Spatially modulated phases in AdS/CFT

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Outline

1. Motivation / Introduction
2. General strategy for finding new phases
3. Examples of Spatially Modulated Phases found via linearised instabilities
4. Helical superconductors
   - Helical superconducting black holes
5. Final comments

Outline

Holographic Phases

- Analyse the behaviour of strongly coupled CFTs when held at finite temperature and charge density and/or in a uniform magnetic field.
  - Construct AdS black hole solutions with relevant asymptotic behaviour
  - Calculate the free energies and deduce the phase diagram
- What type of phases are possible?
- What kind of zero temperature ground states can we have?
- Do we find interesting new behaviour in the far IR? e.g. Lifshitz, Schrodinger, ..., something new??
- Analyse hydrodynamics and transport
- A lot of work on superconducting/superfluid phases

Spatially Modulated Phases

- In condensed matter there is a variety of phases that are spatially modulated, spontaneously breaking translation invariance. For example
  - Charge density waves (CDWs)
  - Spin density waves (SDWs)
- The modulation is fixed by an order parameter associated with non-zero wave-number k.
- The modulation can be in various configurations such as stripes, checkerboards, ... and helical order with pitch $2\pi/k$
- Is QCD in a spatially modulated "chiral density wave" state at high density?

- Can we study spatially modulated phases using holographic techniques? Yes!
- Are they exotic? No! Perhaps typical?
- Several examples known - see later
- So far, all of the spatially modulated phases have been inferred using a linearised perturbative analysis. It seemed that to go beyond this one would need to solve PDEs. However, we have recently shown we can do better for helical order and construct fully back-reacted black holes!

  Plan:
  a) General strategy
  b) Helical p-wave: back-reacted black branes and emerging scaling symmetry in IR

Outline

General strategy

Normal phase

- Critical points exhibiting full relativistic conformal invariance could be described by AdS geometries in string or M-theory
- The boundary field theory at finite temperate is described by black hole solutions asymptoting to AdS
- Finite chemical potential would correspond to a charged black hole with the charge carried by a bulk gauge field

$\rightsquigarrow$ Electric AdS - Reissner - Nordström black brane

- (Can also consider magnetic black branes)

Phase transition

- Certain fields can condense due to an instability of AdS-RN black brane, at a critical temperature T_c, spontaneously breaking a space-time and/or internal symmetry
- Emergence of new black hole branch

Electric AdS RN black hole - unbroken phase

Einstein-Maxwell theory in $d = 4$

$$\mathcal{L}_{EM} = \sqrt{-g}\left(\frac{1}{2}R + 6 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) + \ldots$$

Electrically charged AdS RN black hole

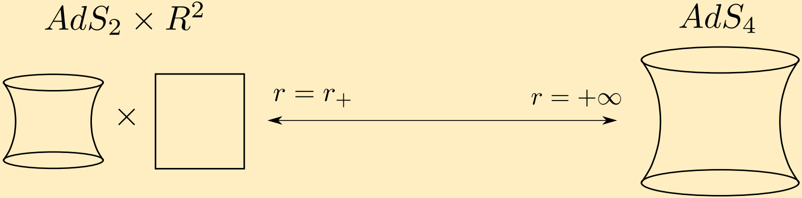
$$ds_4^2 = -g\,dt^2 + g^{-1}\,dr^2 + r^2\,\left(dx_1^2 + dx_2^2\right)$$
$$A = \mu\,\left(1 - \frac{r_+}{r}\right)\,dt$$
$$g = 2r^2 - \left(2r_+^2 + \frac{\mu^2}{2}\right)\frac{r_+}{r} + \mu^2\frac{r_+^2}{2r^2}$$

- Outer horizon located at $r = r_+$
- Temperature $T = (12\,r_+^2 - \mu^2)/(8\pi r_+)$
  Entropy (density) $s = 2\pi r_+^2$

$\Rightarrow$ Non-zero entropy at $T = 0$! Profound? Or always unstable?

The extremal limit - diagnostic for instability

- $T = 0$ the near horizon (IR) limit is $AdS_2 \times \mathbb{R}^2 \rightarrow$ 1D CFT



- The $\mathbb{R}^2$ Fourier modes of bulk fields yield a continuum of dual operators $\mathcal{O}_{\vec{k}}$ in the IR CFT
- The modes $\vec{k} \neq 0$ break translations in the UV CFT
- Check unitarity (BF) bound of the IR CFT for all $\vec{k}$
- If for high T RN bh is stable and the IR CFT is unstable, there must be a T_c for the onset of the instability

Instabilities and critical temperatures

- Some instabilities do NOT show up as violations of AdS2 BF bound and need to consider instabilities in the full AdS-RN black brane solution
- To obtain the critical temperature where the instability sets in we need to construct normalisable static modes of the AdS-RN black brane

Outline

Spatially Modulated Phases via linearised instabilities

Finite charge density

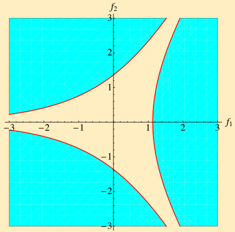
- D=4 model with metric+gauge-field+ pseudo scalar.
  The electric AdS-RN brane is unstable to spatially modulated coupled perturbations ($\lambda << 1$) of the scalar, gauge-field and metric corresponding to striped phase [Donos,Gauntlett]:
  - Scalar operator VeV $\langle \mathcal{O} \rangle \propto \lambda \cos(kx)$
  - Current density wave $\langle \mathcal{J}_y \rangle \propto \lambda \sin(kx)$
  - Momentum density wave $\langle \mathcal{P}_y \rangle \propto \lambda \sin(kx)$
  - Charge density wave $\langle q \rangle \propto q_0 + \lambda^2 q_2 \cos(2kx)$ + ...
- D=5 model with metric + $SU(2) \times U(1)$ gauge fields (imbalanced mixture) with CS coupling leads to competing modulated orders [Donos,Gauntlett]:
  - Helical current order
  - Helical p-wave superconducting order
  - Helical p+$\imath$p superconducting order

Spatially Modulated Phases via linearised instabilities

Finite magnetic field

Dilaton couplings $\tau(\varphi) F^2$ (and generalisations) can lead to modulated current + scalar VEV [Donos,Gautnett,Pantelidou]

- D=5, $\mathcal{N} = 8$ gauged SUGRA with magnetic deformations has a two dimensional moduli space of quantum critical points dual to $1+1$ dim. CFTs including SUSY ones



  - Modulated modes marginally stable around the SUSY ones
  - Become unstable when crossing the red lines

- Similar structure in D=4, $\mathcal{N} = 8$ gauged SUGRA

Questions for perturbation theory

- Higher order perturbative analysis shows that modulated black hole branches exist
- Can be used to study thermodynamics close to T_c

$\rightsquigarrow$ In general they are continuous transitions (second order)

Would still like to ask

- Transport properties of modulated phases?
- Periodicity as function of T?
- What is the low temperature behaviour? Modulation persists at low temps?
- If yes, new emergent IR with modulation?

$\rightsquigarrow$ Can answer for helical order in D=5 models

Outline

Helical p-wave superconductors

Consider the $D=5$ model specified by e, m

$$\mathcal{L} = (R+12) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} \bar{C}_{\mu\nu} C^{\mu\nu} + \frac{\imath}{24m} \varepsilon^{\mu_1 \dots \mu_5} C_{\mu_1\mu_2} \bar{H}_{\mu_3\mu_4\mu_5}$$
$$F = dA, \quad H = dC + \imath\, q_e A \wedge C$$

- Natural class (eg type IIB $q_e = 1/\sqrt{3}$, $m = 1$)
- AdS is a solution dual to some CFT
- C is dual to a charge q_e, $d=4$ self-dual tensor operator with

$$\Delta\left(\mathcal{O}_C\right) = 2 + m$$

  - Operator is charged and has l=1 $\rightarrow$ p-wave superconductor
- The electric RN black brane is a solution with $C = 0$
- To analyse stability of C, simply examine its equation of motion on $AdS_2 \times \mathbb{R}^3$/RN bh background

$$*H = \imath m C$$

2-form perturbations in $AdS_2 \times \mathbb{R}^3$

Consistency of the 2-form equation of motion requires to consider

$$\delta C = (\imath c_1\, dt + c_2\, dr) \wedge (\cos(kx_1)\ dx_2 - \sin(kx_1)\ dx_3) \\ + c_3 dx_1 \wedge (\sin(kx_1)\ dx_2 + \cos(kx_1)\ dx_3)$$

- Solve for c_1 and c_2 in terms of c_3
- The component c_3 satisfies a second order equation
- For the $AdS_2 \times \mathbb{R}^3$ background

$$\left(\mathcal{D}^2 - L^2\ (m^2 + k^2) + \frac{kq_e}{\sqrt{6}m}\right) c_3 = 0, \quad \mathcal{D} = \nabla + \frac{\imath q_e}{\sqrt{3}} A$$

$$\rightsquigarrow \quad m_{eff}^2 = \frac{1}{12}\ (k^2 + m^2) - \frac{q_e^2}{6} - \frac{q_e}{\sqrt{6}m}\, k$$

- Lightest mode at $k_{min} = \sqrt{6} q_e m^{-1}$, unstable if $\sqrt{2}q > m$

Static modes in AdS-RN black hole

To determine T_c we need to construct a static, normalisable mode on the RN background

$$\delta C = (\imath c_1\, dt + c_2\, dr) \wedge (\cos(kx_1)\; dx_2 - \sin(kx_1)\; dx_3) \\ + c_3 dx_1 \wedge (\sin(kx_1)\; dx_2 + \cos(kx_1)\; dx_3)\,,$$

- Solve for c_1 and c_2 in terms of c_3. Find c_3 satisfies a second order equation
- Regular horizon $\rightarrow$ analytic expansion of c_3

$$c_3 = c_3^{(0)} + c_3^{(1)}\,(r - r_+) + \ldots$$

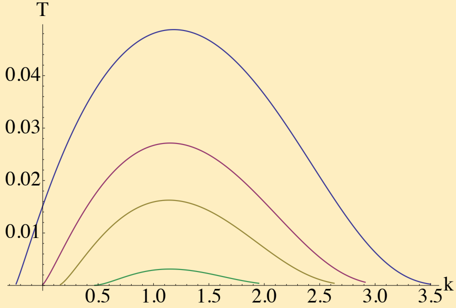
- Set deformations (sources) of c_3 at infinity to zero

$$c_3 = c_d r^{|m|} + c_v r^{-|m|} + \ldots, \quad c_d = 0$$

$\rightsquigarrow$ Parameter counting implies that for a fixed k there exist a discrete set of solutions (if any)

Static normalisable modes

For fixed chemical potential and $m = 1$ construct static spontaneous symmetry breaking modes for $q = 2$, 1.8, 1.7, 1.5

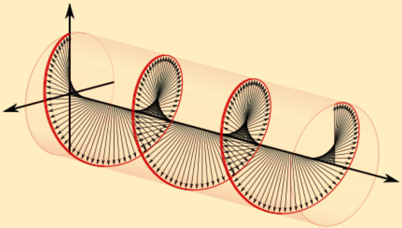


- Instability always at $k \neq 0$, as expected from AdS_2 analysis
- Comment: The same kind of curve appears for all holographic black holes eg s-wave superconductors, but usually centred at $k = 0$.

Helical superconducting black holes

Symmetries of the new phase

- At $T > T_c$ finite temperature and chemical potential preserve $R_t \times ISO\left(3\right) \times U\left(1\right)_g$
- At $T = T_c$ a static mode drives the system to a new phase
- The driving mode has a helical structure!



$$\delta C = \cdots + c_3\, dx_1 \wedge \left(\sin\left(kx_1\right)\, dx_2 + \cos\left(kx_1\right)\, dx_3\right)$$

It preserves the Bianchi VII_0 subgroup of $ISO\left(3\right)$

- Translations in the $x_2 - x_3$ plane
- A particular combination of translations in x_1 and rotations in the $x_2 - x_3$ plane

Helical superconducting black holes

- The generators for the preserved group are

$$L^t = \partial_t, \qquad L^2 = \partial_{x_2}, \quad L^3 = \partial_{x_3},$$
$$L^1 = \partial_{x_1} + k\,(x_3\partial_{x_2} - x_2\partial_{x_3})$$

- Very convenient to use invariant forms

$$\begin{aligned}
&dt \\
&\omega_1 = dx_1 \\
&\omega_2 = \cos(kx_1)\,dx_2 - \sin(kx_1)\,dx_3 \\
&\omega_3 = \sin(kx_1)\,dx_2 + \cos(kx_1)\,dx_3
\end{aligned}$$

to write ansatz for black hole solutions...

Helical superconducting black holes

Natural ansatz based on Bianchi VII_0 invariants

$$ds^2 = -gf^2\,dt^2 + g^{-1}\,dr^2 + h^2\,\omega_1^2 + r^2\,\left(e^{2\alpha}\,\omega_2^2 + e^{-2\alpha}\,\omega_3^2\right)$$
$$C = (\imath\, c_1\,dt + c_2\,dr) \wedge \omega_2 + c_3\,\omega_1 \wedge \omega_3$$
$$A = a\,dt$$

- g, f, h, α, c_i and a are only functions of r
- General enough to capture:
  - (1) AdS_5
  - (2) Electric RN black brane: high temperature, normal phase
  - (3) The static mode when pert. expanding functions around (2)
- Plugging in the $5D$ equations of motion yields a consistent system of non-linear ODEs with k a parameter

Helical superconducting black holes

Modulated black hole phase

- Starting at T_c and finite k we find new black hole branches
- Boundary conditions for ODEs
  - Demand the existence of a horizon $g(r_+) = 0$
  - Demand the solution is analytic on the black hole horizon $r = r_+$
  - Demand expansion at infinity appropriate for spontaneous symmetry breaking

$$
\begin{aligned}
g &= r^2 \left(1 - M\, r^{-4} + \cdots\right), & f &= 1 - c_h\, r^{-4} + \cdots \\
h &= r \left(1 + c_h\, r^{-4}\right), & \alpha &= c_\alpha\, r^{-4} + \cdots \\
a &= \mu + q\, r^{-2} + \cdots, & c_3 &= c_v\, r^{-|m|} + \cdots
\end{aligned}
$$

Helical superconducting black holes

Physical content of constants

- The vev of the superconducting order parameter is determined by both c_v and k

$$\langle \mathcal{O}\left(k\right)_C \rangle \propto c_v$$

- In addition we have charge density

$$\langle J^0 \rangle \propto q$$

- Holographic renormalisation reveals the stress tensor

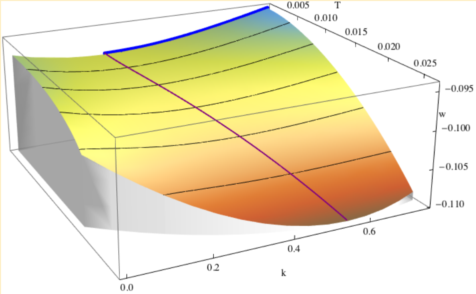
$$\begin{aligned} T_{tt} &= 3M + 8c_h \\ T_{x_1 x_1} &= M + 8c_h \\ T_{x_2 x_2} &= M + 8c_\alpha \cos\left(2kx_1\right) \\ T_{x_3 x_3} &= M - 8c_\alpha \cos\left(2kx_1\right) \\ T_{x_2 x_3} &= 8c_\alpha \sin\left(2kx_1\right) \end{aligned}$$

$\rightsquigarrow$ Spatially modulated pressure and shear in the $x_2 - x_3$ plane

- Free energy density $w\,\mathrm{vol}_3 \equiv T\left[I_{Tot}\right]_{OS}$ with $w = w_k\left(T, \mu\right) = -M$

Helical superconducting black holes

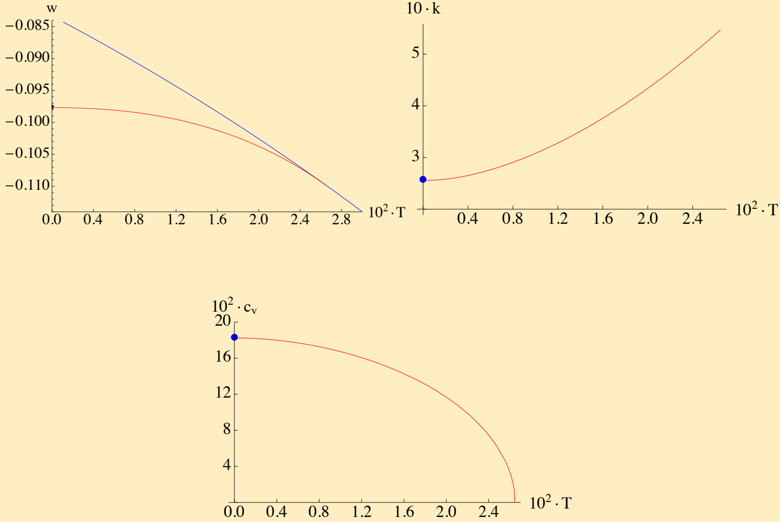
- A parameter count reveals that, for fixed μ, there exists a two parameter family of black holes labeled by T and k.
- For fixed T and μ minimise the free energy with respect to k



- Free energy minimised on the red curve
- All black holes have $c_v \neq 0$ and hence are helical superconductors
- All black holes have $w_{helical} < w_{RN}$

Helical superconducting black holes

For the thermodynamically preferred $k(T)$



Helical superconducting black holes

Zero temperature limit

- As $T \to 0$ our black hole solutions approach smooth domain walls interpolating between AdS_5 in the UV and a new scaling solution in the IR

$$ds^2 = -r^{2z}\,dt^2 + r^{-2}\,dr^2 + h_0^2\,\omega_1^2 + r^2\,\left(e^{2\alpha_0}\,\omega_2^2 + e^{-2\alpha_0}\,\omega_3^2\right)$$
$$C = \left(\imath\,c_{1(0)}\,r^{z+1}\,dt + c_{2(0)}\,dr\right) \wedge \omega_2 + c_{3(0)}\,r\,\omega \wedge \omega_3$$
$$A = a_0\,r^z\,dt$$

- z, h_0, α_0, $c_{i(0)}$, a_0 are constants
- Scaling $t \to \lambda^z t$, $x_{2,3} \to \lambda x_{2,3}$, $x_1 \to x_1$, $r \to \lambda^{-1} r$

Outline

Final comments

- Instabilities of electrically and magnetically charged black holes imply there is a very wide range of spatially modulated black holes
- Constructed the first black holes dual to spatially modulated phases $\rightarrow$ helical p-wave superconducting order, novel scaling symmetry in the IR
- Interesting to calculate transport coefficients using linear response theory. c.f. chiral nematic phase of liquid crystals

Final comments

- Other examples of helical order:
  - We looked at p-wave order. There is also $p + \imath p$ wave order. Which one wins?
  - $D = 5$ Einstein-Maxwell Chern-Simons : similar story, BUT the nature of the T=0 limit is currently unclear [AD,JPG]
  - $D = 5$ with $SU(2) \times U(1)$: expect a similar story (in progress)
- Other examples mentioned before (stripes, magnetic field ..) involve solving PDE's - expect new features!
- Rich set of examples with couplings natural in string/M-theory. Are they generic ground states of deformed CFTs?