An Overview of AdS/CFT

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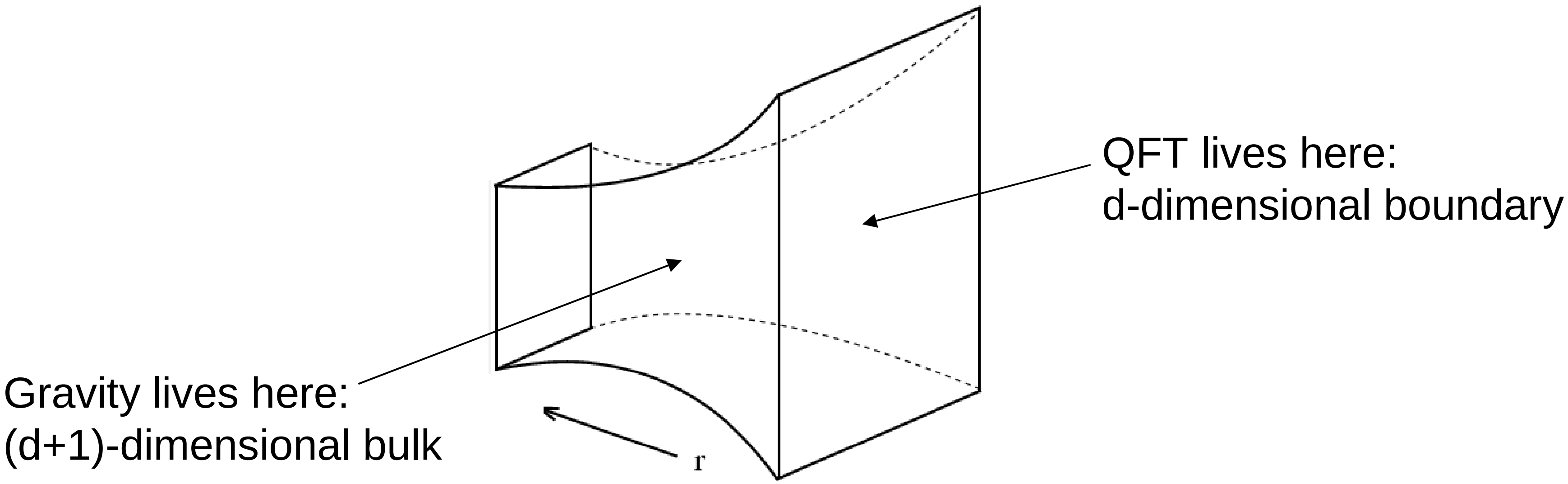
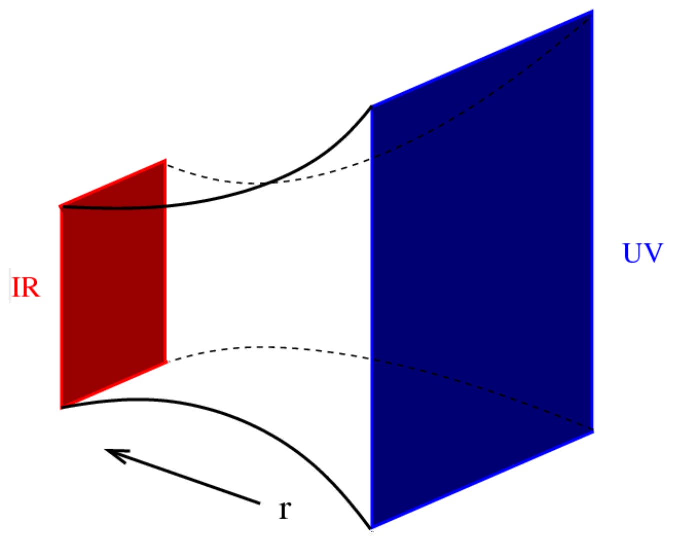
The AdS/CFT Correspondence

Strongly interacting QFT in d-dimensions ⟺ General relativity in (at least) (d+1)-dimensions



[G,R]=0

- The extra direction, r, should be thought of as *energy scale*.
- Objects occuring on different scales live in different *r*-slices of bulk
- AdS/CFT is the geometrization of Wilsonian RG flow.



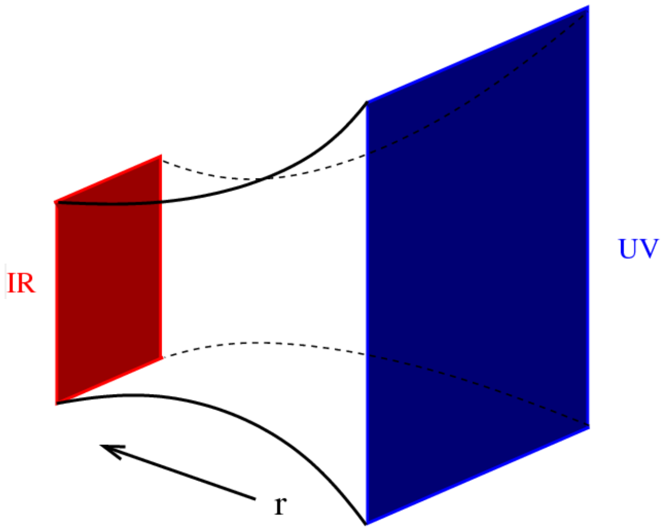
What is AdS?

- AdS = Anti-de Sitter Space

slice of Minkowksi space

$$ds^2 = \frac{L^2}{r^2}\left(dr^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$

horizon at
$r \rightarrow \infty$

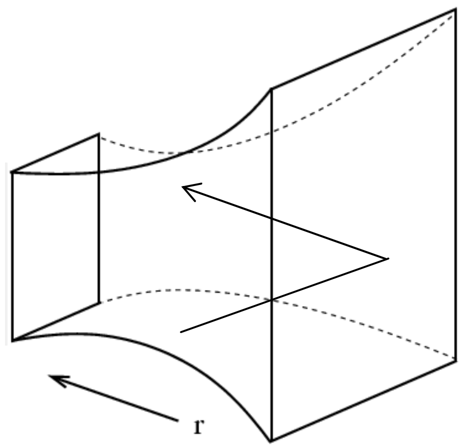


boundary at $r = 0$

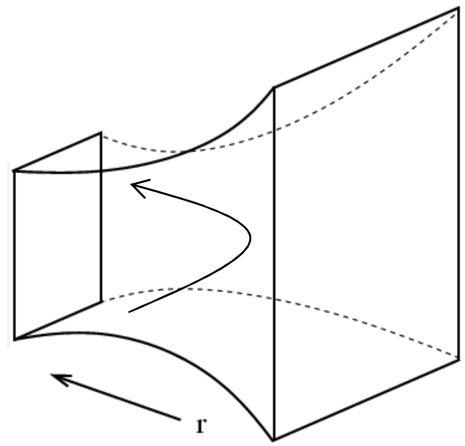
Properties of AdS

$$ds^2 = \frac{L^2}{r^2}(dr^2 + \eta_{\mu\nu}dx^\mu dx^\nu)$$

- Solution of Einstein's equations with *negative* cosmological constant.
- Isometry group is translations + *SO(d,2)*
- Geodesics



lightlike geodesics hit the boundary



timelike geodesics don't

The Basics of AdS/CFT

Generating Function

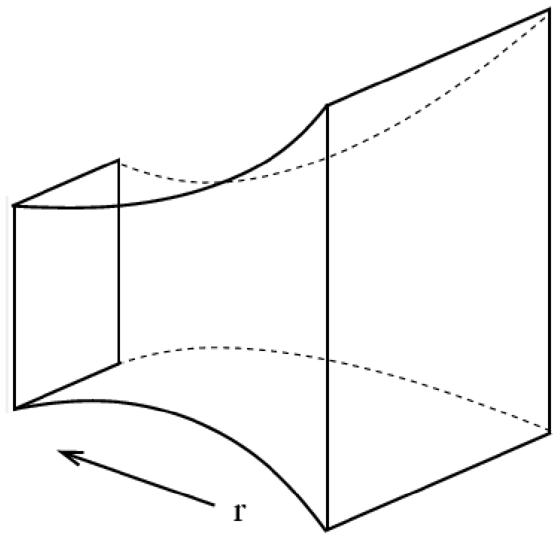
$$Z_{\mathrm{QFT}}[\phi_0] = \int \mathcal{D}A \, \exp\left(\frac{i}{\hbar} S_{\mathrm{QFT}}[A] + \phi_0 \mathcal{O}(A)\right)$$

source

operator

Idea: Make the Sources Come Alive

$$\phi(\vec{x}, r) \to \phi_0(\vec{x})$$



How to Calculate: GKPW Formula

(Gubser, Klebanov, Polyakov; Witten)

$$Z_{\mathrm{QFT}}[\phi_0] = Z_{\mathrm{Quantum\ Grav}}[\phi \to \phi_0]$$

$$\approx e^{iS_{\mathrm{gravity}}(\phi)}\Big|_{\phi \to \phi_0}$$

when classical gravity is valid

on-shell action for AdS bulk

boundary conditions

AdS/CFT Will Not Solve Your Favourite Theory

- Tricky Part: Find the map

$$S_{\mathrm{QFT}} :\rightarrow S_{\mathrm{gravity}}$$

- We have several classes of well explored examples, but no proof.
  - e.g. Maximally supersymmetry Yang-Mills = IIB string theory on $AdS_5 \times S^5$

- Simpler method: take your favourite gravity theory to *define* the QFT on the boundary

- Classical gravity means large N
  - Matrix large N, not vector large N

Building the Dictionary

The Dictionary

Bulk	Boundary
Fields, $\phi(\vec{x}, r)$	Operators, $\mathcal{O}(\vec{x})$
Mass, m	Dimension, $\Delta(\Delta - d) = m^2 L^2$
Spin, Charge	Spin, Charge

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Some special fields

Metric, $g_{\mu\nu}$	Stress Tensor, $T_{\mu\nu}$
Gauge Fields, A_μ	Conserved Current, J_μ

A Scalar Field

- The Lagrangian for a scalar field in AdS is

$$S_{\text{scalar}} = -\frac{1}{2}\int d^{d+1}x\sqrt{-g}\,\left(g^{AB}\partial_A\phi\partial_B\phi + m^2\phi^2\right)$$

- Special property of AdS: stability requires

$$m^2 \geq -\frac{d^2}{4L^2}$$

(Breitenlohner-Freedman bound)

  - Relevant boundary operator: $m^2 < 0$
  - Marginal boundary operator: $m^2 = 0$
  - Irrelevant boundary operator: $m^2 > 0$

Solution near the Boundary

- Solve the equation of motion near *r=0*

$$\phi(\vec{x}, r) \rightarrow \left(\tfrac{r}{L}\right)^{\Delta_-} [\phi_0(\vec{x}) + \ldots] + \left(\tfrac{r}{L}\right)^{\Delta_+} [\phi_1(\vec{x}) + \ldots]$$

$$\Delta_\pm = \tfrac{d}{2} \pm \sqrt{\left(\tfrac{d}{2}\right)^2 + m^2 L^2}$$

- $\phi_0(\vec{x})$ is source. (I lied before!)
- What is interpretation of $\phi_1(\vec{x})$

Response

- Can use GKPW formula to show that

$$\phi_1(\vec{x}) = \langle \mathcal{O}(\vec{x}) \rangle$$

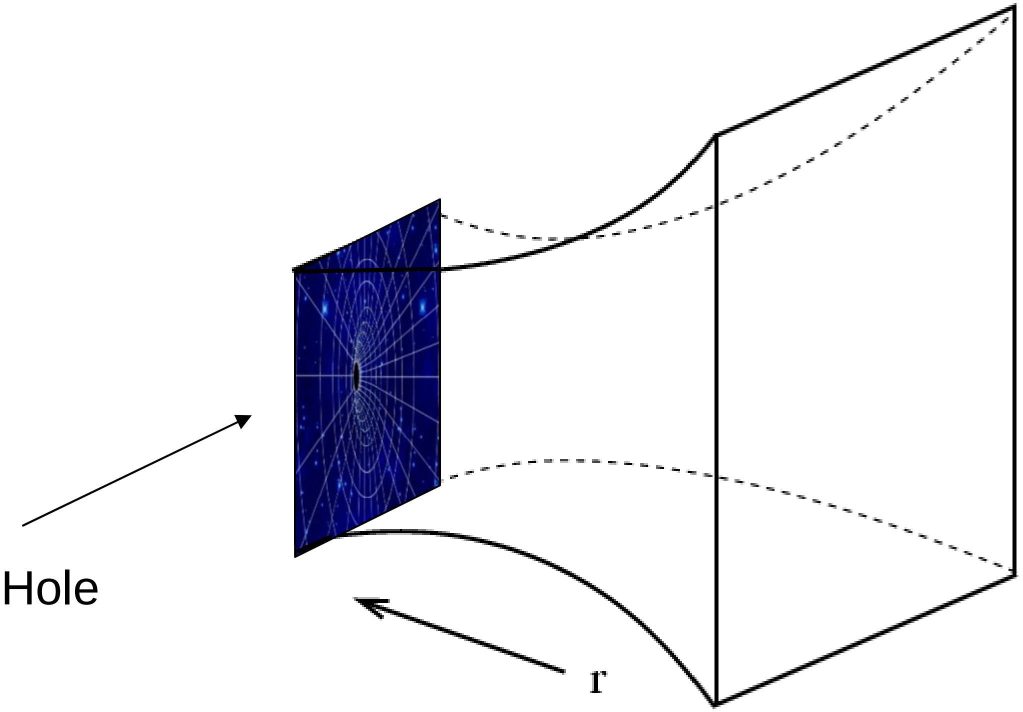
  - i.e. response in presence of source $\phi_0(\vec{x})$

Every Single “Holographic”Calculation Ever...

- Fix the source $\phi_0(\vec{x})$
- Fix another boundary condition in the interior of space
  - regularity
  - ingoing boundary condition at horizon
- Compute the response $\phi_1(\vec{x})$

What AdS/CFT is Good For

Finite Temperature

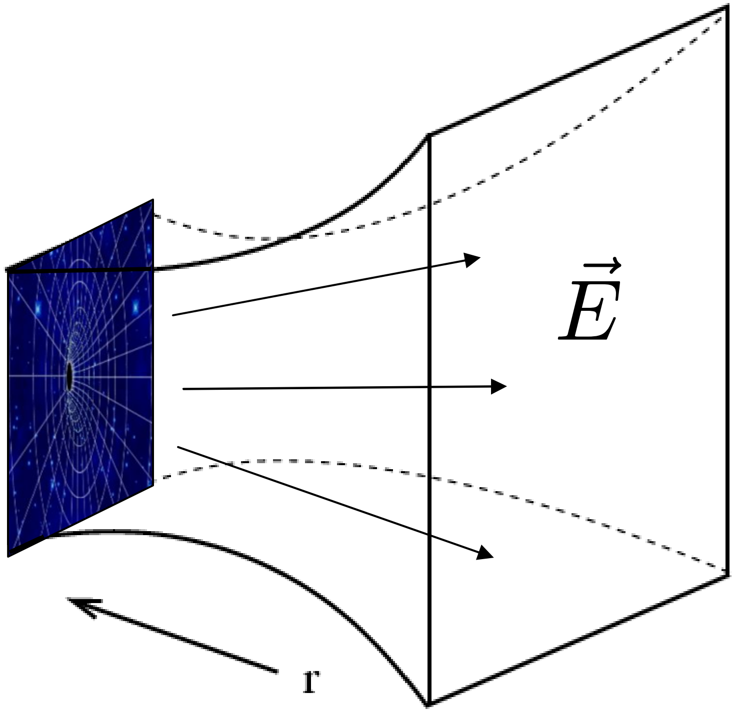


Black Hole

Boundary theory at
Hawking temperature, *T*

Euclidean and *Lorentzian* signatures

Finite Density



Temperature, *T*

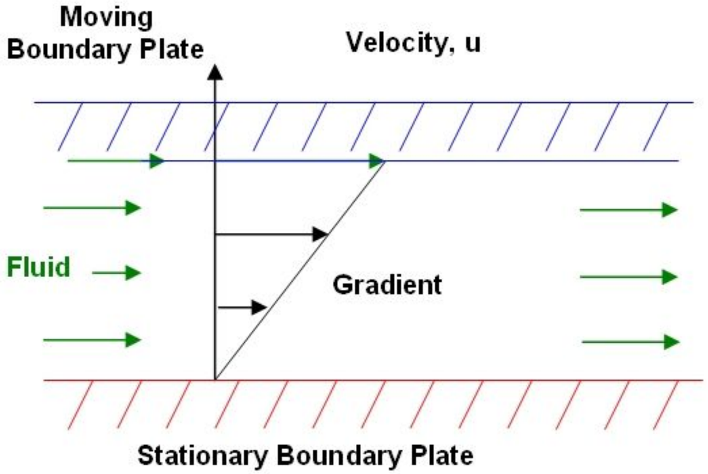
Chemical Potential, μ

Reissner-Nordström
Black Hole

Transport

Kovtun, Policastro, Son, Starinets

e.g. shear viscosity



$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Many Other Phenomena

- Superconductivity
- Non-Fermi Liquids
- Quantum Oscillations
- Quantum Hall Transitions
- Dynamical Lattice Formation
- Band Structure
- …