The “old” marginal Fermi liquid “theory”

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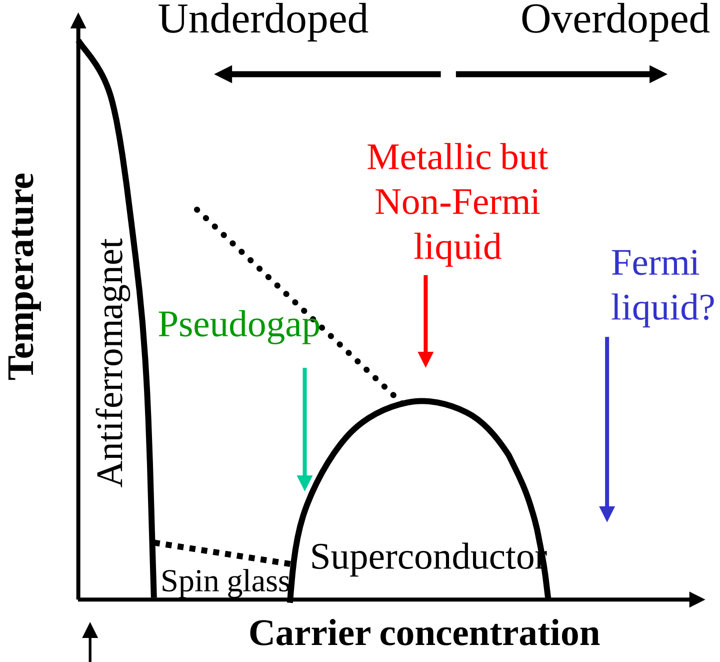
Review: Les Houches, Session LVI (1991), ed Doucot and Zinn-Justin, p 69-148, Elsevier (1995)

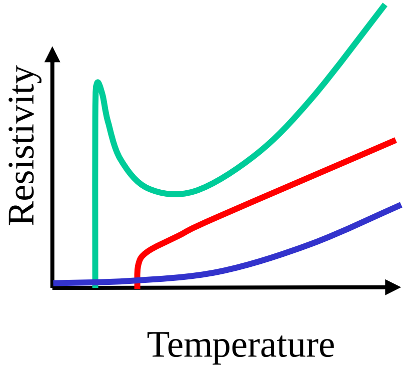
Outline

- **Selected** experimental data on cuprate superconductors
  - Phase diagram
  - Resistivity, tunnelling, Raman scattering, optical conductivity, superconductivity,
- Phenomenology of a non Fermi liquid
- Remarks

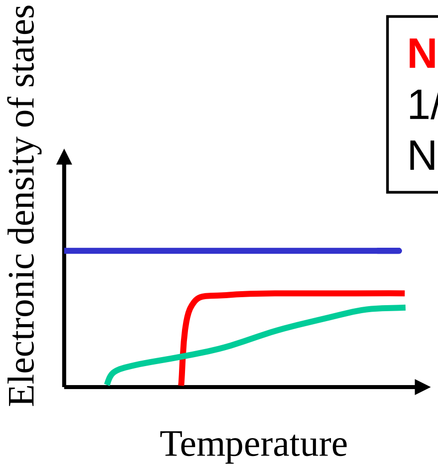
Phenomenology of high-Tc superconductors



Mott insulator
La_2CuO_4



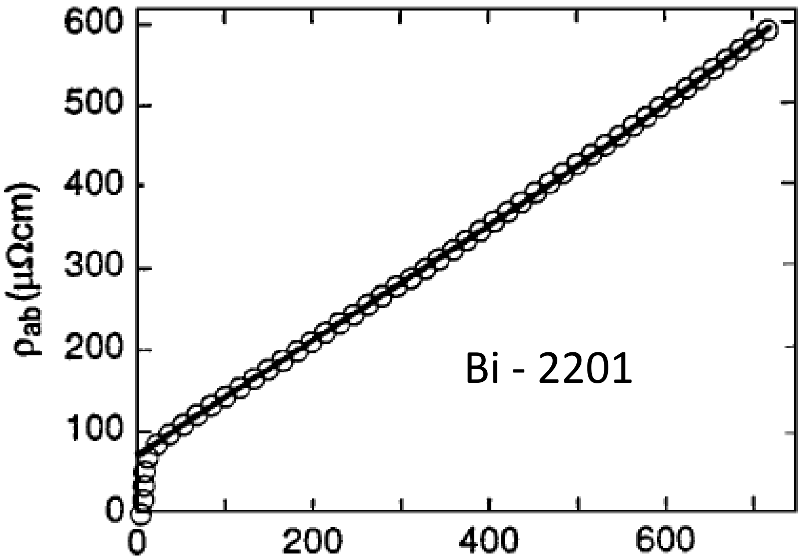
Fermi liquid
$\rho \sim T^2$
$\chi , \gamma \sim$ const



Non-Fermi liquid
$1/\tau = \max(T,\omega)$
No long-lived quasiparticles

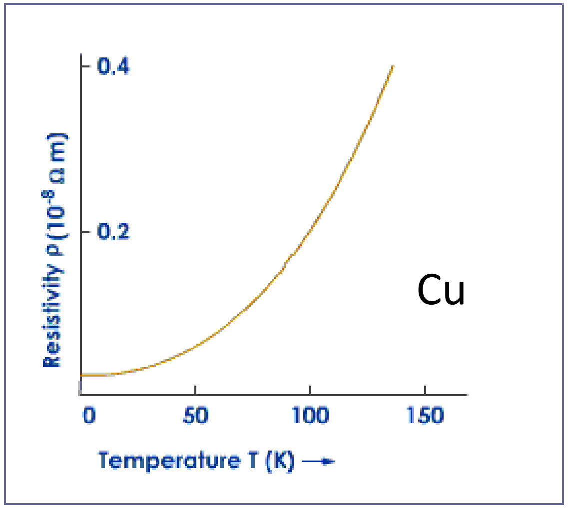
Pseudo-gap
destroys portions of Fermi surface

Resistivity



Martin et al *Phys. Rev. B 41:846* (1990)

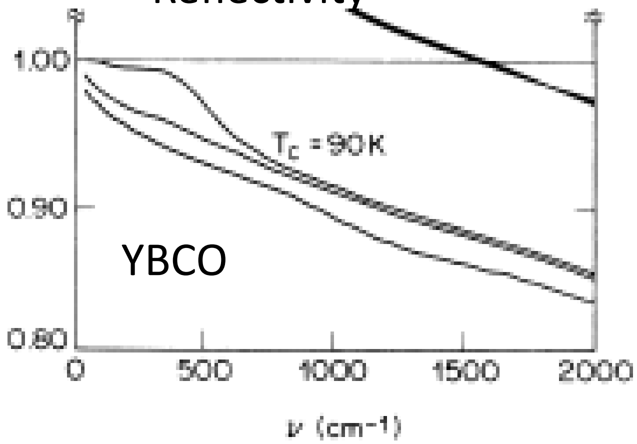
Resistivity is very large, linear in T, non-saturating
Implies strong local (back-)scattering
Mean free path is a lattice constant or less --- is this a meaningful concept?



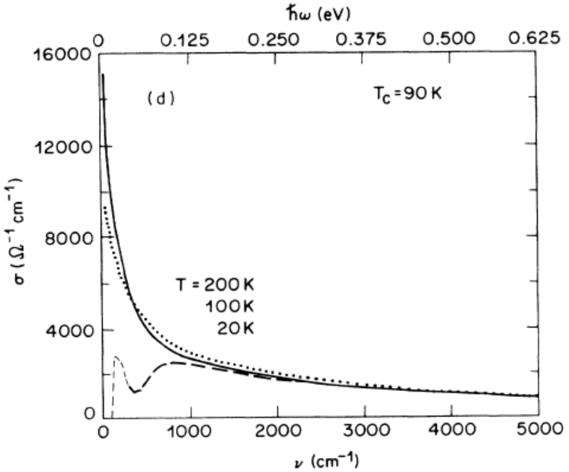
Optical conductivity

Orenstein et al. PRB 42, 6342 (1990)

Reflectivity



YBCO



$$\sigma = \frac{\omega_p^2 \tau}{1 + i\omega\tau}$$

Drude metal

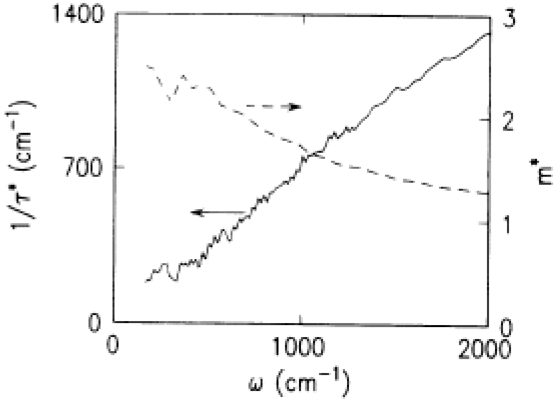
$$R \approx 1 - \frac{2}{\omega_p \tau}, \quad \tau^{-1} \ll \omega \ll \omega_p$$

Scattering rate is frequency dependent

$$1/\tau \propto \omega; \quad \Re\sigma \propto 1/\omega$$

Divergent effective mass (Kramers-Kronig)

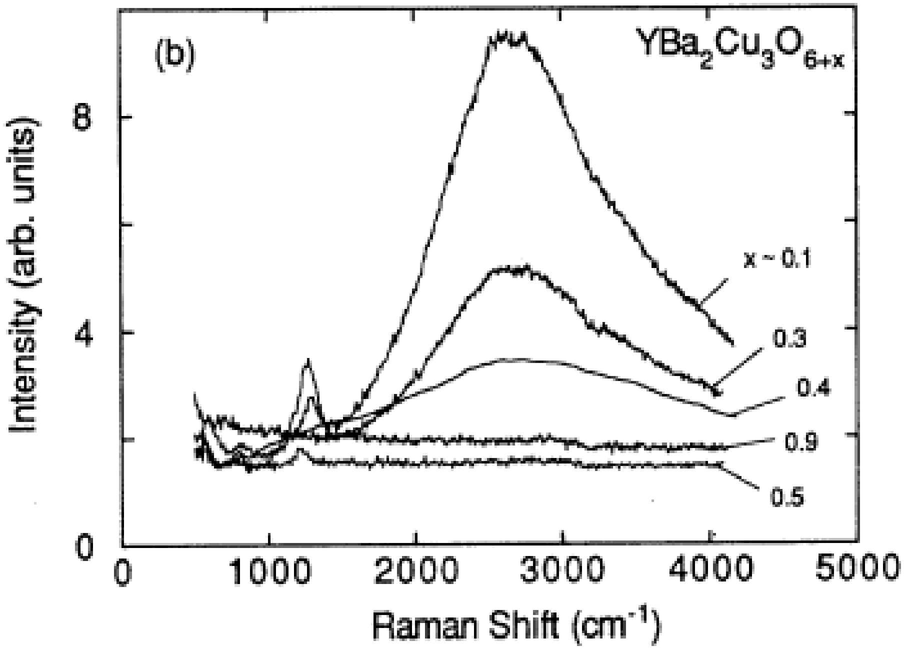
$$m^* \propto \log \omega$$



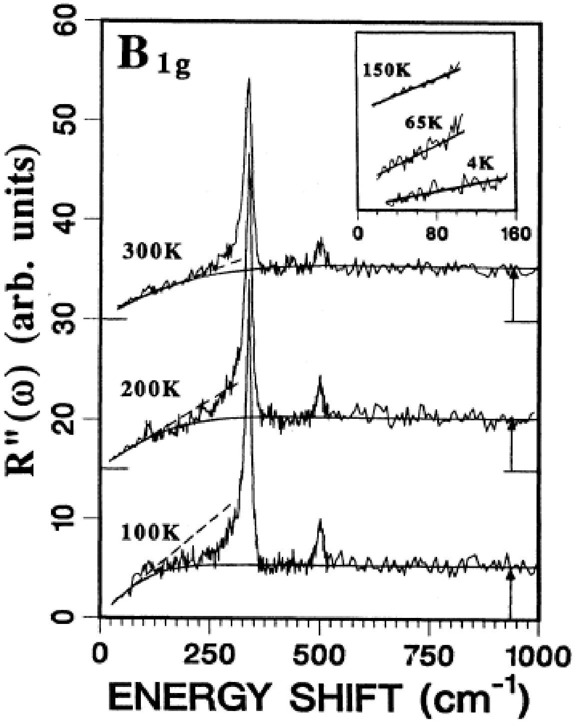
Schlesinger et al. PRL 65, 801 (1990)

Inelastic light scattering

Cooper et al PRB 47, 8233 (1993)



Slakey et al., PRB 42, 3764 (1991)



"2-magnon" light scattering peak in insulator evolves to a flat continuum in metal

Lower cutoff of spectrum is k_B T

Much larger than conventional metal ~ q^2 (Galilean invariance)

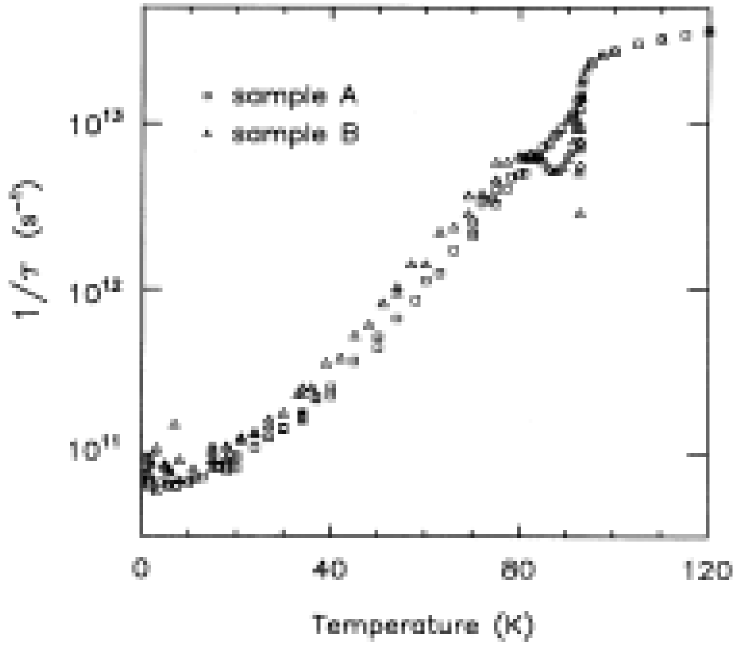
$$R'' = -\frac{q^2}{4\pi e^2}\Im\epsilon^{-1}(q\omega)$$

But with strong local scattering

$$R'' \propto \omega\sigma(\omega) \simeq \mathrm{const.}$$

Microwave conductivity below Tc

Bonn et al PRB 47, 11314 (1993)



Contribution to the low-frequency conductivity from thermally excited quasiparticles in the superconducting state

- Scattering rate drops off below T_c
- Gap in quasiparticle spectrum
- Dominant scattering mechanism is then quasiparticle-quasiparticle
- Not scattering from well-defined collective modes

Recap

- Quasiparticle scattering rate

$$\hbar/\tau = \alpha(\pi k_B T + \hbar\omega) \quad , \alpha \approx 1$$

- Response functions
  - Optical conductivity
  - Inelastic light scattering
  - Spin fluctuations

$$\begin{aligned} \mathrm{Im}\,[P(q,\omega,T)] &= (\omega/T)F(q), \quad \text{for } \omega < T \\ &= F(q) \quad \text{for } T < \omega < \omega_c \end{aligned}$$

- F(q) smooth function (at small q); determined by electronic bandstructure (large q)
- Cutoff ω_c ~ few tenths of eV
- Pauli susceptibility, specific heat unremarkable, not strongly renormalised
- Spin fluctuation response close to prediction from bandstructure (Lindhard)

Ad hoc phenomenology

- Postulate a scattering spectrum

$$P(q,\omega) = \frac{\tanh \omega/T}{1 + (\omega/\omega_c)^2}$$

- Quasiparticle self-energy (Born, one-loop)
  - Linear scaling
  - Logarithmic mass renorm
  - Weak function of momentum

$$\mathrm{Im}\Sigma(p\nu) = \lambda\, \mathrm{max}(\nu, \pi T)$$

$$\Sigma(\nu, T) \approx \lambda \left[(2\nu/\pi) \log \left(\frac{\pi T + i\nu}{\omega_c} \right) + i\pi T \right]$$

- Quasiparticle spectral function is 1/ω on-shell – not a δ-function
  - “Marginal” Fermi liquid

$$A(k\nu) = -\mathrm{Im}[\nu - (\epsilon_k - \mu) - \Sigma_k(\nu)]^{-1}$$

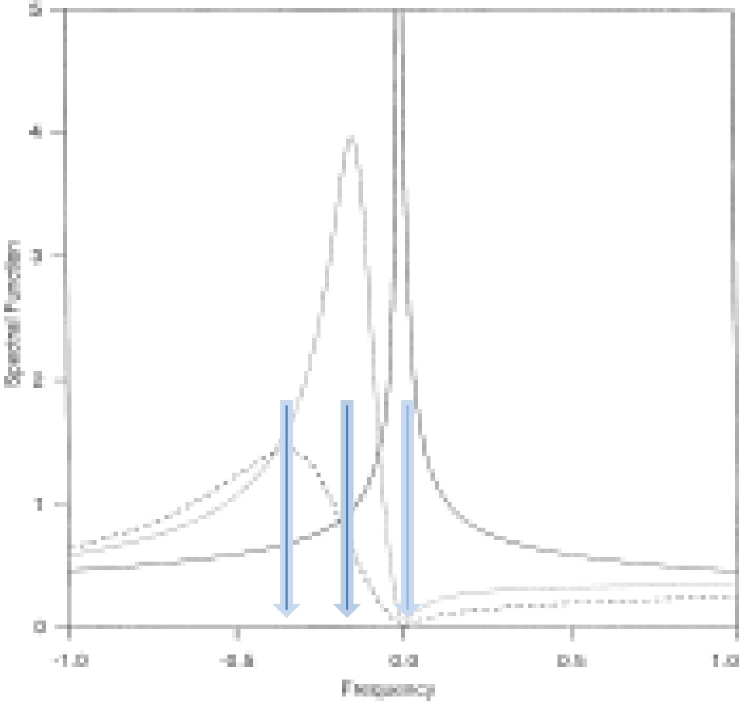
- Response functions now calculated again at the one-loop level
  - For q~0 get the expected forms
  - For large q, see bandstructure effects (nesting etc.)

$$\sigma_{MFL}(\omega) = \frac{-i\omega_p^2}{\omega - \Sigma(\omega/2)}$$

Consequences, generalisations

Electron spectral function

$$A(k\nu) = -\mathrm{Im}[\nu - (\epsilon_k - \mu) - \Sigma(\nu)]^{-1}$$



Spectral function sharpens to 1/ω peak at $k=k_f$

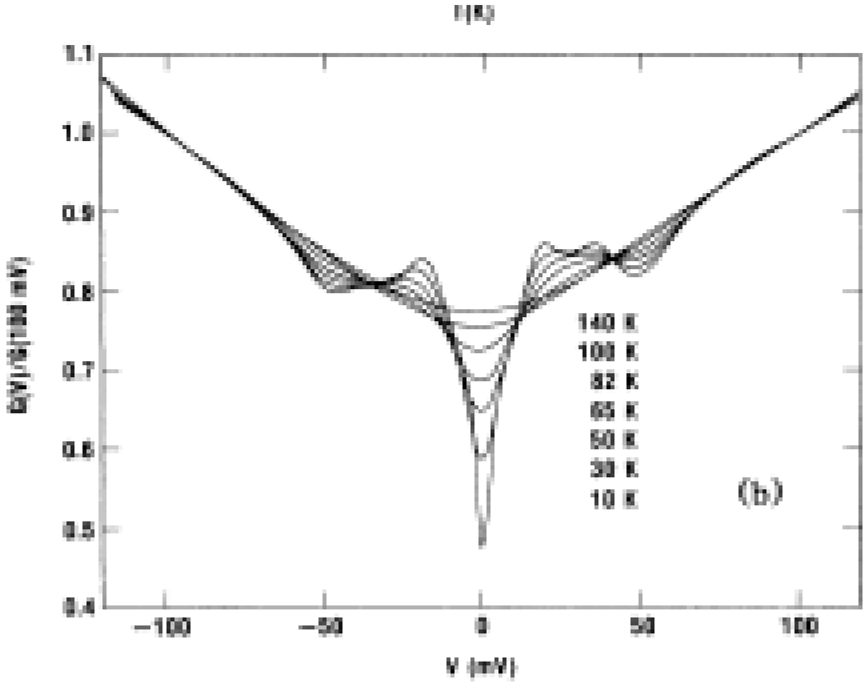
At low energy, there is a contribution to the spectral weight from all k-states, with weight ~ $|\nu|$

Tunnelling conductance gives access to spectral weight at low energies and momenta far from the Fermi surface

Tunnelling

Tunnelling conductance gives access to spectral weight at low energies and momenta far from the Fermi surface

“c-axis” tunnelling – quasiparticles are injected at momenta far from k_F

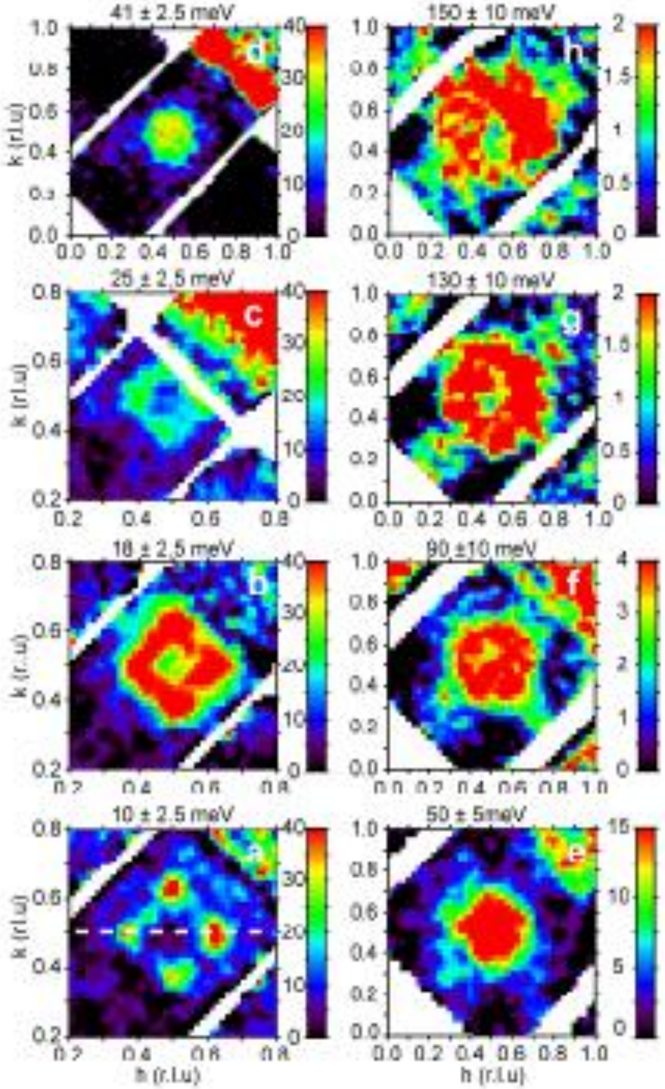


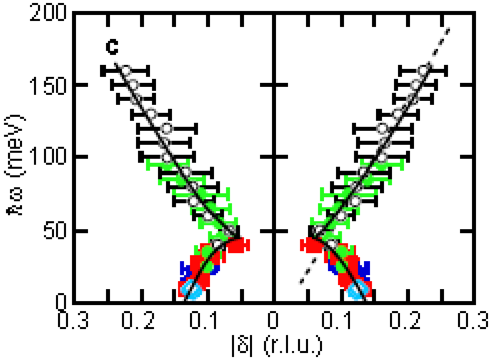
$$g(V) \approx g_0 \mathrm{Im}\Sigma(eV)$$

Gurvitch et al PRL 63, 1008 (1989)

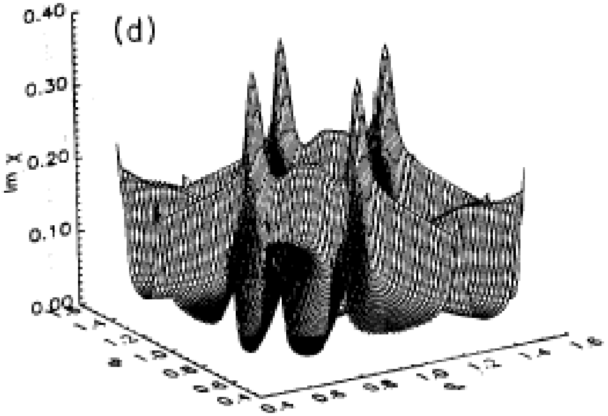
Spin fluctuations

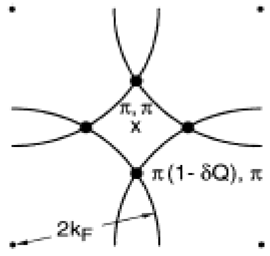
Vignolle et al 2007





Strongly dispersing magnetic fluctuations are most easily explained by details of bandstructure



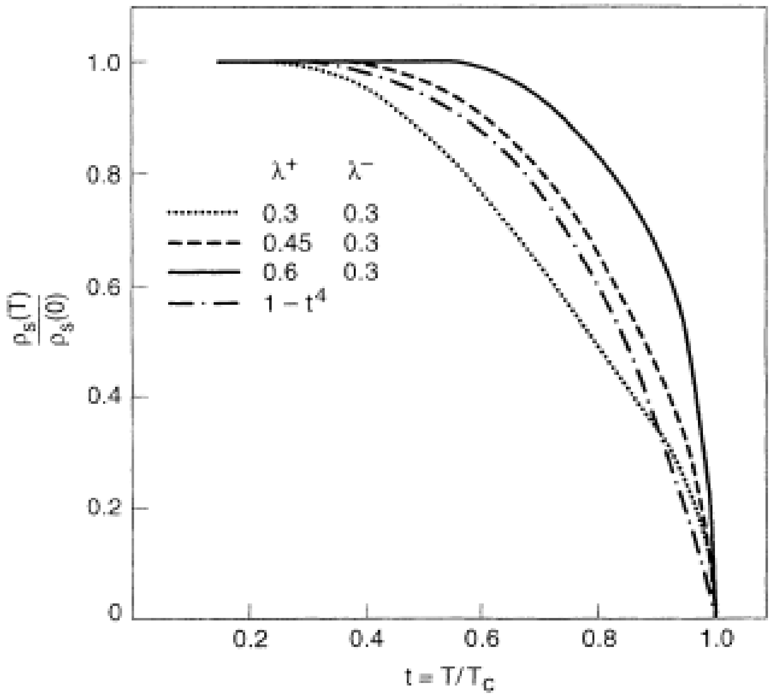


Generalisation to the superconducting state

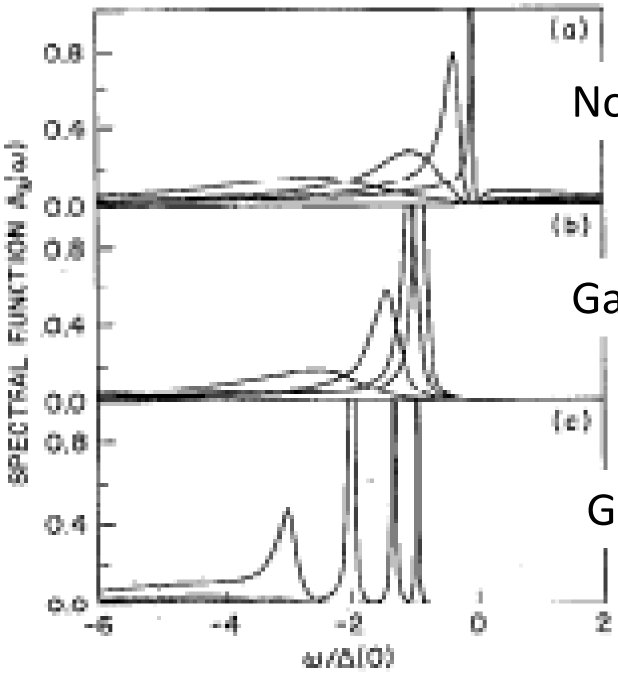
- Utilise the "bubble" diagram as the pairing spectrum?
- NB – low frequency scattering (ω < 2Δ) is pairbreaking
  - Superconductivity is itself "marginal" $1/\tau(T_c) \simeq T_c$
  - Superconductivity avoids pair-breaking because gap self-consistently opens as pairbreaking is suppressed
  - $2\Delta/T_c$ large (whatever the scale of T_c)
  - Fermi liquid restored in superconducting state
  - Homes reanalysis of "Uemura plot"
    - phase fluctuations are not dominant except at very low doping
    - Superconductive transition is "BCS"-like far into the underdoped regime

Generalisation to the superconducting state

Superfluid density



Electron spectral function for k~k$_f$



Normal

Gapless

Gapped

NB. Model with s-wave pairing

Remarks

- No explicit Hamiltonian
  - short-range Coulomb implicated
  - dangerous to expect that Hubbard or t-J model (with fixed parameters) can explain both metallic and insulating phases
  - phenomena are robust – details should not matter
- Not a self-consistent theory: one loop only.
  - in higher order, logarithms exponentiate ...
  - but superconductivity emerges first? – a "medium"-energy theory
- Is this a "critical" theory? $\chi(q,\omega) = \bar{\chi}(q/q_o)F(q/\omega^{1/z});\ \ q_o \sim (x - x_c)^{\eta}$
  - expect scaling nearby
  - there are crossovers ("pseudogap") but very little evidence for "conventional" QCP
  - certainly dominated by quasiparticle fluctuations rather than low energy collective modes (i.e. pairbreaking not Hertz-Millis)
  - small fermi pockets now resolved in very underdoped regime