AdS/CMT Far From Equilibrium

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Outline of Talk

- Perspectives on AdS/CMT and open challenges
- Far from equilibrium processes in correlated systems
- Utility of AdS/CFT
- Quantum quenches
- Experiment, QFT/CMT, AdS/CFT
- Dynamical phase diagram for a BCS superconductor
- Non-equilibrium AdS superconductors
- Current status and future developments

Progress in AdS/CMT

Transport Coefficients

Viscosity, Conductivity, Hydrodynamics, Bose–Hubbard, Graphene

Strange Metals

Non-Fermi liquid theory, instabilities, cuprates

Holographic Duals

Superconductors, Fermi Liquid, Ising Model

Castro *et al*, *"The Gravity Dual of the Ising Model"*
arXiv:1111.1987

$$Z_{grav} = \int \mathcal{D}g \, e^{-S_E[g]}$$

Equilibrium or close to equilibrium

Open Challenges

Far From Equilibrium

Condensed matter   Cold atoms   Heavy ion collisions

US DOE Grand Challenge # 5

"How do we characterize and control matter away - especially very far away - from equilibrium"

Lack of broadly applicable techniques, particularly for handling interacting quantum many body systems

Current approaches are either technically cumbersome or incapable of handling more generic situations

Tractable Examples

Utility of Gauge-Gravity Duality

$\boxed{\text{Quantum dynamics}}$ $\boxed{\text{Classical Einstein equations}}$

$\boxed{\text{Finite temperature}}$ $\boxed{\text{Black holes}}$

Real time approach to finite temperature quantum dynamics in interacting systems, with the possibility of anchoring to $1+1$ and generalizing to higher dimensions

Non-Equilibrium Beyond linear response

Temporal dynamics in strongly correlated systems

Combine analytics with numerics

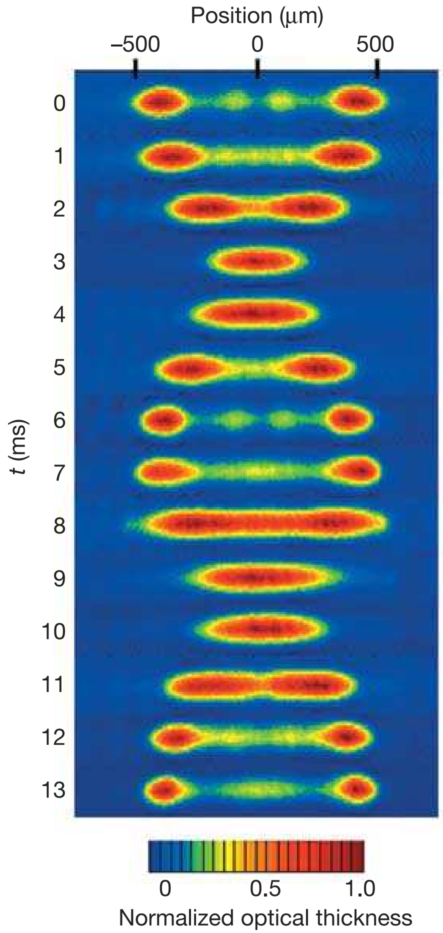
Dynamical phase diagrams

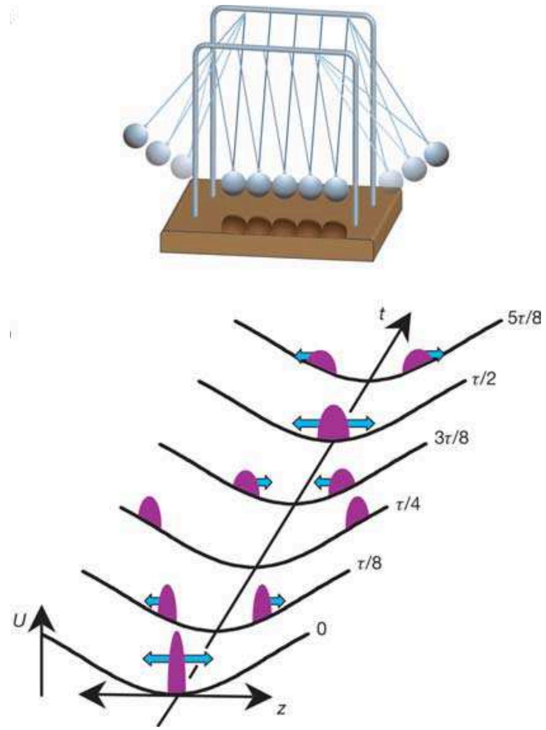
$\boxed{\text{Organizing principles out of equilibrium}}$

Experiment

Weiss *et al* "A quantum Newton's cradle", Nature **440**, 900 (2006)

Non-Equilibrium 1D Bose Gas

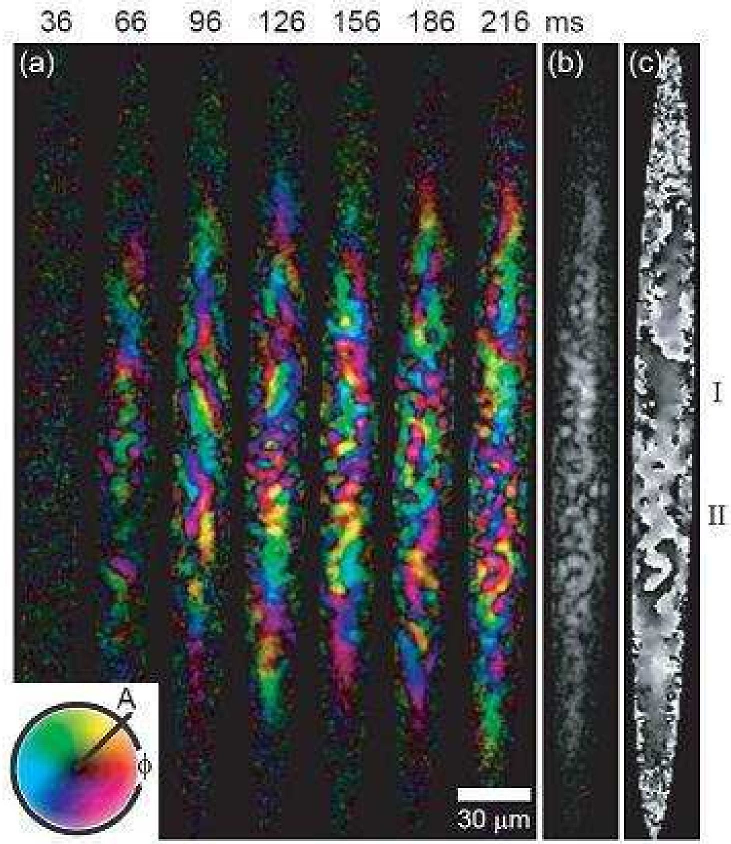




Integrability and Conservation Laws

Experiment

Stamper-Kurn *et al* "Spontaneous symmetry breaking in a quenched ferromagnetic spinor Bose–Einstein condensate", Nature **443**, 312 (2006)



Domain formation

Progress on the CMT Side

Simple protocals and integrability

Methods of integrability and CFT have been invaluable in classifying equilibrium phases and phase transitions in 1+1

Do do these methods extend to non-equilibrium problems?

Quantum quench Parameter in H abruptly changed

$$H(g) \to H(g')$$

System prepared in state $|\Psi_g\rangle$ but time evolves under $H(g')$

Quantum quench to a CFT

Cardy & Calabrese, PRL **96**, 136801 (2006)

Quantum quenches in quantum spin chains

e.g. Calabrese, Essler & Fagottini, PRL **106**, 227203 (2011)

Quantum quench in BCS pairing Hamiltonian

Andreev, Gurarie, Radzihovsky, Barankov, Levitov, Yuzbashyan...

Quantum Quench to a CFT

Calabrese and Cardy, *"Time Dependence of Correlation Functions Following a Quantum Quench"*, PRL **96**, 136801 (2006)

$$\boxed{\langle \Phi(t) \rangle = A_b^\Phi \left[\frac{\pi}{4\tau_0} \frac{1}{\cosh[\pi t/2\tau_0]} \right]^x \sim e^{-\pi x t/2\tau_0}}$$

Scaling dimension x Non-universal decay $\tau_0 \sim m^{-1}$

Ratios of observables exhibit universality

$$\boxed{\langle \Phi(r,t)\Phi(0,t) \rangle \sim e^{-\pi x r/2\tau_0}} \qquad t > r/2$$

Emergent length scale or temperature scale

AdS/CFT gives access to higher dimensional interacting critical points and the possibility of universal results

AdS Quenches

Chesler & Yaffe, *"Horizon formation and far-from-equilibrium isotropization in supersymmetric Yang–Mills plasma"*, PRL (2009)

$$\boxed{ds^2 = -dt^2 + e^{B_0(t)} d\mathbf{x}_\perp^2 + e^{-2B_0(t)} d\mathbf{x}_\parallel^2}$$

The dependent shear of the geometry $\quad B_0(t) = \frac{1}{2}c\,[1 - \tanh(t/\tau)]$

Jan de Boer & Esko Keski-Vakkuri *et al*, *"Thermalization of Strongly Coupled Field Theories"*, PRL (2011)

$$\boxed{ds^2 = \frac{1}{z^2}\left[-(1 - m(v))z^d dv^2 - 2dzdv + d\mathbf{x}^2\right]}$$

Vaidya metric quenches $\quad m(v) = \frac{1}{2}M[1 + \tanh(v/v_0)]$

Aparício & López, *"Evolution of Two-Point Functions from Holography"*, arXiv:1109.2571

Albash & Johnson, *"Evolution of Holographic Entanglement Entropy after Thermal and Electromagnetic Quenches"*, NJP (2011)

Basu & Das, *"Quantum Quench across a Holographic Critical Point"*, arXiv:1109.3309

BCS Quench Dynamics

Barankov, Levitov and Spivak, *"Collective Rabi Oscillations and Solitons in a Time-Dependent BCS Pairing Problem"*, PRL **93**, 160401 (2004)

Time dependent BCS Hamiltonian

$$\boxed{H = \sum_{\mathbf{p},\sigma} \epsilon_{\mathbf{p}} a^{\dagger}_{\mathbf{p},\sigma} a_{\mathbf{p},\sigma} - \frac{\lambda(t)}{2} \sum_{\mathbf{p},\mathbf{q}} a^{\dagger}_{\mathbf{p},\uparrow} a^{\dagger}_{-\mathbf{p},\downarrow} a_{-\mathbf{q},\downarrow} a_{\mathbf{q},\uparrow}}$$

Pairing interactions turned on abruptly

$$\boxed{\lambda(t) = \lambda\theta(t)}$$

Generalized time-dependent many-body BCS state

$$\boxed{|\Psi(t)\rangle = \prod_{\mathbf{p}} \left[u_{\mathbf{p}}(t) + v_{\mathbf{p}}(t) a^{\dagger}_{\mathbf{p},\uparrow} a^{\dagger}_{-\mathbf{p},\downarrow} \right] |0\rangle}$$

Integrable

Collective Oscillations

Baranskov, Levitov and Spivak, PRL **93**, 160401 (2004)

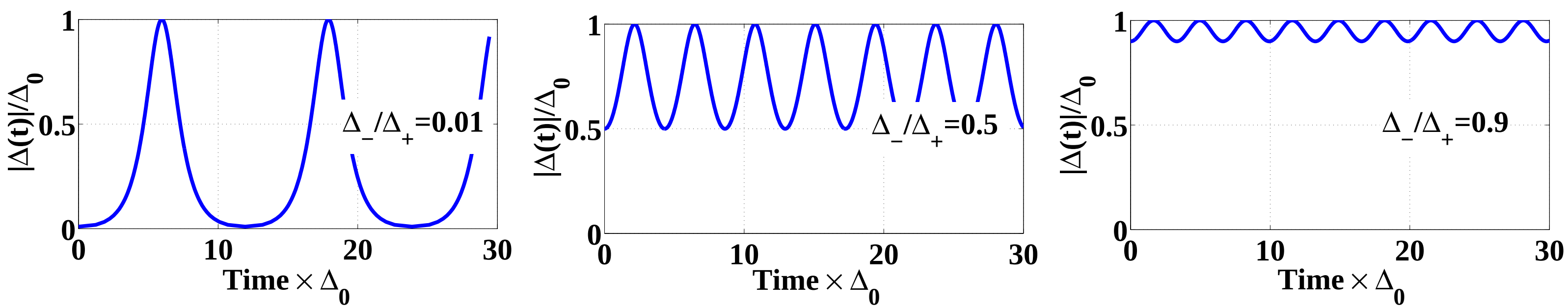
Time-dependent pairing amplitude

$$\boxed{\Delta(t) \equiv \lambda \sum_{\mathbf{p}} u_{\mathbf{p}} v^*_{\mathbf{p}}(t) = e^{-i\omega t}\Omega(t)}$$

Equation of Motion

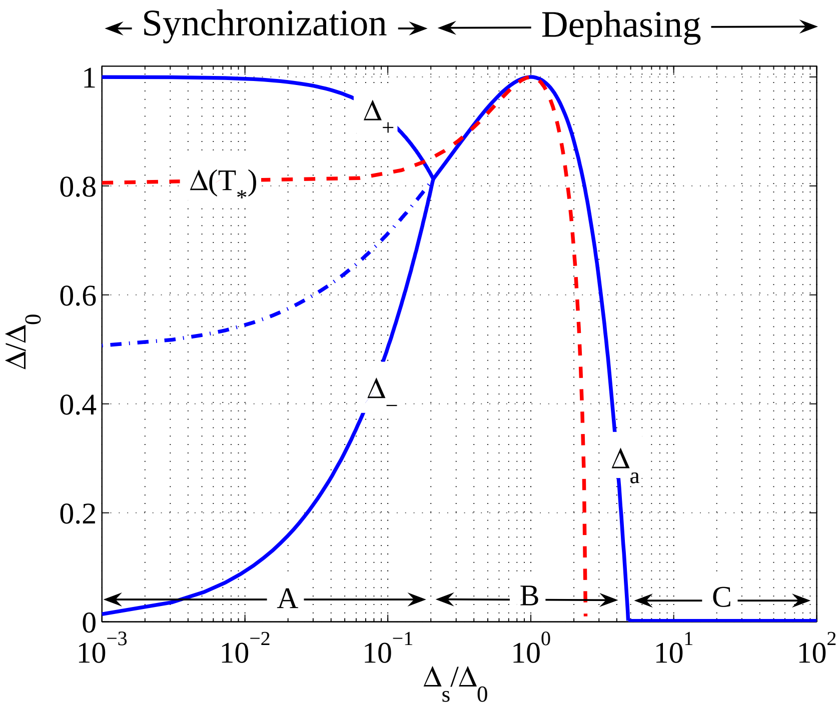
$$\boxed{\dot{\Omega}^2 + (\Omega^2 - \Delta_-^2)(\Omega^2 - \Delta_+^2) = 0}$$

Oscillations between Δ_- and Δ_+



Regimes of BCS Quench Dynamics

Barankov and Levitov, *"Synchronization in the BCS Pairing Dynamics as a Critical Phenomenon"*, PRL **96**, 230403 (2006)



Initial pairing gap $\boxed{\Delta_s}$ Final pairing gap $\boxed{\Delta_0}$

(A) Oscillations between $\Delta_\pm$ (B) Underdamped approach to Δ_a

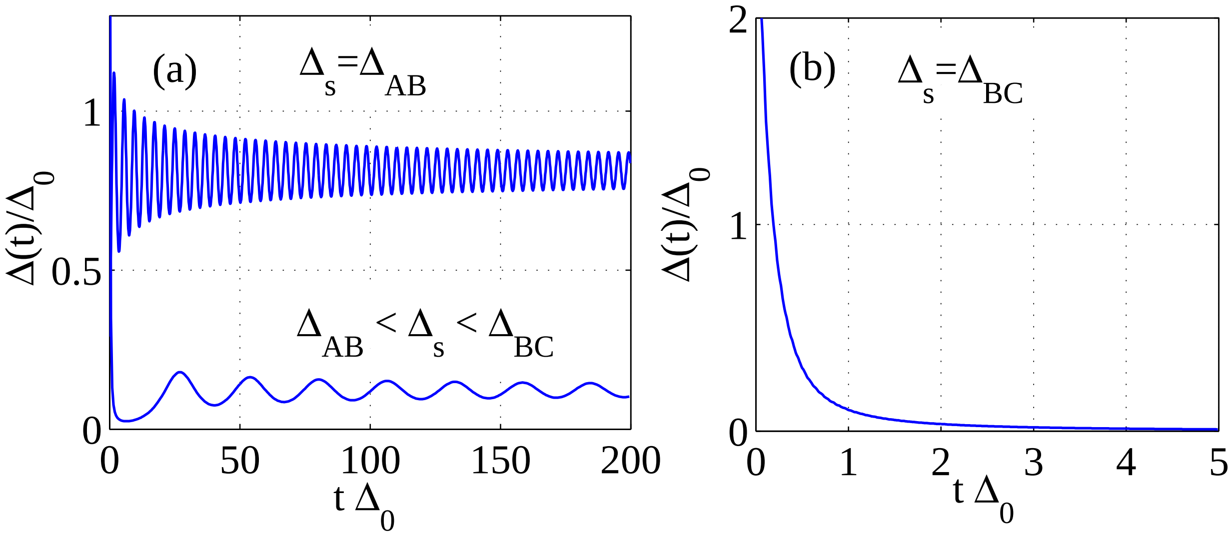
(C) Overdamped approach to $\Delta = 0$

Emergent temperature T^* and gap $\Delta(T^*)$

Bloch Dynamics

Anderson Pseudo-Spins

$$H = -\sum_{\mathbf{p}} 2\epsilon_{\mathbf{p}} s^z_{\mathbf{p}} - \lambda(t) \sum_{\mathbf{pq}} s^-_{\mathbf{p}} s^+_{\mathbf{q}}$$



$$\Delta(t) = \Delta_a + A(t)\sin(2\Delta_a t + \alpha)$$

$$A(t) \propto t^{-1/2}$$

$$\Delta(t) \propto (\Delta_s t)^{-1/2} e^{-2\Delta_s t}$$

Non-Equilibrium Dynamical Phase Diagram

Yuzbashyan *et al* Andreev *et al*

Holographic Superconductor

Gubser, *"Breaking an Abelian Gauge Symmetry Near a Black Hole Horizon"*, Phys. Rev. D. **78**, 065034 (2008)

Hartnoll, Herzog, Horowitz, *"Building a Holographic Superconductor"*, PRL **101**, 031601 (2008)

Abelian Higgs Coupled to Einstein–Hilbert Gravity

$$\boxed{S = \int d^D x \sqrt{-g} \left[R - 2\Lambda - \frac{1}{4} F_{ab} F^{ab} - |D_a \psi|^2 - m^2 |\psi|^2 \right]}$$

$$F_{ab} = \partial_a A_b - \partial_b A_a \quad D_a = \partial_a - iqA_a \quad \Lambda = -\frac{D(D-1)}{4L^2}$$

Finite temperature **Black hole**

Below critical temperature black hole with $\psi = 0$ unstable

Black hole with charged scalar hair $\psi \neq 0$ becomes stable

Spontaneous U(1) **symmetry breaking**

Non-Equilibrium AdS Superconductors

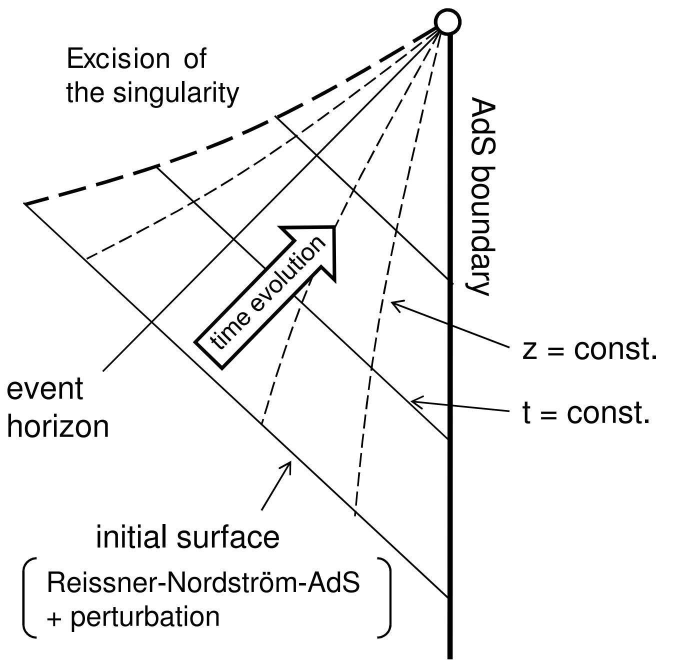
Murata, Kinoshita & Tanahashi, *"Non-Equilibrium Condensation Process in a Holographic Superconductor"*, JHEP 1007:050 (2010)

Start in unstable normal state described by a Reissner–Nordström black hole and perturb

Temporal dynamics from supercooled to superconducting

AdS Geometry

Murata, Kinoshita & Tanahashi, JHEP 1007:050 (2010)



$$\psi(t,z) = \psi_1(t)z + \psi_2(t)z^2 + \psi_3(t)z^3 + \cdots \equiv \psi_1(t)z + \tilde{\psi}(t,z)z^2$$

Gaussian perturbation of Reissner–Nordström–AdS

$$\boxed{\tilde{\psi}(t=0,z) = \frac{\mathcal{A}}{\sqrt{2\pi}\delta} \exp\left[-\frac{(z-z_m)^2}{2\delta^2}\right]}$$

$$\mathcal{A} = 0.01,\ \delta = 0.05,\ z_m = 0.3$$

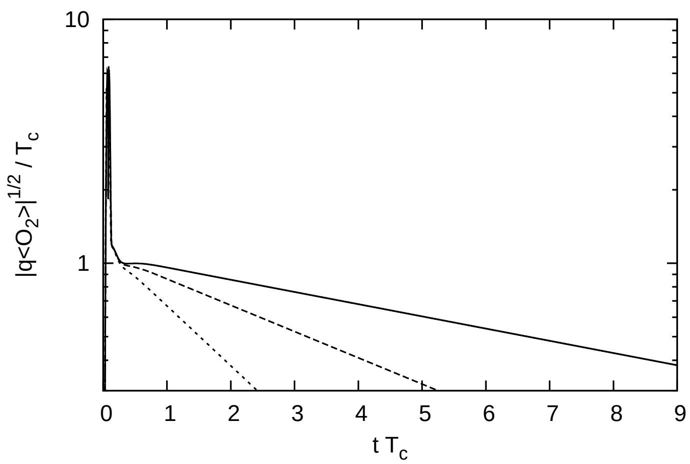
Time Evolution of the Order Parameter

Murata, Kinoshita & Tanahashi, JHEP 1007:050 (2010)

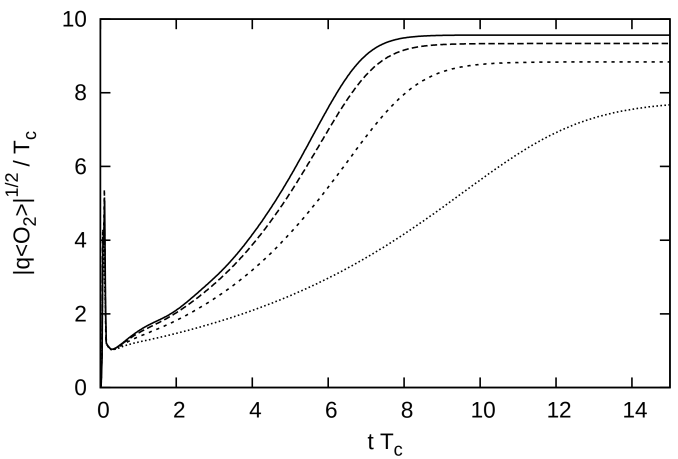
Start in normal state (Reissner–Nordström) & perturb

Numerical solution of Einstein equations

$$\boxed{\langle \mathcal{O}_2(t) \rangle \equiv \sqrt{2}\psi_2(t)}$$



$T/T_c = 1.1, 1.2, 1.4$



$T/T_c = 0.2, 0.4, 0.6, 0.8$

$T > T_c$ **Relaxation to N** $|\langle \mathcal{O}_2(t) \rangle| = C \exp(-t/t_{\rm relax})$

$T < T_c$ **Evolution to SC** $|\langle \mathcal{O}_2(t) \rangle| = C_1 \exp(-t/t_{\rm relax}) + C_2$

Questions

Where are the oscillations?

- Is there always damping or are there different regimes?
- Are the dynamics of holographic superconductors related to condensed matter systems or intrinsically different?
- What is the role of coupling to a large number of critical degrees of freedom?
- What is the role of large N and strong coupling?
- What is their influence on the emergent timescales?
- Do thermal fluctuations reduce the amplitude of oscillations?
- Beyond BCS and mean field dynamics using AdS/CFT?
- What is the dynamics at short, intermediate and long times?
- What happens if one quenches from a charged black hole?

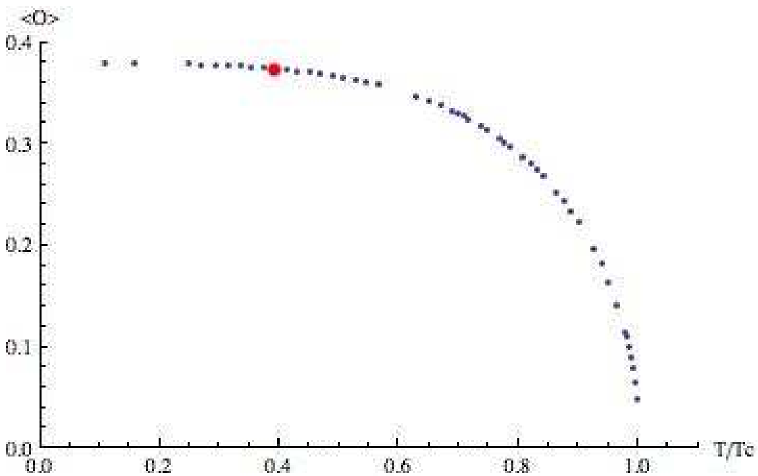
AdS/CMT far from equilibrium?

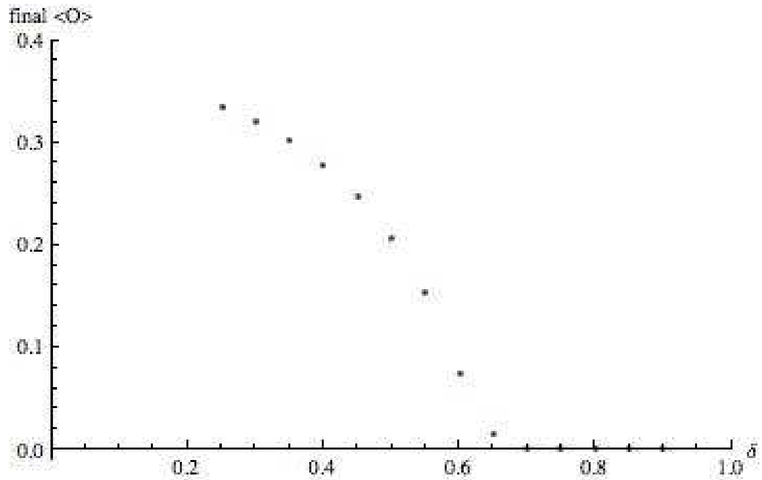
Preliminary Dynamical Phase Diagram

Conjugate field pulse

$\boxed{\mathrm{AdS}_4/\mathrm{CFT}_3}$ $\qquad \psi(t,r) = \psi_0(t)/r + \psi_1(t)/r^2 + \ldots$

$$\boxed{\psi_0(t) = -\delta e^{-100(t-1)^2}}$$





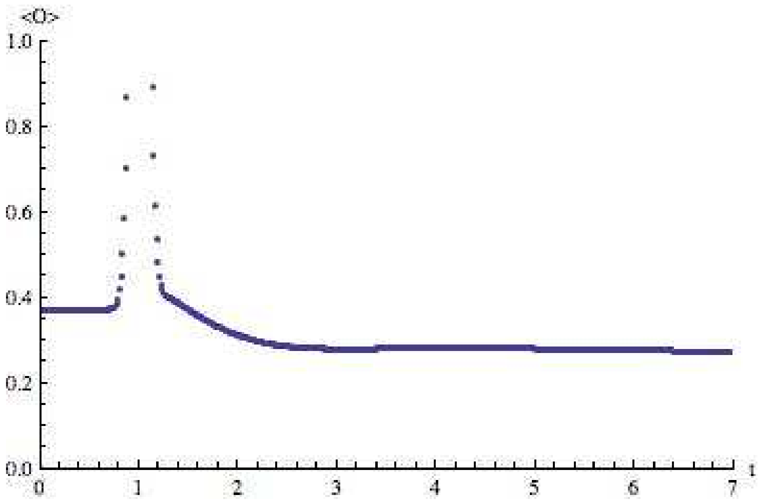
Time Evolution

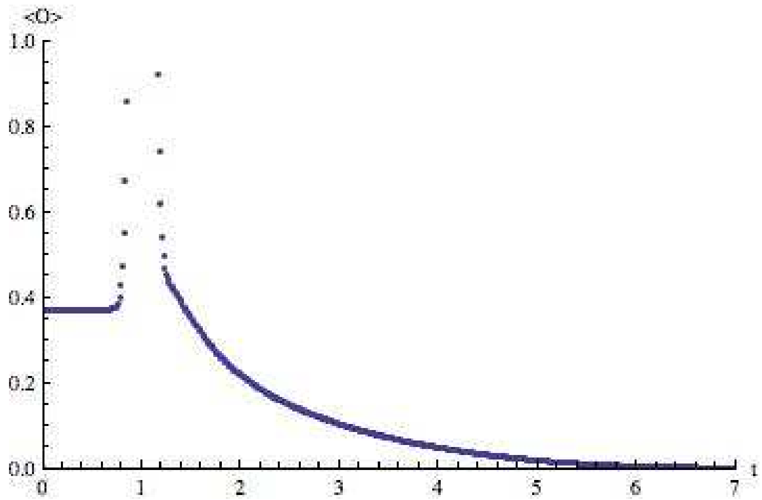
Conjugate field pulse

AdS$_4$/CFT$_3$ $\qquad \psi(t,r) = \psi_0(t)/r + \psi_1(t)/r^2 + \dots$

$$\psi_0(t) = -\delta e^{-100(t-1)^2}$$

Weak pulse $\boxed{\delta = 0.4}$ $\qquad$ Strong pulse $\boxed{\delta = 0.9}$





Remains superconducting $\qquad$ **Approaches normal**

Conclusions

- AdS/CFT is a potential tool for non-equilibrium dynamics
- Allows access to real time dynamics over entire time interval
- Amenable to both numerical and analytical treatments
- Emergent temperatures, black hole formation and equilibration
- AdS_3 allows contact with integrable systems in 1+1 dimensions
- Extensions to higher dimensional systems

Developing new tools and organizing principles

Acknowledgements

DAMTP AdS/CMT discussion group

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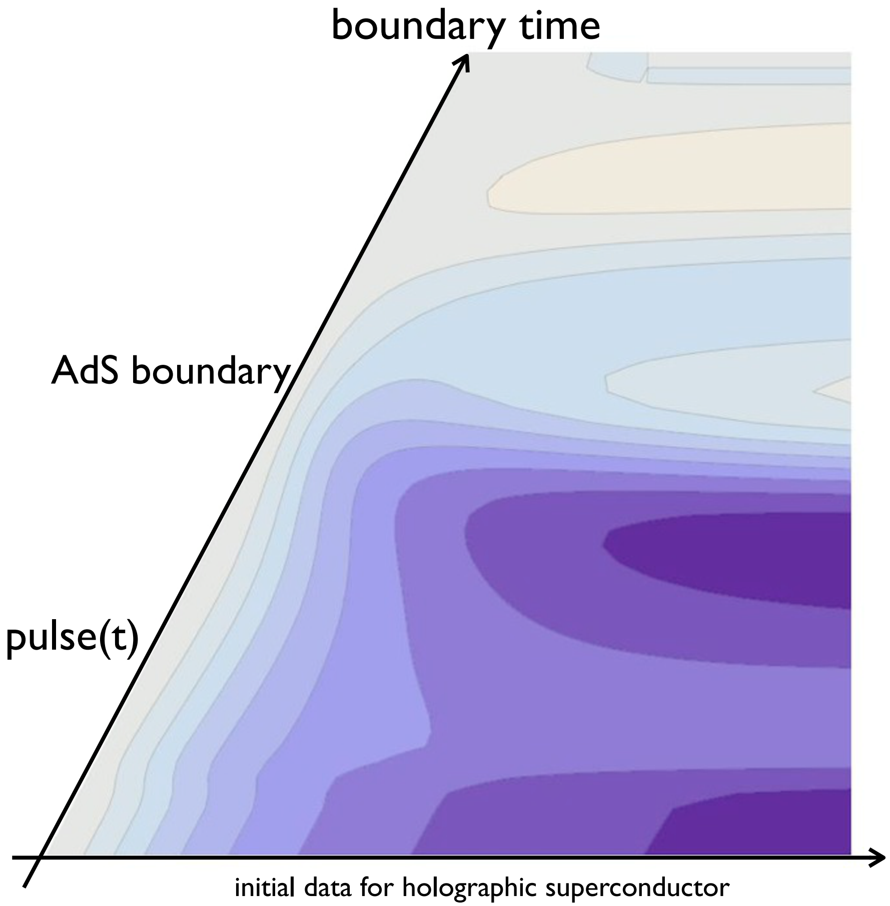
Research Council

Work in Progress

Chemical potential quench in a holographic superconductor

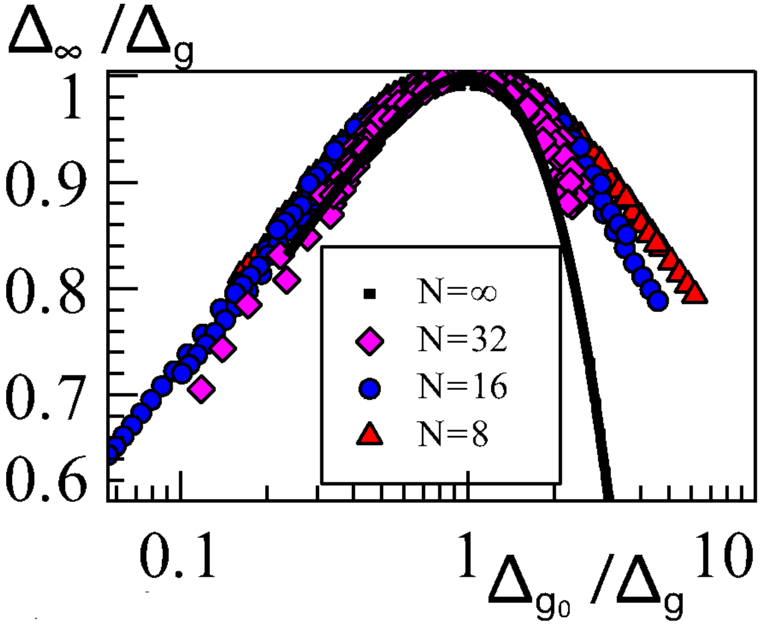
$\boxed{\mathrm{AdS}_4/\mathrm{CFT}_3}$ $\quad \mathbf{A} \leftrightarrow \mathbf{J} \quad A_0(t,z) = A_0^{\mathrm{I}}(t) + zA_0^{\mathrm{II}(t)} + \ldots$

Short top hat pulse of $A_0^{\mathrm{I}}(t)$ on boundary and monitor $\psi(t,z)$



Bethe Ansatz + Numerics

Faribault, Calabrese and Caux, *Quantum quenches from integrability: the fermionic pairing model*, J. Stat. Mech. (2009) P03018



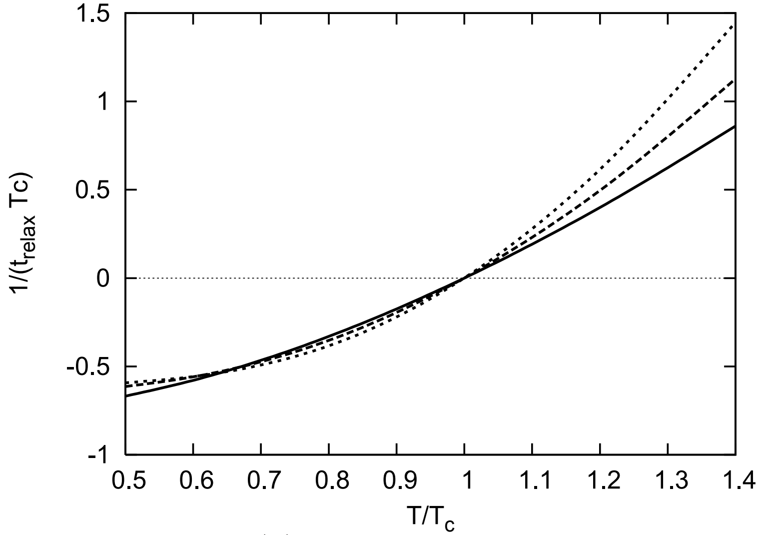
$$g_0 \to g \qquad |\psi(t)\rangle = e^{-iH_g t}|\psi^{\mu}_{g_0}\rangle \qquad |\psi(t)\rangle = \sum_\nu e^{-i\omega^\nu_g t}\langle\psi^\nu_g|\psi^\mu_{g_0}\rangle|\psi^\nu_g\rangle$$

$$\langle\mathcal{O}(t)\rangle \equiv \langle\psi(t)|\mathcal{O}|\psi(t)\rangle = \sum_{\nu,\xi} e^{i(\omega^\nu_g-\omega^\xi_g)t}Q^{\mu\nu}_{g_0 g}Q^{\xi\mu}_{g g_0}\langle\psi^\nu_g|\mathcal{O}|\psi^\xi_g\rangle$$

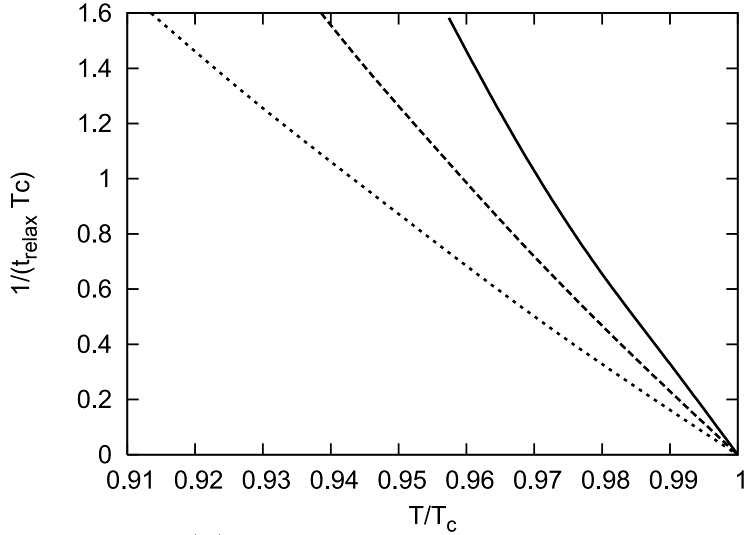
Quench Matrix $\boxed{Q^{\nu\mu}_{g g_0} \equiv \langle\psi^\nu_g|\psi^\mu_{g_0}\rangle}$

Growth and Decay Rates

Murata, Kinoshita & Tanahashi, JHEP 1007:050 (2010)



(a) Normal phase



(b) Superconducting phase

Quasi Normal Modes

Imado *et al*, *"Hydodynamics of Holographic Superconductors"*, JHEP '09

Konoplya, *"Decay of a charged scalar field around a black hole: Quasi normal modes of RN, RNAdS and dilaton black holes,"* PRD '02

Miranda *et al*, *"Quasinormal modes of plane-symmetric black holes according to the AdS/CFT correspondence"*, JHEP '08