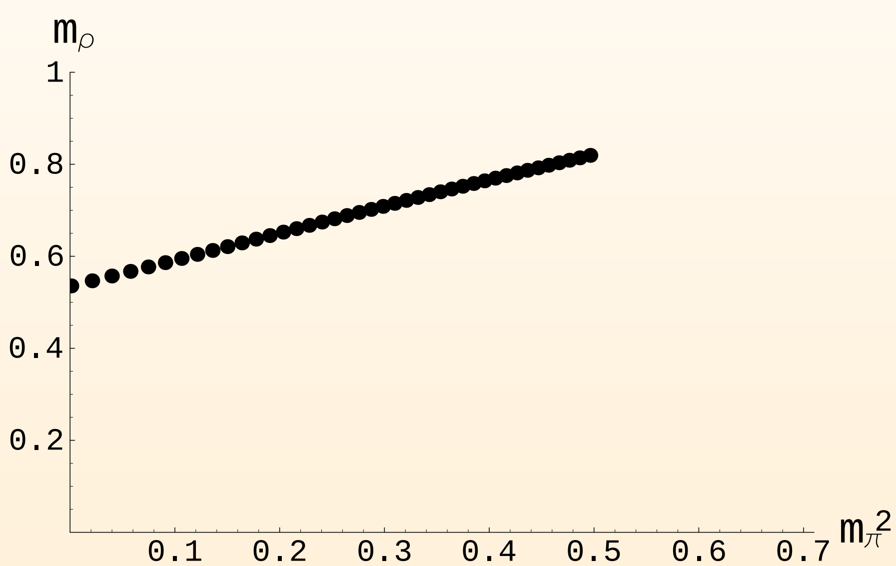


New ground state given by quark condensate $\langle \bar{\psi} \psi \rangle$

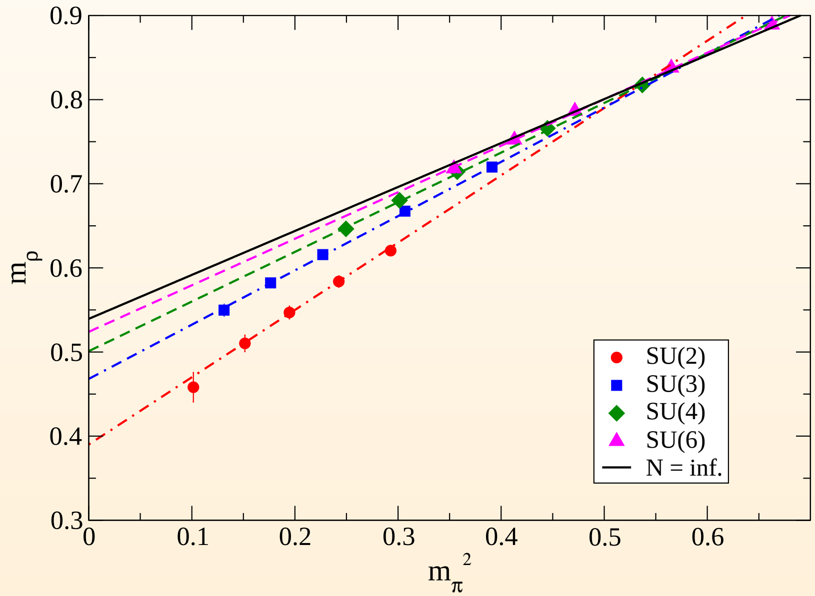
Spontaneous symmetry breaking $\to$ Goldstone bosons (Mesons)

Comparison to lattice gauge theory

Mass of ρ meson as function of π meson mass^2 (for $N \to \infty$)



J.E., Evans, Kirsch, Threlfall '07, review EPJA



Lattice: Lucini, Del Debbio, Patella, Pica '07

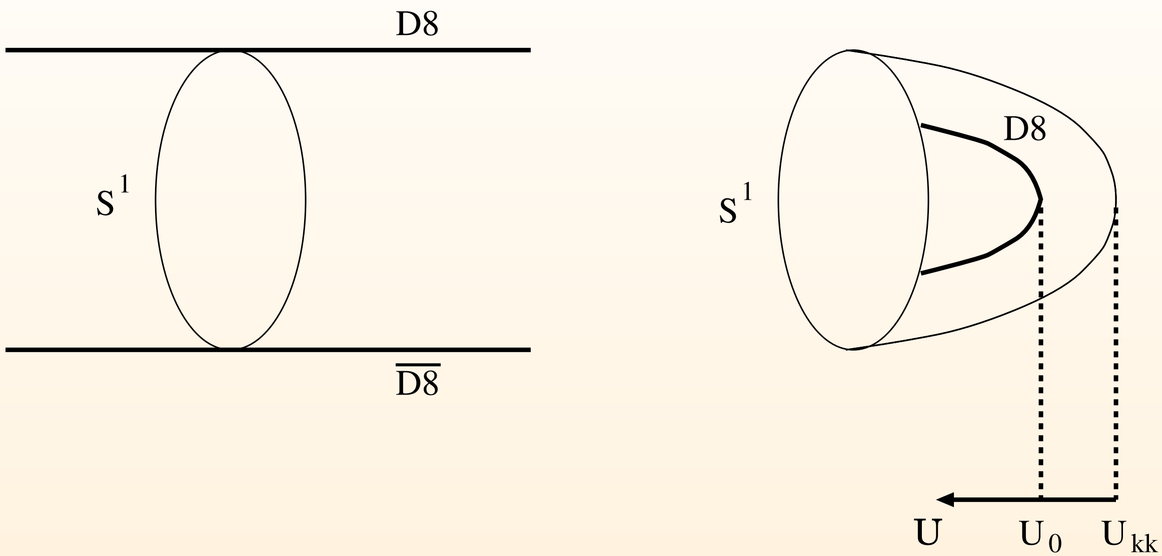
AdS/CFT result:

$$\frac{m_\rho(m_\pi)}{m_\rho(0)} = 1 + 0.307 \left(\frac{m_\pi}{m_\rho(0)}\right)^2$$

Lattice result (from Bali, Bursa '08): slope 0.341 ± 0.023

Sakai-Sugimoto model

Sakai, Sugimoto hep-th/0412141



Involves D4 branes as well as D8 and $\bar{\mathrm{D}}8$ brane probes

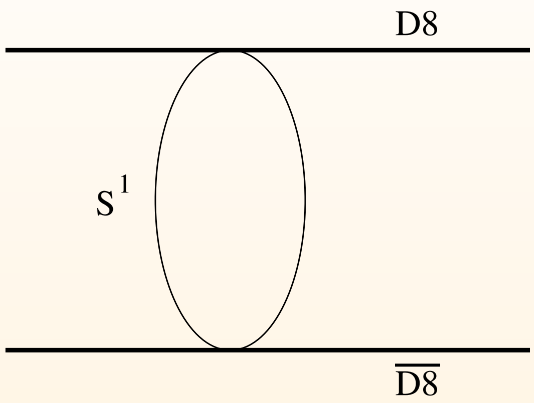
Realisation of $SU(N_f)_L \times SU(N_f)$ chiral symmetry

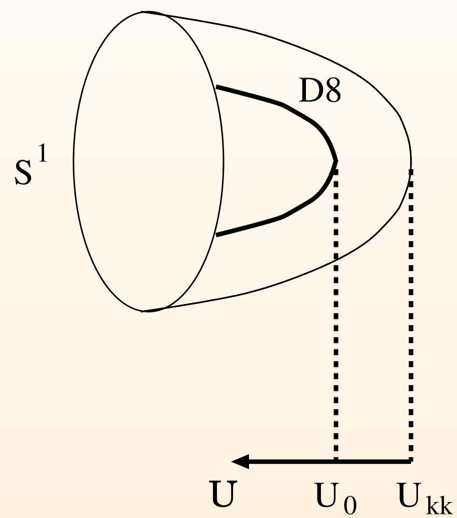
+ spontaneous breaking to diagonal $SU(N_f)$

Meson phenomenology

Sakai-Sugimoto model

Sakai, Sugimoto hep-th/0412141





Involves D4 branes as well as D8 and $\bar{\mathrm{D}}8$ brane probes

Realisation of $SU(N_f)_L \times SU(N_f)$ chiral symmetry

+ spontaneous breaking to diagonal $SU(N_f)$

Meson phenomenology

UV theory five-dimensional, quark mass not tunable

2. Finite Temperature and Chemical Potential

Quark-gluon plasma:

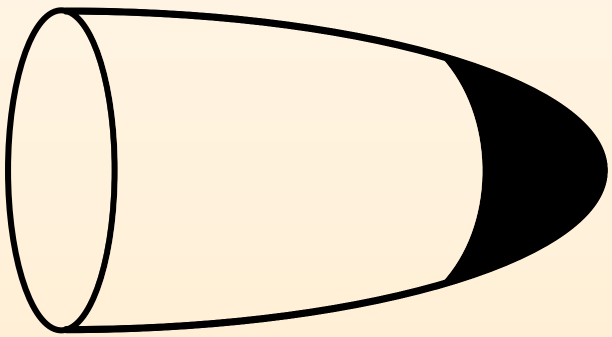
Strongly coupled state of matter above deconfinement temperature T_d

2. Finite Temperature and Chemical Potential

Quark-gluon plasma:

Strongly coupled state of matter above deconfinement temperature T_d

AdS/CFT dual of field theory at finite temperature: AdS black hole

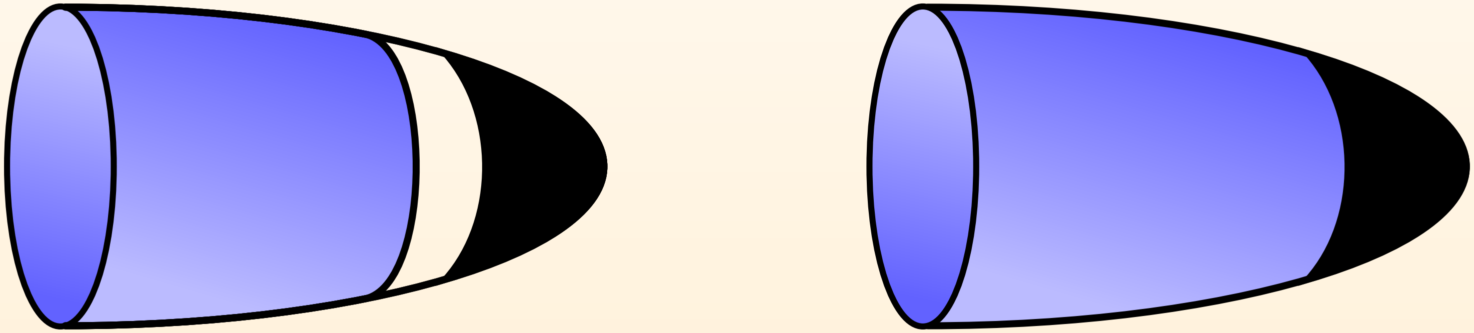


Hawking temperature $\Leftrightarrow$ temperature in the dual quantum field theory

Quarks and mesons at finite temperature

1st order phase transition at $T = T_f$ where mesons dissociate

$T_f \sim 0.12 M_{\text{meson at } T=0}$



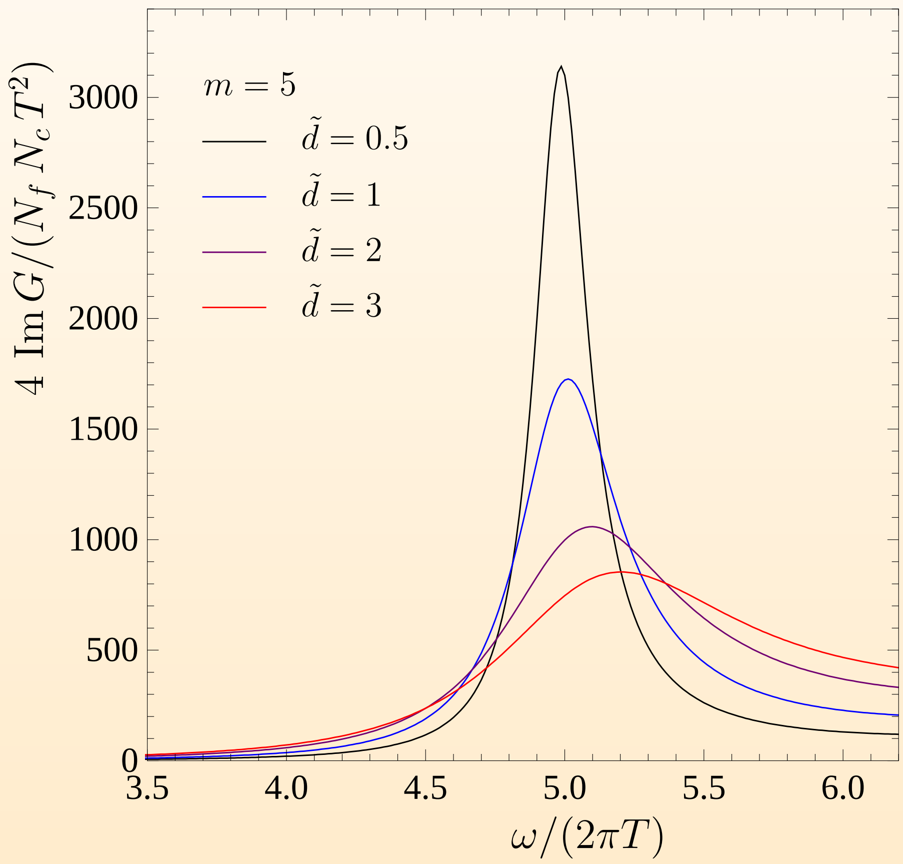
Large m/T: No dissipation, spectral function contains delta peaks

Small m/T:
Energy absorbed by black hole $\Rightarrow$ dissipation, mesons have finite width

Finite baryon chemical potential

Non-trivial radial profile for time component of gauge field on brane

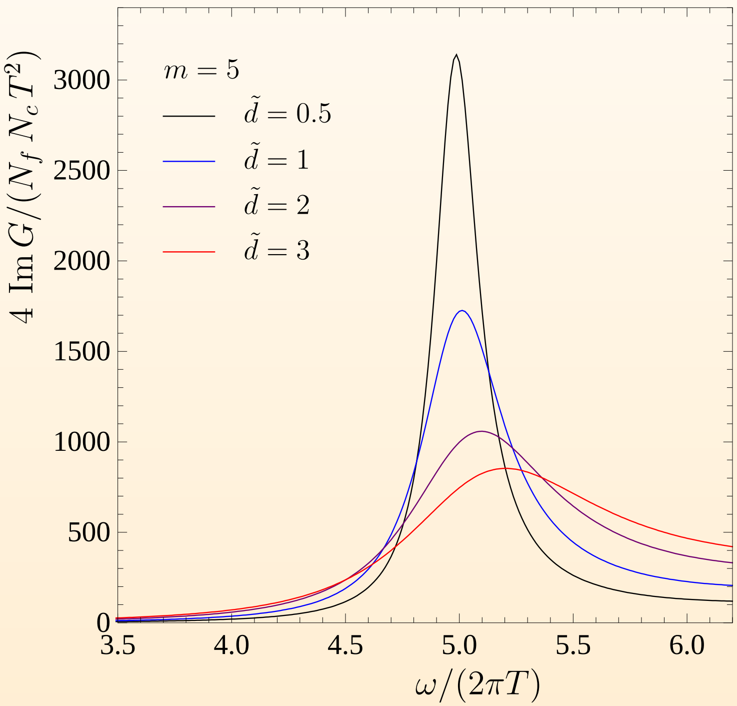
ρ vector meson spectral function in dense hadronic medium



AdS/CFT result (J.E., Kaminski, Kerner, Rust 2008)

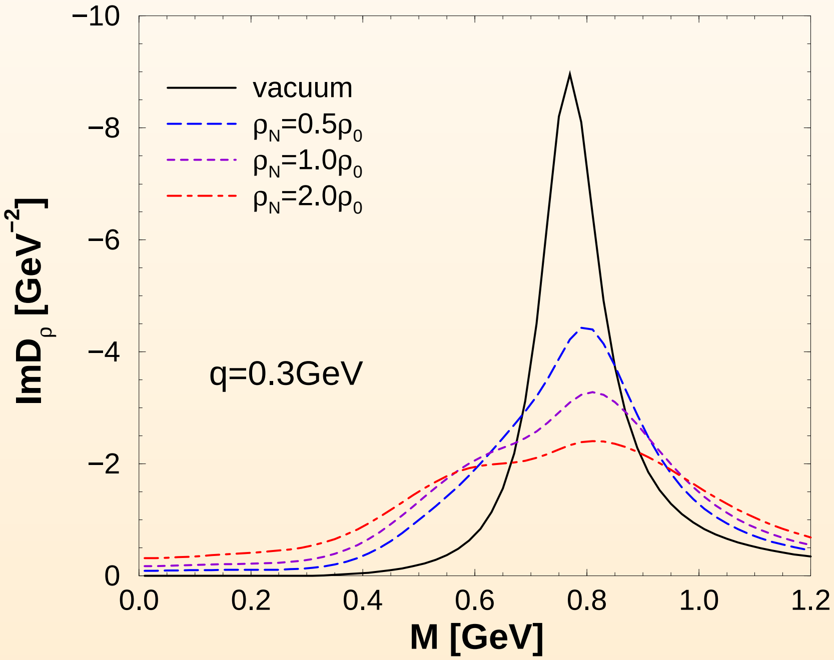
Spectral function at finite baryon density

ρ vector meson spectral function in dense hadronic medium



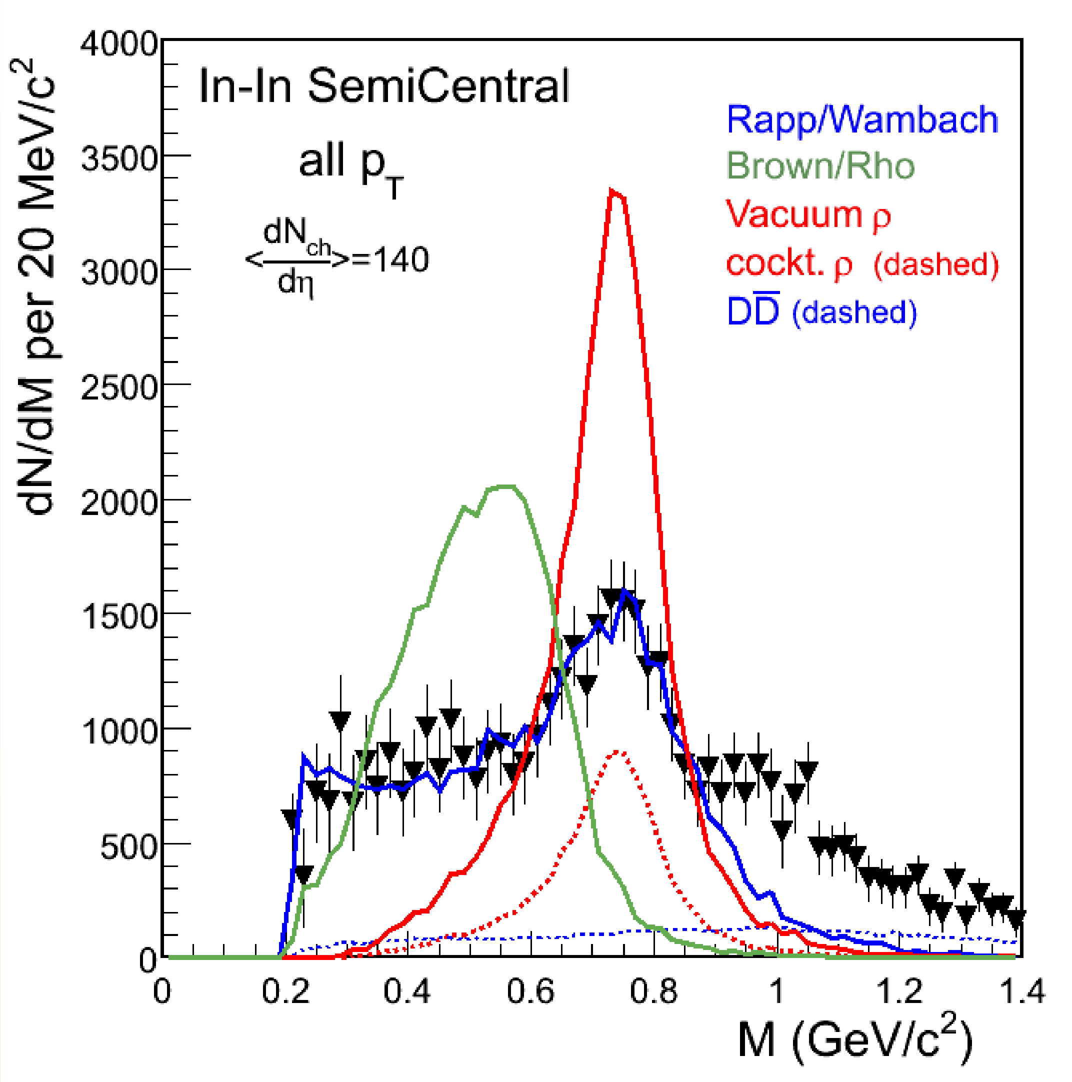
AdS/CFT result

(J.E., Kaminski, Kerner, Rust 2008)



Field theory (Rapp, Wambach 2000)

Application to NA 60 data



From NA 60 collaboration (EPJC 49 (2007) 235)

Theory: R. Rapp (2003)

(also Renk,Ruppert

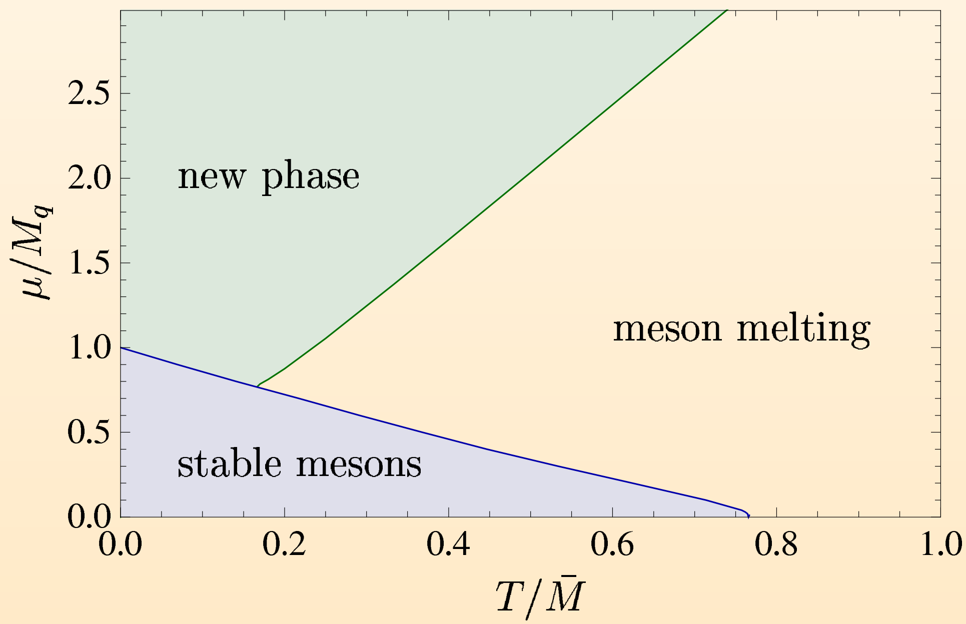
Dusling, Zahed)

Ammon, J.E., Kaminski, Kerner ’08, ’09

Chemical Potential and Finite Density for Isospin ($SU(2)$)
u- and d-Quarks

Two coincident D7 brane probes $\Rightarrow$ Non-abelian DBI action

Phase diagram



3. Superfluidity and Superconductivity from probe branes

3. Superfluidity and Superconductivity from probe branes

New solution to the equations of motion with lower free energy

New solution to the equations of motion with lower free energy

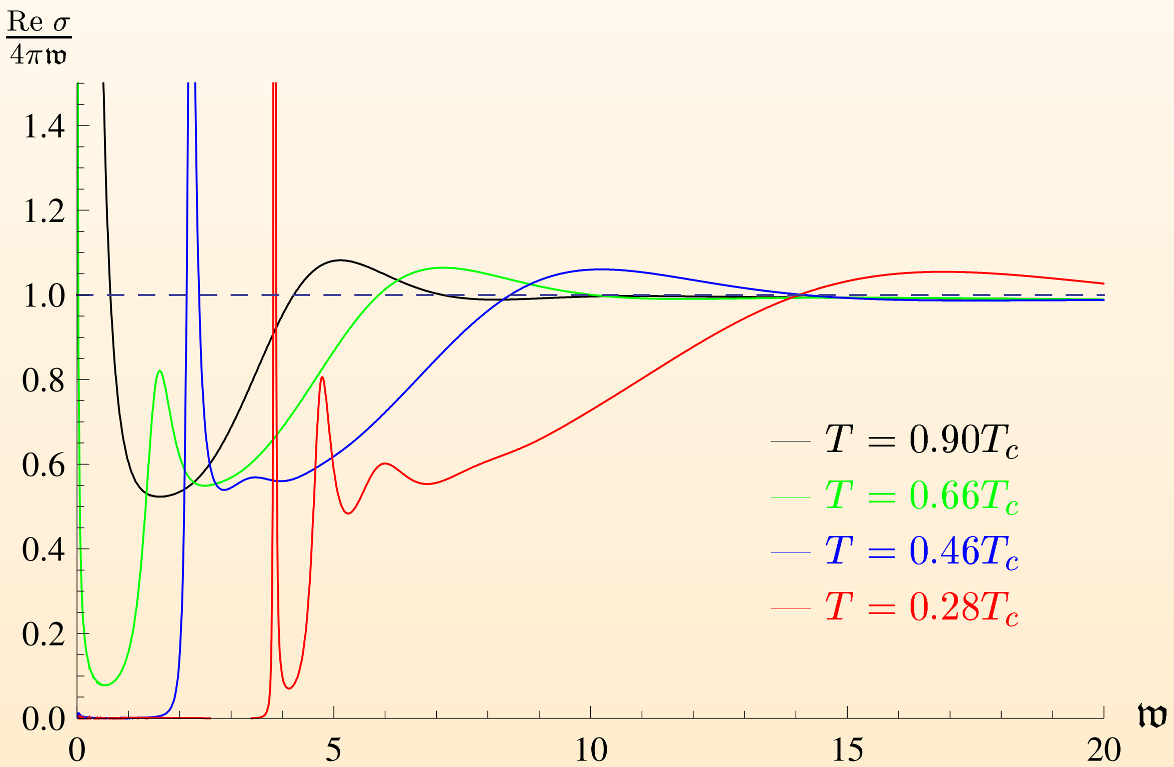
New solution contains a condensate $\langle \bar{\psi}_u \gamma_3 \psi_d + \bar{\psi}_d \gamma_3 \psi_u + \ bosons \,\rangle$

ρ meson condensate (p-wave, triplet pairing)

Ammon, J.E., Kaminski, Kerner 2008

The new ground state is a superfluid.

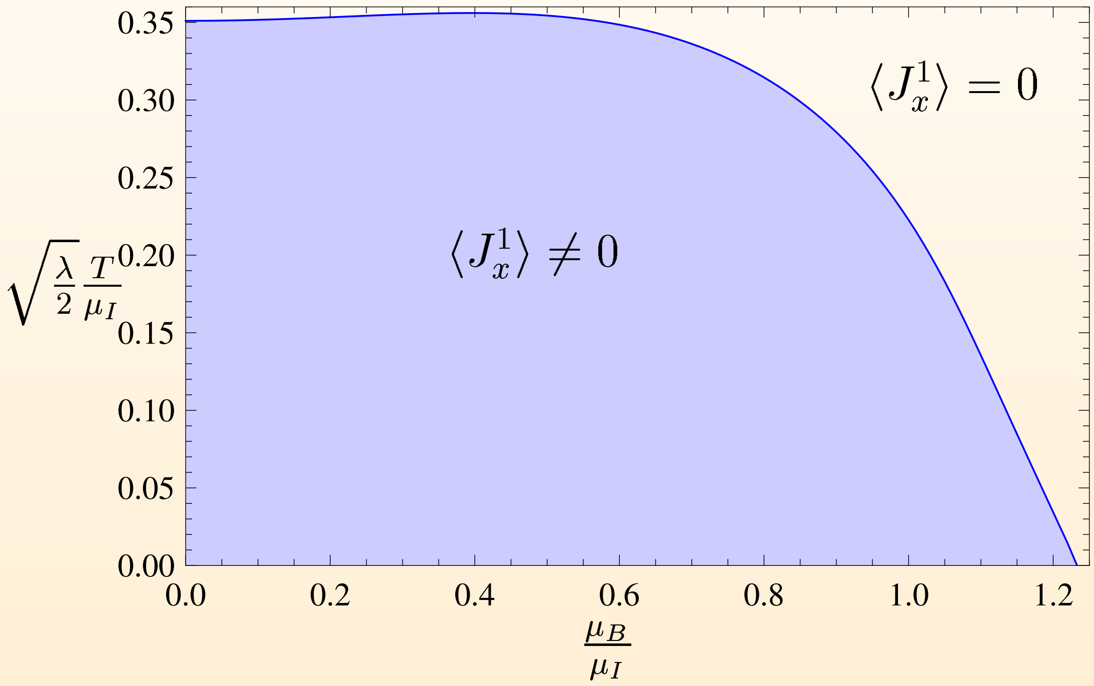
Frequency-dependent conductivity from spectral function



$$\mathfrak{w} = \omega/(2\pi T)$$

Prediction: Frictionless motion of mesons through the plasma

Two chemical potentials: Isospin and Baryon Chemical Potential



J.E., Grass, Kerner, Ngo 2011

Example for Quantum Phase Transition

Shear viscosity in presence of ρ meson condensate

Universal result which relies on space-time isotropy:
Kovtun, Son, Starinets '04

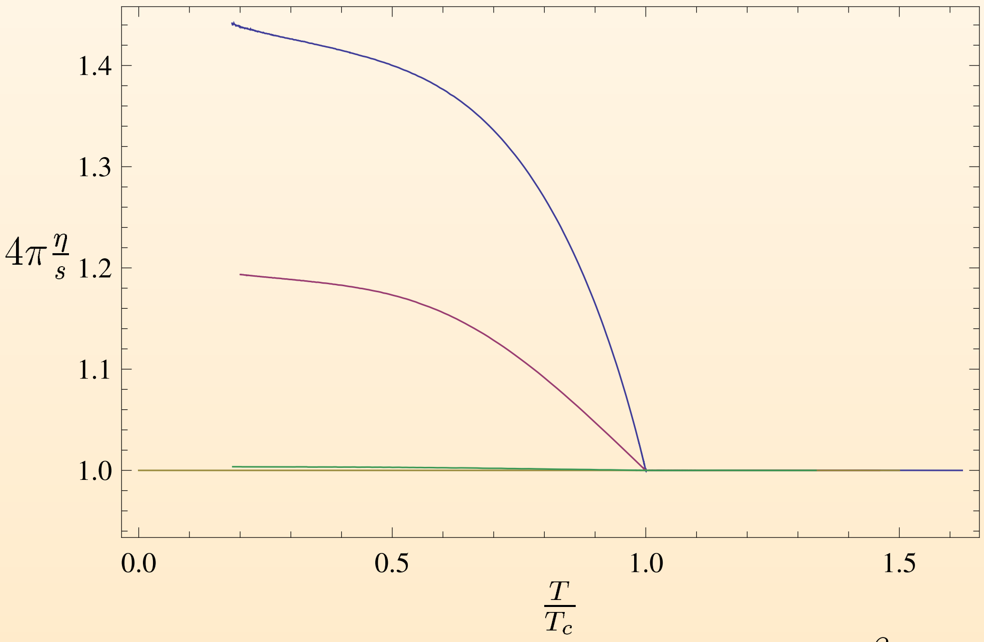
$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

Shear viscosity in presence of ρ meson condensate

Universal result which relies on space-time isotropy:
Kovtun, Son, Starinets '04

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

ρ meson condensate: Anisotropy, Lorentz symmetry broken
Shear viscosity becomes tensor;
one of the components becomes temperature dependent (non-universal)



J.E., Kerner, Zeller
PLB 2010, JHEP 2011

Critical Exponent: $1 - 4\pi\frac{\eta xy}{s} \propto \left(1 - \frac{T}{Tc}\right)^{\beta}$ with $\beta = 1.00 \pm 3\%$

Fermions from probe branes

Fermions

Ammon, J.E., Kaminski, O'Bannon 1003.1134 (JHEP)

Fermions

Ammon, J.E., Kaminski, O'Bannon 1003.1134 (JHEP)

- D5 brane probe: Fundamental matter in (2+1) dimensions
- Mass of bulk fermions fixed ($m = 0$)
- Fermionic field theory operator: $\Psi = \bar{\psi} q$
- Fermi surface forms pockets in the superconducting phase (p-wave)

Operators

D5-brane Mode	Δ	$SU(2)_H$	$SU(2)_V$	Operator
Ψ_l^-	$l+3/2$	$l+1/2, \quad l \geq 0$	$1/2$	$\mathcal{F}_l$
Ψ_{l-1}^+	$l+5/2$	$l-1/2, \quad l \geq 1$	$1/2$	$\mathcal{G}_l$

$$\mathcal{F}_l^{I_1 \dots I_l \, im} = \bar{\psi}^i \left(X_H^l\right)^{I_1 \dots I_l} q^m + q^{\dagger m} \left(X_H^l\right)^{I_1 \dots I_l} \psi^i$$

$$\mathcal{G}_l^{I_1 \dots I_{l-1} \, im} = \bar{\psi}^j \left(X_H^{l-1}\right)^{I_1 \dots I_{l-1}} \lambda^{im} \psi_j + q^{\dagger n} \left(X_H^{l-1}\right)^{I_1 \dots I_{l-1}} \lambda^{im} X_{H,I} \, \sigma^I_{np} \, q^p$$

Fermions from probe branes

Operators

D5-brane Mode	Δ	$SU(2)_H$	$SU(2)_V$	Operator
Ψ_l^-	$l+3/2$	$l+1/2, \quad l \geq 0$	$1/2$	$\mathcal{F}_l$
Ψ_{l-1}^+	$l+5/2$	$l-1/2, \quad l \geq 1$	$1/2$	$\mathcal{G}_l$

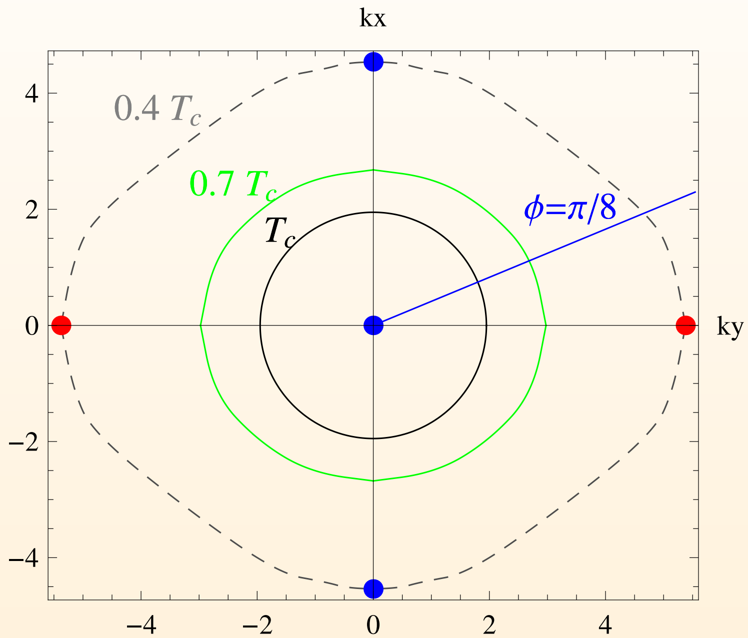
$$\mathcal{F}_l^{I_1\ldots I_l\, im} = \bar{\psi}^i \left(X_H^l\right)^{I_1\ldots I_l} q^m + q^{\dagger m} \left(X_H^l\right)^{I_1\ldots I_l} \psi^i$$

$$\mathcal{G}_l^{I_1\ldots I_{l-1}\, im} = \bar{\psi}^j \left(X_H^{l-1}\right)^{I_1\ldots I_{l-1}} \lambda^{im} \psi_j + q^{\dagger n} \left(X_H^{l-1}\right)^{I_1\ldots I_{l-1}} \lambda^{im} X_{H,I}\, \sigma^I_{np}\, q^p$$

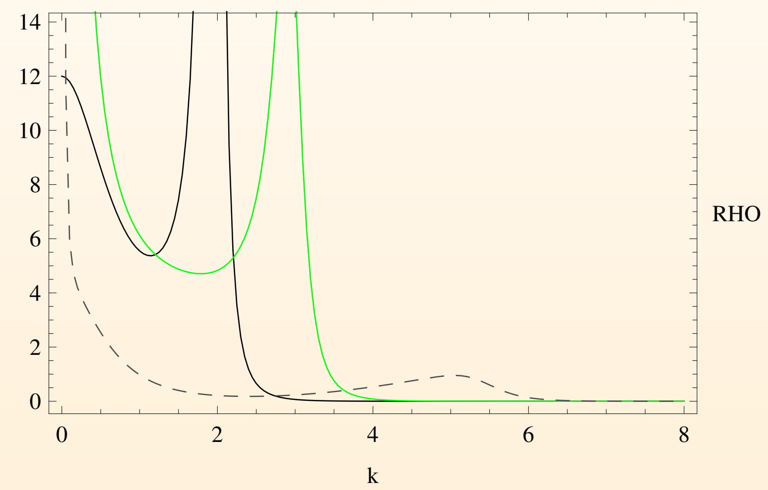
Supergravity side: Fermionic contribution to DBI action: (Martucci, Kirsch)

$$S_{Dp} = N_f T_{Dp} \int d^{p+1}\xi \sqrt{-g_{Dp}}\, \frac{1}{2} Tr \left[\bar{\hat{\mathcal{J}}} P_- \Gamma^{\hat{A}} \left(D_{\hat{A}} + \frac{1}{8} \frac{i}{2*5!} F_{\hat{N}\hat{P}\hat{Q}\hat{R}\hat{S}} \Gamma^{\hat{N}\hat{P}\hat{Q}\hat{R}\hat{S}} \Gamma_{\hat{A}} \right) \hat{\mathcal{J}} \right]$$

Non-Fermi liquid



(a)



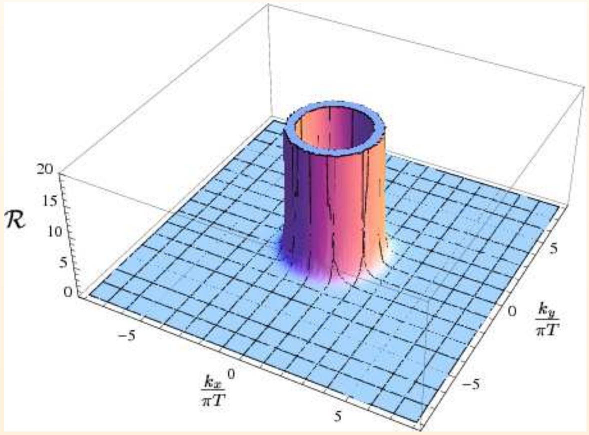
(b)

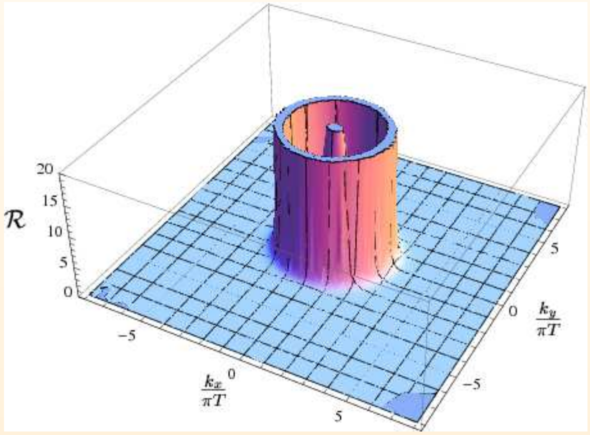
$$\omega - \omega^* \sim (k - k^*)^z, \quad z = 1.00 \pm 0.01$$

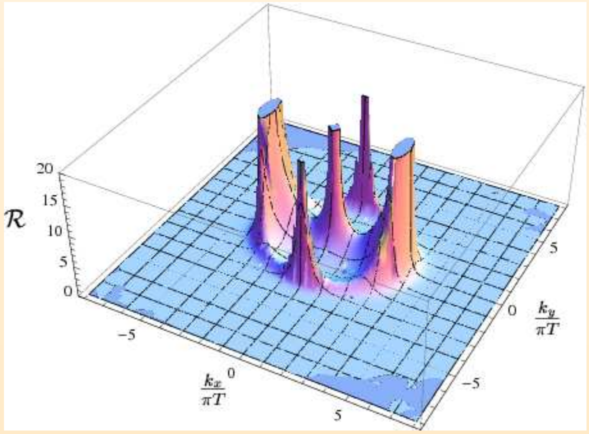
$$\mathcal{R}_{11} \sim (k - k^*)^{-\alpha}, \quad \alpha = 2.0 \pm 0.1$$

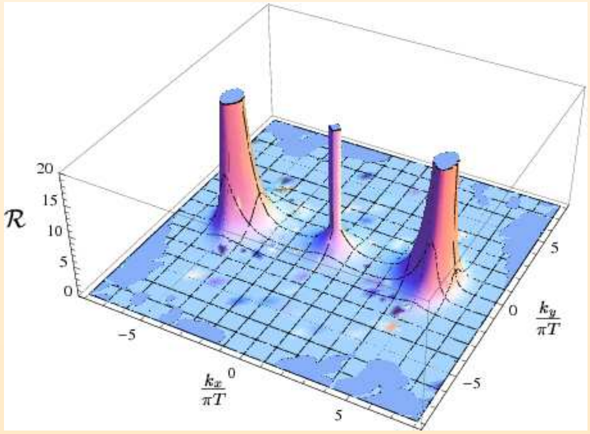
Fermionic Excitations in p-wave Superconductors

Ammon, J.E., Kaminski, O'Bannon 2010









Conclusions

Probe branes:

- Simple models within 10d supergravity
- Dual field theory and dual operators known
- QCD: Chiral symmetry breaking, finite temperature phase transitions
- Meson spectral functions
- Superfluids and superconductors
- Quantum phase transitions
- Fermions