Optical Conductivity from a Holographic Lattice

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Based on work with Gary Horowitz and Jorge Santos

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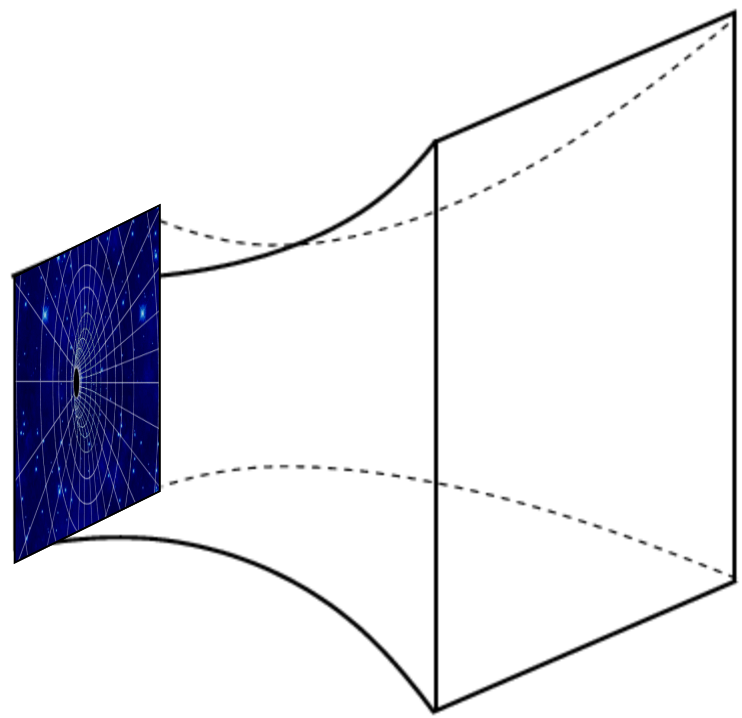
Strongly Interacting Matter at Finite Temperature and Density

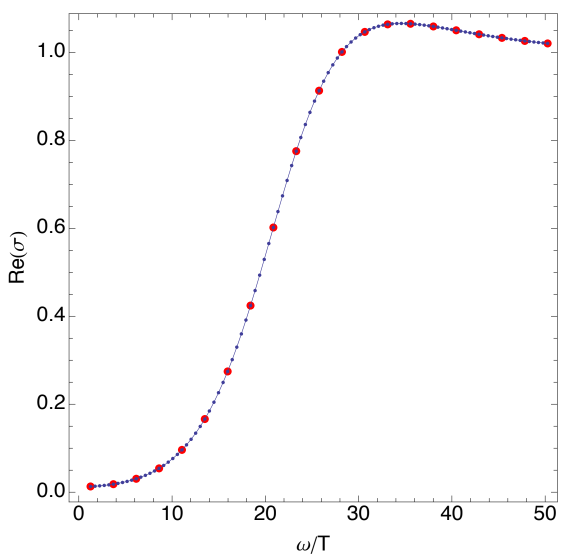


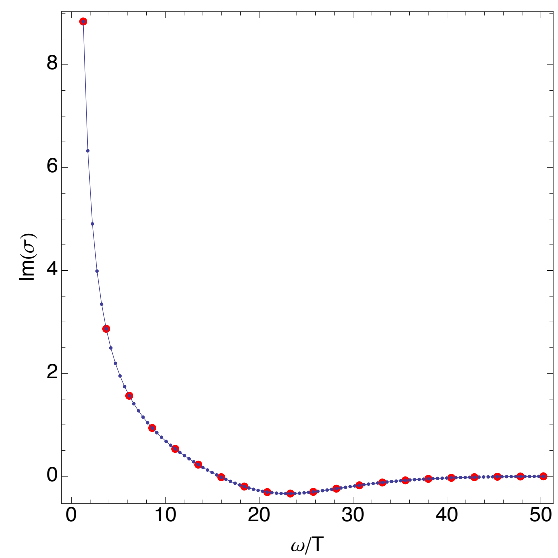
Boundary field theory
d=2+1

- Bulk d=3+1 black hole
- Hawking radiation = finite temperature
- Electrically charged (Reissner-Nordstrom) = finite density

Optical Conductivity

$$j(\omega) = \sigma(\omega) E(\omega)$$



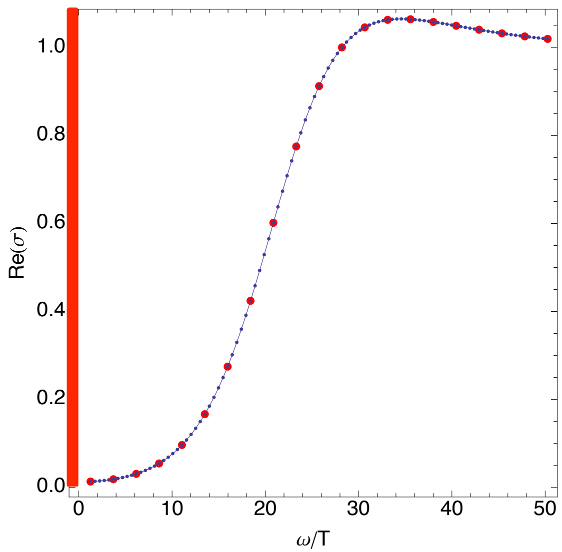


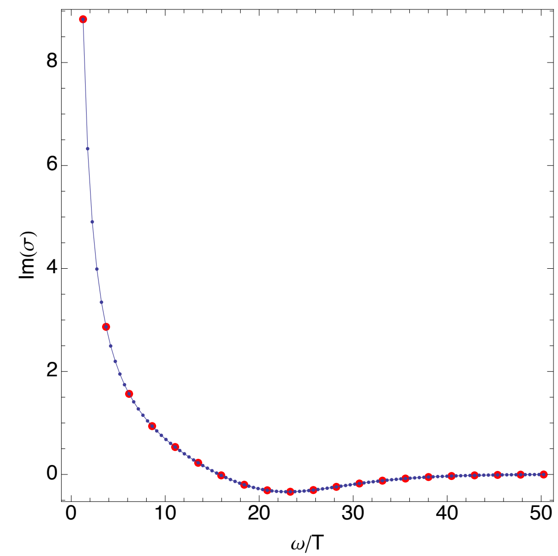
$$\operatorname{Im} \sigma(\omega) \to \frac{K}{\omega}$$

Herzog, Kovtun, Sachdev and Son; Hartnoll

Optical Conductivity

$$j(\omega) = \sigma(\omega) E(\omega)$$





$$\mathrm{Re}\,\sigma(\omega) \sim K\,\delta(\omega)$$

$$\mathrm{Im}\,\sigma(\omega) \to \frac{K}{\omega}$$

Herzog, Kovtun, Sachdev and Son; Hartnoll

Resolving the Delta-Function

$$\mathrm{Re}\,\sigma(\omega) \sim K\,\delta(\omega)$$

Due to:
- Finite density of charge carriers
- Translational invariance

Goal: Break translational invariance

Building the Lattice

$$\mathcal{L} = \mathcal{L}_{\rm CFT} + \mu \mathcal{Q} + \phi_0(x,y)\mathcal{O}$$

Introduce a neutral, bulk scalar field: $\Phi \longleftrightarrow \mathcal{O}$

Pick $m_\Phi^2 L^2 = -1 \Longrightarrow$ relevant operator:

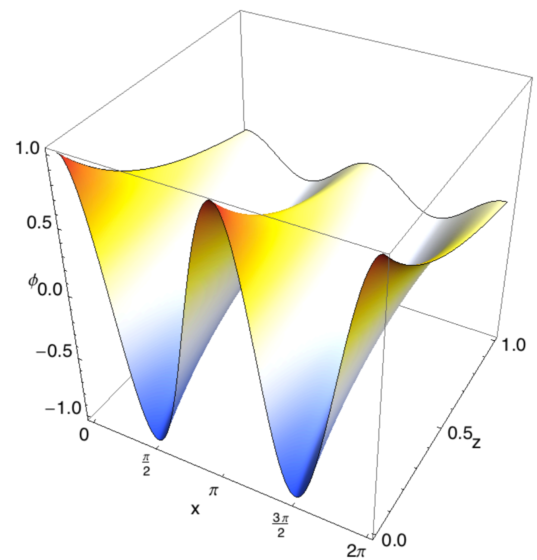
$$\Phi \to z\phi_0 + z^2\phi_1 + \ldots$$

source: $\phi_0 = A\cos(k_L x)$

The Lattice

Solve Einstein equations subject to lattice boundary conditions: $A_0(z,x)$, $\Phi(x,z)$ and

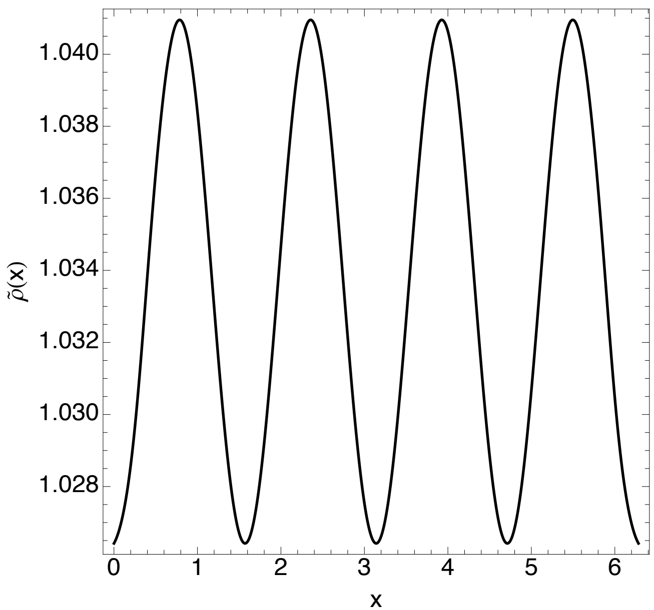
$$ds^2 = \frac{L^2}{z^2}\Big[-g_{tt}(z,x)dt^2 + g_{zz}(z,x)dz^2 + g_{xx}(z,x)(dx + a(z,x)dz)^2 + g_{yy}(z,x)dy^2\Big]$$



Note: $\mathcal{O}$ relevant; $e^{ik_L x}\mathcal{O}$ irrelevant

parameters
$T,\ \mu,\ k_L,\ A$

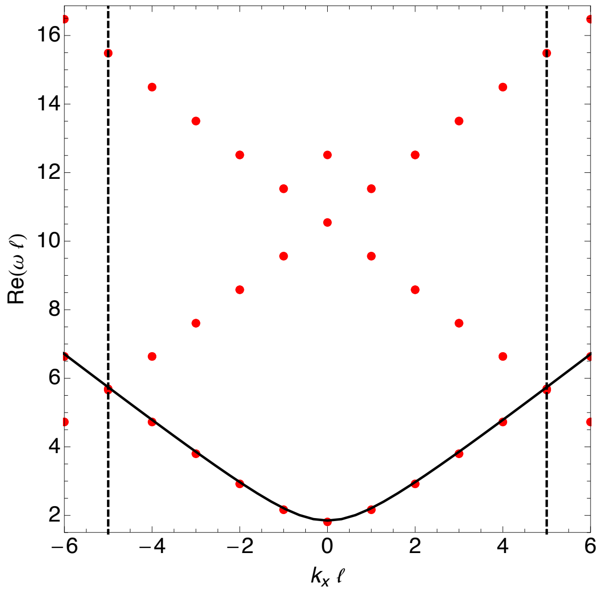
Charge Density

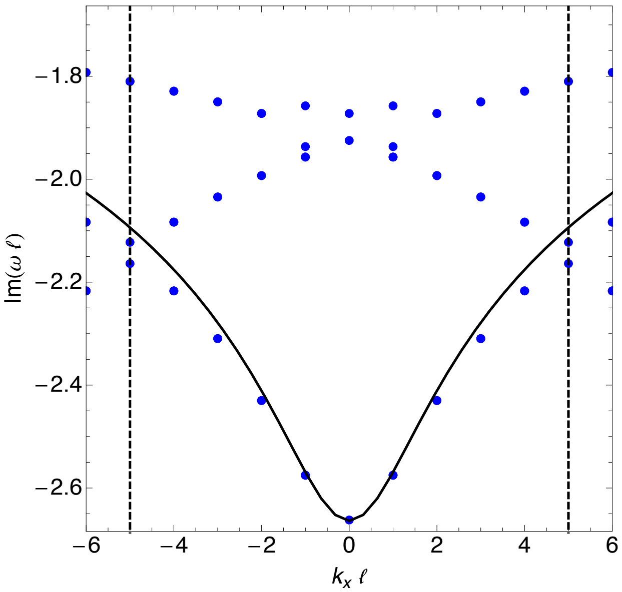


$$\text{Charge Density} \sim k_L^2$$

Band Structure

Add a probe scalar field in the lattice background





Band Structure

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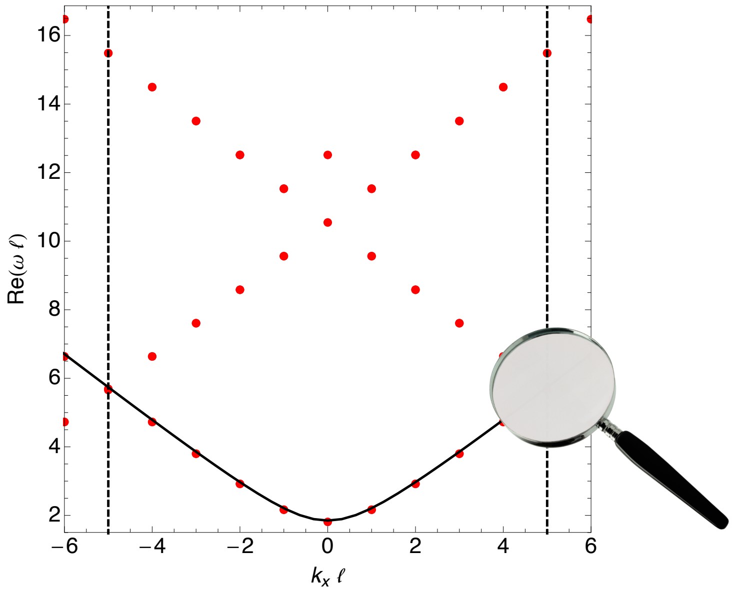
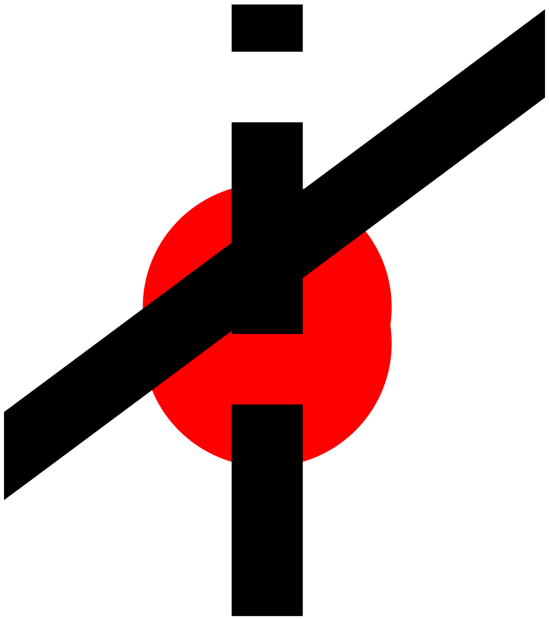




Band gaps negligible

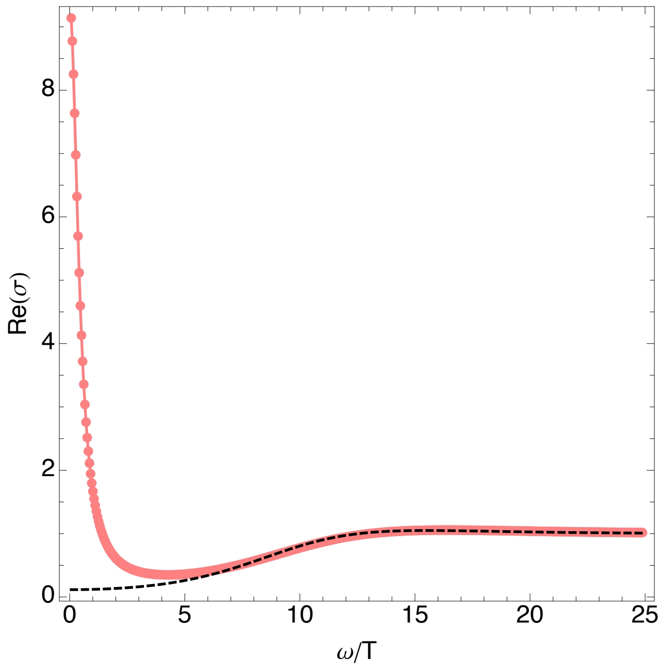
Perturbing the Lattice

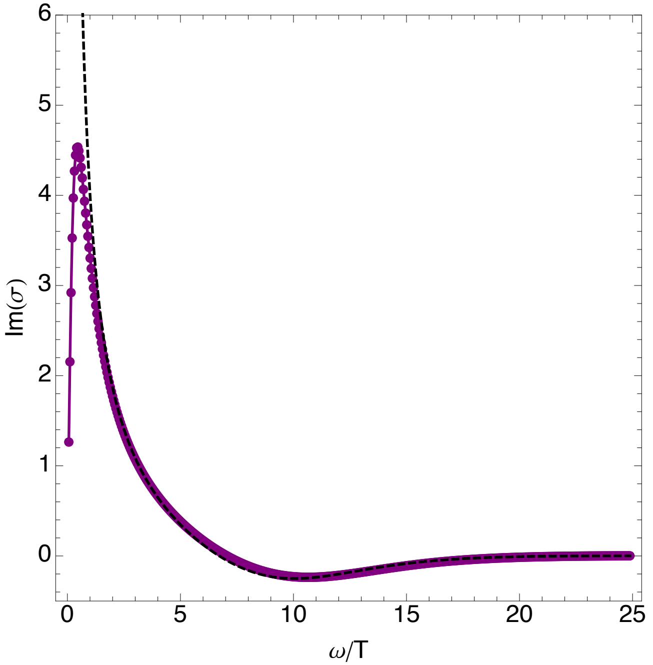
$$\delta g_{tt},\ \delta g_{tz},\ \delta g_{tx},\ \delta g_{zz},\ \delta g_{zx},\ \delta g_{xx},\ \delta g_{yy}$$

$$\delta A_t,\ \delta A_z,\ \delta A_x,\ \delta \Phi$$

Optical Conductivity

$$j(\omega) = \sigma(\omega)E(\omega)$$

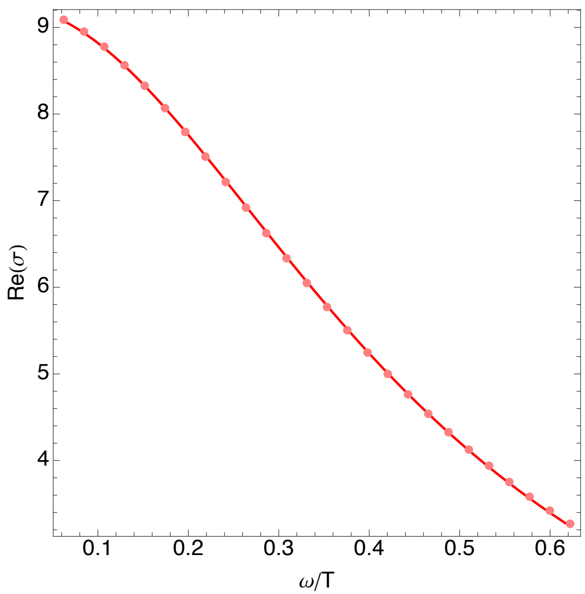


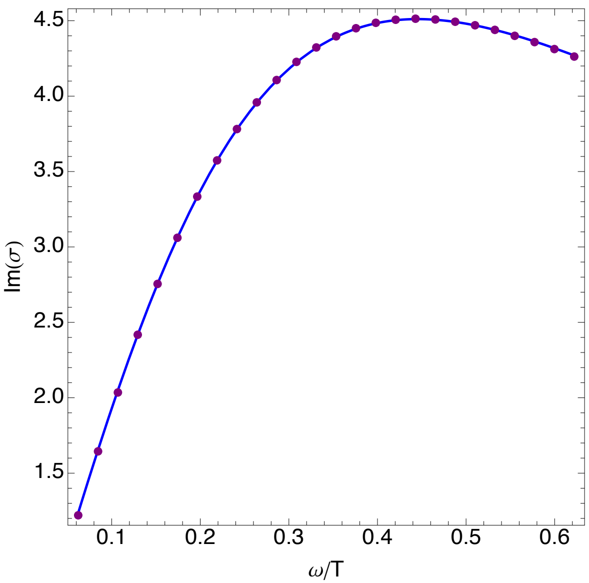


$\mu = 1.4$ $\quad T = 0.115\mu$ $\quad k_L = 2$ $\quad A = 1.5$

Low Frequency Behaviour

$\omega \lesssim T$

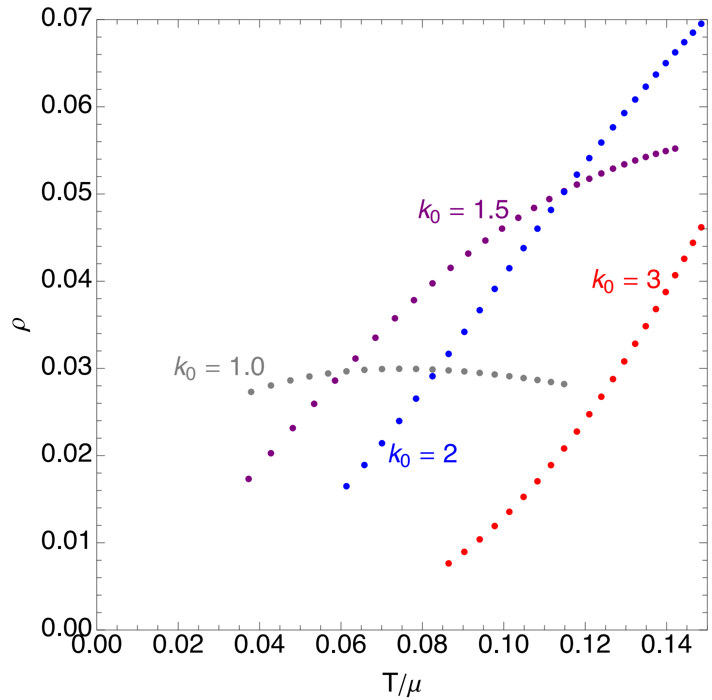




$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$

Note: No quasiparticles

DC Resistivity



$$\rho = \frac{1}{K\tau}$$

Nearly all temperature dependence in τ

DC Resistivity

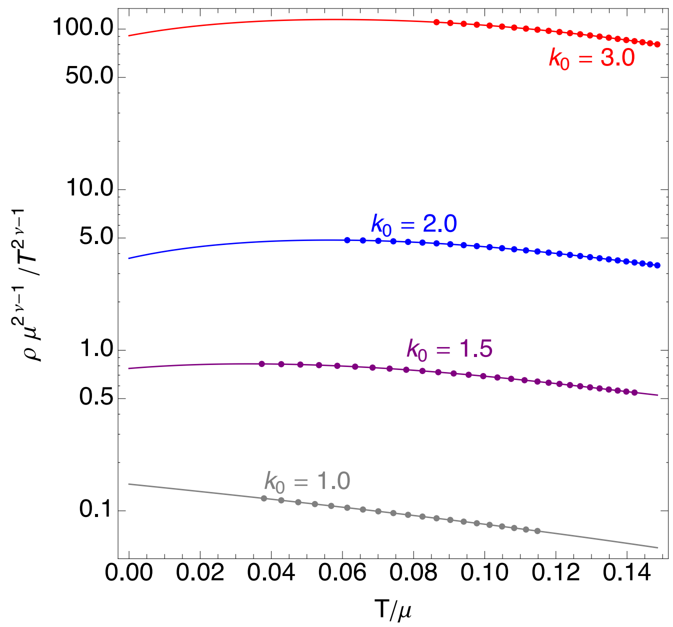
Low energy excitations around black hole governed by *locally critical* theory

- Field Theory: $z \to \infty$
- Geometry: $AdS_2 \times \mathbf{R}^2$

Hartnoll and Hofman: $\rho \sim T^{2\nu-1}$

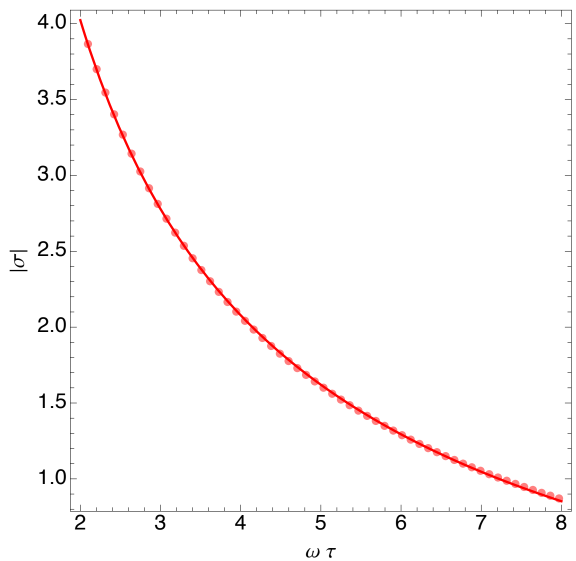
$$\nu = \frac{1}{2}\sqrt{5 + 2(k/\mu)^2 - 4\sqrt{1 + (k/\mu)^2}}$$

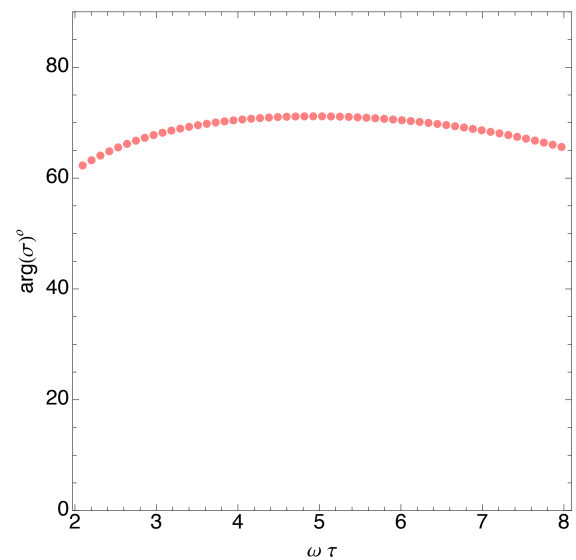
DC Resistivity



Mid-Frequency Behaviour

$$\omega \gtrsim T$$
$$2 < \omega\tau < 8$$



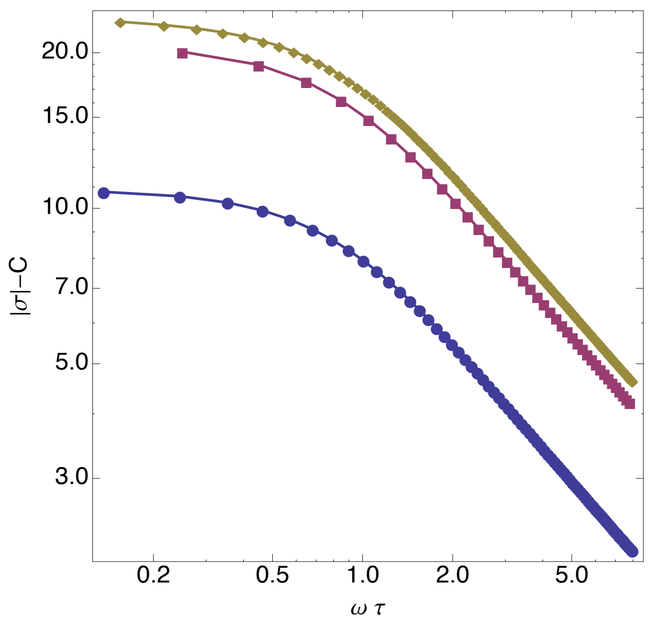


$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

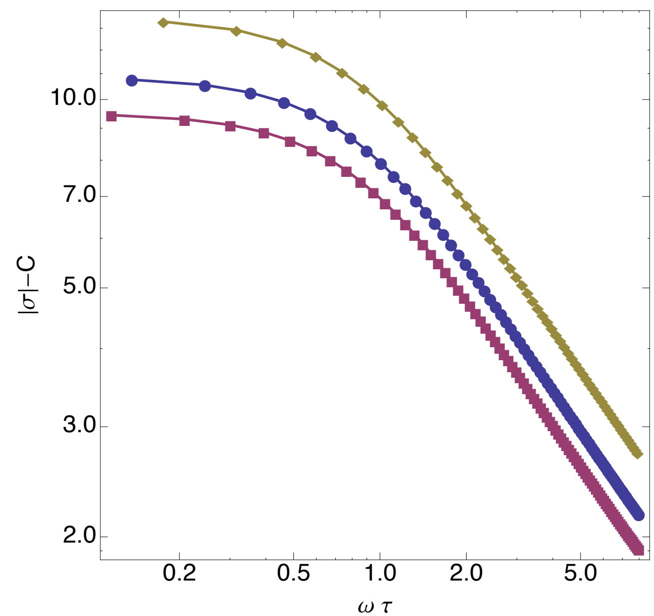
Not arising from near horizon geometry alone

Robust Power-Law

Log-log plots



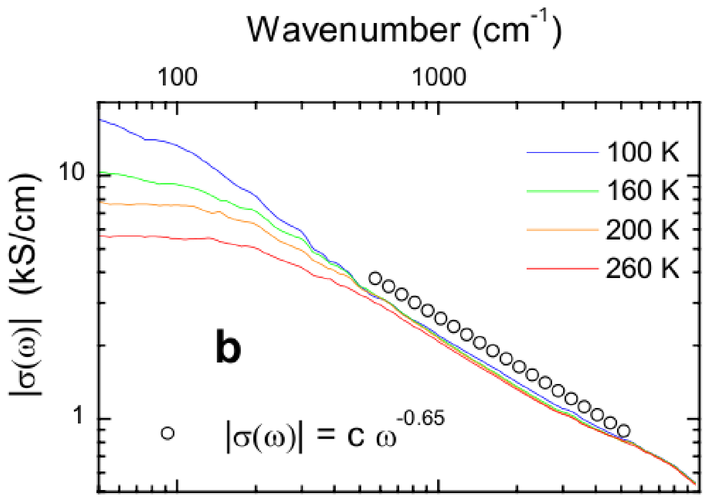
$k_L = 3,\ k_L = 1,\ k_L = 2$

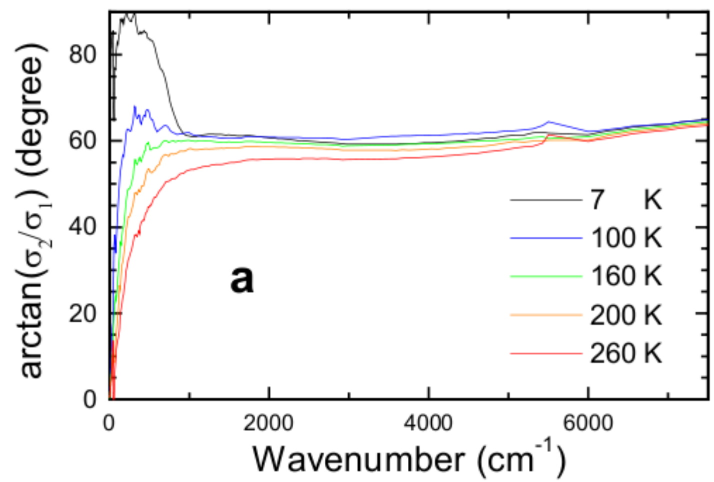


$T = 0.98\mu,\ T = 0.115\mu,\ T = 0.13\mu$

Comparison to Cuprates

van der Marel et al.





$$Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$$

Note: No offset, *C*

Summary

Why?