

Evaporation of Schwarzschild Black Holes in Matrix Theory

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Recently, in collaboration with Susskind, we proposed a model of Schwarzschild black holes in Matrix theory. A large Schwarzschild black hole is described by a metastable bound state of a large number of D0-branes which are held together by a background, whose structure has so far been understood only in 8 and 11 dimensions. The Hawking radiation proceeds by emission of small clusters of D0-branes. We estimate the Hawking rate in the Matrix theory model of Schwarzschild black holes and find agreement with the semiclassical rate up to an undetermined numerical coefficient of order 1.

1. Introduction

In a recent paper [1] Susskind and the authors presented a model of Schwarzschild black holes in Matrix Theory [2]. The key feature of the model was the notion of a *Boltzmann gas of D0-branes* (which we will review briefly below).¹ We showed that the Matrix model for M theory in 11 noncompact dimensions, and also for its toroidal compactification to 8 noncompact dimensions, contained a set of states which obeyed (up to a numerical coefficient which we could not calculate) the Bekenstein-Hawking relation between entropy and transverse area for a Schwarzschild black hole. The relation between energy and entropy for these states also obeyed the black hole formula.² We were also able to compute the long distance Newtonian gravitational force between equal mass black holes, with an answer in agreement with classical gravity.

The purpose of the present note is to calculate the rate of Hawking radiation from our model black holes. In [1] we showed that the individual bound D0-branes in the model, had the kinematic properties of Hawking radiation in the boosted frame in which we examine our black hole. The D0-branes are “tethered” to a classical background by harmonic forces. In this note we argue that the probability for the classical variables which produce these forces on an individual D0-brane to fluctuate to zero is independent of the mass of the black hole in the large mass limit. We show that when this estimate is combined with the proper phase space integral, it gives a decay rate for the boosted black hole which is just the Lorentz transform of the rest frame Hawking evaporation rate.

To briefly summarize our black hole model: we consider the Hilbert space of the matrix theory which represents Discrete Light Cone Quantized M theory compactified on some manifold Y [10] in the sector with N units of longitudinal momentum ($DLCQ_N$). The radius of the light-like circle will be denoted by R . We will also choose the noncompact transverse spacetime dimension D to be greater than or equal to 6. The model contains a set of variables which includes matrices X^i representing the transverse positions of N D0-branes in a weakly coupled Type IIA string theory called “the analog model.” We emphasize that these are not the Boltzmann D0-branes of which our black hole is constructed. The matrices also describe creation and annihilation operators for strings stretching between the D0-branes.

¹ For related work, see [3], [4], [5], [6]

² In the supersymmetric Yang-Mills formulation on the dual torus the corresponding results were obtained in [7], [8], [9].

We consider a semiclassical configuration of the variables of the model, which includes a semiclassical background X_{cl}^i for the transverse position matrices. This background configuration must satisfy a number of constraints, which were described in [1]. Boltzmann D0-brane positions are defined as perturbations of the background

$$X_{cl}^i \rightarrow X_{cl}^i + \sum_{n=1}^N r_n^i \delta_n \quad (1.1)$$

where the δ_n are a set of independent commuting matrices chosen to minimize the quantity $|Tr[X_{cl}^i, \delta_n]^2|$. This is a term in the matrix model Hamiltonian which gives rise to a harmonic potential binding these Boltzmann D0-branes to the classical configuration. In [1] we argued that for $D \geq 6$ we could choose our classical configuration so that this harmonic potential did not interfere with the scaling argument we review in the next paragraph.

In matrix theory, the Bose or Fermi statistics of particles arises as a residual gauge symmetry. Since the entire gauge symmetry of the model is broken by the classical background, this means that the variables r_n^i should be treated as the coordinates of distinguishable, or Boltzmann, particles. The approximately degenerate configurations of these particles then have an entropy of order N . In [1] we argued that the effective Hamiltonian of these degrees of freedom gave rise to bound states whose transverse area is of order $G_D N$ and whose light cone energy is of order $NR(G_D N)^{-2/(D-2)}$. When translated into invariant mass, this gives precisely the mass/entropy/area relations of a Schwarzschild black hole. We were unable to calculate the numerical coefficients in these relations because we did not have the full effective Hamiltonian of the Boltzmann gas³ and because the mean field approximation which we used was very crude.

According to the above discussion, the light cone energy per particle of the Boltzmann gas is $\sim R(G_D N)^{-2/(D-2)}$, where G_D is the Newton constant in D non-compact dimensions. The transverse momentum per particle is $\sim (G_D N)^{-1/(D-2)}$ and the longitudinal momentum is just $1/R$. We showed that these were precisely the kinematical properties of the Hawking particles when boosted into a frame where the black hole has longitudinal momentum $P_- = N/R$. This suggests the following attractive and simple picture of

³ Recently, Liu and Tseytlin [11] have proposed to describe the interactions of the Boltzmann D0-branes by a Hamiltonian containing terms of all orders in the velocity [12]. Perhaps this can be used to make some progress on the numerical coefficients.

the Hawking evaporation process: the classical background provides a harmonic potential which binds the Boltzmann D0-branes to the black hole. The background itself should be represented as a coherent quantum state centered around a periodic solution of the classical equations of motion of the matrix model. According to the wave function of this state, there is a certain amplitude for the part of the classical configuration which interacts with a given D0-brane, to fluctuate to zero. From the point of view of a basis in which the particular D0-brane of interest (say δ_1) occupies the lowest right hand corner of the matrix, the relevant part of the classical solution is the part which occupies the last row and column. There are thus $o(N)$ possible degrees of freedom which might be excited.

However, in the explicit classical backgrounds which we constructed in $D = 11, 8$ [1], only $o(1)$ of these possible background degrees of freedom were actually utilized. In $D = 11$, this is a consequence of approximate locality of the action on the world volume of the classical membrane. In $D = 8$ the classical background for a typical black hole could be viewed as a lattice of D0-branes connected by strings on a 3-torus, with the lattice spacing of order $N^{-1/3}$ [7]. Energetic considerations ensured that only strings attaching a given D0-brane to $o(1)$ nearest neighbors on this lattice were excited. We will assume that similar pictures work for all $D \geq 6$.

As a consequence, it seems reasonable to assume that the amplitude to “liberate a D0-brane” from the black hole is independent of N as $N \rightarrow \infty$. It is the value of the wave function of the system on a submanifold of codimension which is $o(1)$.

2. Calculation of the Hawking Evaporation Rate

Given the estimate of the D0-brane liberation amplitude in the previous section, we can proceed to calculate the Hawking rate. The quantum fluctuation of the background described in the previous section gives rise to a single D0-brane wave function which is, by the estimates of [1], a smooth function $A(y)$ of rapid decrease in the variable

$$y = \frac{|p_\perp|}{(G_D N)^{-1/(D-2)}} , \tag{2.1}$$

where p_\perp is the transverse momentum and G_D is the D -dimensional Newton constant. Thus, the amplitude to produce a D0-brane with momentum much larger than the Hawking momentum is highly suppressed. If we assume that the fluctuations which liberate any of

the N D0-branes are independent and incoherent, then the probability per unit time to emit a Hawking particle is given by

$$\frac{dN}{dx^+} \sim N \frac{1}{R} \sum_{n>0} \int_0^\infty dp_+ \int d^{D-2} p_\perp \delta \left(\frac{n}{R} - \frac{p_\perp^2}{p_+} \right) |A(n, y)|^2 \quad (2.2)$$

where y is the variable defined above. In this equation we have generalized our considerations to include processes in which a cluster of n D0-branes, with n finite and independent of N , is emitted. Such systems also have the kinematic properties of Hawking radiation. However, since the cluster of D0-branes is connected to the classical background by $o(n)$ degrees of freedom, we should expect the matrix element to fall off exponentially in n . The overall factor of N in equation (2.2) represents the incoherent sum over processes in which a particular bound D0-brane is liberated. Finally, we have used relativistic phase space to integrate over the final states of the outgoing D0-brane. Although our calculation is done in a particular frame, chosen so that the geometrical structure of the black hole just fits inside the $DLCQ_N$ quantization volume, we expect that the matrix element which we have estimated is in fact, for large N , approximately the invariant S matrix element of a Lorentz covariant system. Of course, the crucial, as yet unproven, assumption of Matrix Theory is that the S matrix computed by the theory is indeed Lorentz invariant. We cannot prove this claim at present, nor can we show that our approximate evaluation of the matrix element is accurate. However, assuming that these claims are valid, the correct rate is obtained by integrating our matrix element against relativistic phase space.

Our measure thus contains an extra factor of light cone energy, p_+ , compared to the nonrelativistic phase space of the D0-brane quantum mechanics. This factor, which endows the measure with the correct boost transformation properties, has been absorbed into the longitudinal momentum delta-function in (2.2) .

Finally, we need to estimate the square of the matrix element for n D0-branes to be liberated, $|A(n, y)|^2$. As we have explained, this quantity is appreciable only if n and y are of order 1. $|A(n, y)|^2$ has dimensions of length $^{D-2}$. The only dimensionful quantities at our disposal are R , l_{11} and the radii of the compactification torus, L_i . Since the measure in (2.2) transforms properly under boosts, any dependence of $|A(n, y)|^2$ on R would violate Lorentz invariance. Furthermore, we will assume that the dependence on l_{11} and the radii is through the D -dimensional Newton constant only.⁴ Thus, we are led to

$$|A(n, y)|^2 \sim G_D ,$$

⁴ This assumption is plausible because G_D is the only quantity that appears in the D-brane interaction Hamiltonian, but its better justification is clearly necessary.

for n and y of order 1. Estimating the Hawking rate (2.2) with this assumption, we find

$$\frac{dN}{dx^+} \sim R(G_D N)^{-2/(D-2)} . \quad (2.3)$$

By way of comparison, we now compute the Hawking radiation rate according to the conventional semiclassical formulae. First, let us write down the Hawking rate in the usual equal-time quantization. The answer can be written as

$$\frac{dN}{dx^0} \sim \int_0^\infty dp_+ \int_0^\infty dp_- \int d^{D-2} p_\perp \delta(p_+ p_- - p_\perp^2) e^{-p_0/T_H} \mathcal{A} p_0 , \quad (2.4)$$

where $\mathcal{A} = 4G_D S$ is the horizon area. We have included the thermal factor appropriate for the Boltzmann statistics. The Hawking temperature is related to the Schwarzschild radius R_S by

$$T_H \sim \frac{1}{R_S} , \quad (2.5)$$

and

$$R_S \sim (S G_D)^{1/(D-2)} \sim (N G_D)^{1/(D-2)} . \quad (2.6)$$

An explicit factor of p_0 is needed in (2.4) because the measure

$$\int_0^\infty dp_+ \int_0^\infty dp_- \int d^{D-2} p_\perp \delta(p_+ p_- - p_\perp^2) \quad (2.7)$$

is Lorentz invariant. The left-hand side, however, contains a derivative with respect to x^0 , hence transforms in the same way as p_0 .

Now we perform a parallel computation in the light-cone frame. Here, the number of particles radiated per unit light-cone time is

$$\frac{dN}{dx^+} \sim \int_0^\infty dp_- \int_0^\infty dp_+ \int d^{D-2} p_\perp \delta(p_+ p_- - p_\perp^2) e^{-p_+/T_+} e^{-p_-/T_-} \mathcal{A} p_+ . \quad (2.8)$$

The factor of p_+ is needed for correct boost invariance, since the left-hand side contains a derivative with respect to x^+ . In the rest frame of the black hole, we have

$$T_+^{\text{rest}} = T_-^{\text{rest}} \sim T_H \sim 1/R_S . \quad (2.9)$$

If we carry out a boost

$$x^- = \frac{R}{R_S} x_{\text{rest}}^- , \quad x^+ = \frac{R_S}{R} x_{\text{rest}}^+ , \quad (2.10)$$

then the new temperatures are

$$T_- = T_-^{\text{rest}} \frac{R_S}{R} \sim 1/R, \quad T_+ = T_+^{\text{rest}} \frac{R}{R_S} \sim R/R_S^2. \quad (2.11)$$

Doing the integrals, we find

$$\frac{dN}{dx^+} \sim \mathcal{A}(T_+ T_-)^{(D-2)/2} T_+ \sim T_+ \sim R(G_D S)^{-2/(D-2)}. \quad (2.12)$$

If we compactify x^- on a circle of radius R , and work with $S \sim N$, then the integral over p_- is replaced by sum,

$$\frac{dN}{dx^+} \sim \frac{1}{R} \sum_{n>0} \int_0^\infty dp_+ \int d^{D-2} p_\perp \delta\left(\frac{n}{R} - \frac{p_\perp^2}{p_+}\right) e^{-p_+/T_+} e^{-n/(RT_-)} G_D N. \quad (2.13)$$

While this changes the normalization, the result (2.12) still holds. Thus, our formula (2.3) for the rate of emission of D0-branes from one of our matrix theory black holes coincides (up to uncalculated numerical factors of order one) with the semiclassical Hawking evaporation rate in the light cone frame, (2.12).

3. The Rate of Mass Loss

Using arguments analogous to those that led to (2.2), we may obtain from the matrix model a formula for the rate of light cone energy loss per unit light cone time,

$$\frac{dE}{dx^+} \sim N \frac{1}{R} \sum_{n>0} \int_0^\infty dp_+ \int d^{D-2} p_\perp \delta\left(\frac{n}{R} - \frac{p_\perp^2}{p_+}\right) p_+ |A(n, y)|^2. \quad (3.1)$$

Estimating the integral using the previously stated assumptions, we find

$$\frac{dE}{dx^+} \sim T_+^2 \sim \frac{R^2}{R_S^4}. \quad (3.2)$$

Now using

$$M^2 = 2E \frac{N}{R}, \quad (3.3)$$

we have

$$\frac{dM}{dx^+} = \frac{N}{MR} \frac{dE}{dx^+} + \frac{M}{2N} \frac{dN}{dx^+} \sim \frac{R}{R_S^3}. \quad (3.4)$$

Boosting back to the rest frame, we obtain

$$\frac{dM}{dx_{\text{rest}}^+} = \frac{dM}{dx^+} \frac{dx^+}{dx_{\text{rest}}^+} \sim 1/R_S^2, \quad (3.5)$$

where we used the fact that

$$\frac{dx^+}{dx_{\text{rest}}^+} = \frac{R_S}{R} . \tag{3.6}$$

The Matrix theory result for the rate of mass loss in the rest frame, (3.5), is consistent with the standard semiclassical result.

What we have shown in this note is that plausible assumptions about the D0-brane emission process from the metastable bound state describing the black hole lead to the radiation rate consistent with the semiclassical calculations, up to a constant of proportionality of order 1. Just as in the semiclassical analysis, suppression of the rate for large entropy N comes from the smallness of the one-particle phase space available at an energy comparable to the Hawking temperature (T_H scales as $N^{-1/(D-2)}$). Our analysis should be regarded as a plausibility argument. In particular, we need a microscopic argument for why at low momenta the square of the matrix element for liberating a D0-brane is of order G_D . Nevertheless, we believe that we have described the correct mechanism for evaporation of Schwarzschild black holes in Matrix theory.

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