Supergravity and The Large N Limit of Theories With Sixteen Supercharges

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Abstract

We consider field theories with sixteen supersymmetries, which includes U(N) Yang-Mills theories in various dimensions, and argue that their large N limit is related to certain supergravity solutions. We study this by considering a system of D-branes in string theory and then taking a limit where the brane worldvolume theory decouples from gravity. At the same time we study the corresponding D-brane supergravity solution and argue that we can trust it in certain regions where the curvature (and the effective string coupling, where appropriate) are small. The supergravity solutions typically have several weakly coupled regions and interpolate between different limits of string-M-theory.

1 Introduction

String theory contains D-branes which are solitonic objects [1]. When we consider the full theory in the presence of these solitons we have modes that propagate in the bulk and modes that propagate on the solitons. The modes on the soliton interact with each other and with the bulk modes. It is possible, however, to define a limit of the full theory in which the bulk modes decouple from the modes living on the D-brane. This is typically a low energy limit, in which we tune the coupling constant so as to keep only the interactions among the modes living on the D-brane. In this limit the D-brane theory becomes super-Yang-Mills (for $p \leq 3$). Separating the branes by some distance corresponds in the field theory to giving Higgs expectation values to some fields. Since we want to keep these expectation values finite when we take the limit, we should consider the branes at substringy distances [2].

Since D-branes carry some mass and charge they excite the bulk gravity modes and we can find supergravity solutions carrying the same mass and charges. Naively the supergravity solution describes only the long range fields of the D-branes, since we do not expect supergravity to be valid at short distances. General covariance, however, tells us that we can trust the supergravity solution as long as curvatures are locally small compared to the string scale (or the Planck scale). A more careful analysis shows that for a system with a large number of branes, large N, the curvatures are small and we can trust the supergravity solutions even at the substringy distances involved in the decoupling limit described above. The situation is similar to the one studied in [3] for conformal field theories (see also [4, 5]). In particular for the 4D $N = 4$ $U(N)$ super-Yang-Mills theory associated with N D3-branes, it has been argued in [3] that it is "dual" to type IIB string theory on $AdS_5 \times S^5$ in the large N limit.

The aim of this paper is to explore analogous connections in the more general case of non-conformal field theories. The supergravity solutions corresponding to $p + 1$ super-Yang-Mills are black p-brane solutions. They are extended along $p + 1$ spacetime dimensions. We interpret the radial variable as being related to the energy scale of the process involved. One of the reasons for this interpretation is the fact that a Dp-brane sitting at some position r corresponds to giving a Higgs expectation value to some fields which break the gauge group $U(N + 1) \rightarrow U(N) \times U(1)$. Large values of r correspond to very large Higgs expectation values, which have dimension of energy, and therefore correspond to large energy scales. Hence, large values of r correspond in the field theory to the UV region and small values to the IR region. The curvature and the value of the dilaton depend on the radial variable r. The radial dependence of the dilaton represents the running

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of the effective coupling constant (which is these theories is simply given by dimensional analysis). For $p < 3$ the solutions have large curvatures for large values of r, and we cannot trust them in this region. This is not a problem since in the UV we can trust the perturbative description and we do not expect perturbation theory and gravity to be valid at the same time. As we move to smaller values of r, i.e. lower energies, we find a region with small curvature and small string coupling. In this region we can trust a string theory description on the corresponding background. All Yang-Mills theories considered here contain strings in this sense (as in the $p = 3$ case [3]). These examples realize the general description of large N gauge theories proposed by Polyakov [6]. In the language of [6] the radial variable is related to the Liouville field. This string theory description is valid for intermediate values of r. In some cases we can use a dual description for the small r region. In general we have a reliable supergravity description in the large N limit only for certain energy scales (which is to be expected since the coupling depends on the energy).

For $p > 3$ we have a similar situation with the coupling running in the reverse direction, small r corresponds to weakly coupled super-Yang-Mills and for large r we will have to use dual descriptions.

Since the conformal case $p = 3$ is the borderline case we shall discuss first the $p < 3$ theories and then the $p > 3$ theories. The paper is organized as follows. In Sec.2 we describe some general properties which are common to all Dp-branes. In Sec.3 we consider D2-branes and obtain a relation with the 2+1 conformal field theory at low energies. In particular, the flow of the super-Yang-Mills in 2+1 dimensions to a superconformal field theory with SO(8) R-symmetry is realized by the supergravity solution of the D2 branes. We also relate the super-Yang-Mills with small temperature to the near extremal M2-branes solutions. In Sec.4 we discuss D1-branes. We show that for large N there is an intermediate region between perturbative super-Yang-Mills in the UV and the orbifold $(R^8)^N/S_N$ CFT with the Dijgraff-Verlinde-Verlinde vertex operators [7] in the IR. This intermediate region is described by type IIB on a non-trivial background. In Sec.5 we study D0 branes and the relation to black holes in matrix theory [25, 8, 26, 27]. In Sec.6 we consider D4 branes and the relation with the six-dimensional (0,2) field theory compactified on a circle. The flow of the (0,2) theory on a circle to the 4+1 dimensional super-Yang-Mills in the IR has a counterpart in the supergravity solution. In Sec.7 we briefly discuss the case of D5 branes and their relation to IIB NS 5branes and we make, in Sec.8, a digression into IIA NS fivebranes. Finally, in Sec.9 we consider D6 branes where we conclude (as in [9, 11, 12]) that the theory does not decouple from the bulk. We show that a finite temperature configuration is described by a Schwarzschild black hole in five

dimensions. To make the outcome of the analysis of each case clearer, we give a short summary of the conclusions at the end of each section.

The connection between Yang-Mills theories and supergravity solutions was explored following a different method in [13].

2   Generalities

We study Dp-branes in the field theory limit[1][10, 11, 12]

$$g_{YM}^2 = (2\pi)^{p-2} g_s \alpha'^{(p-3)/2} = \text{fixed}, \qquad \alpha' \to 0, \tag{1}$$

where $g_s = e^{\phi_\infty}$, and g_{YM} is the Yang-Mills coupling constant. We keep the energies fixed when we take the limit. For $p \leq 3$ this limit implies that the theory decouples from the bulk since the ten dimensional Newton constant goes to zero. It also suppresses higher order corrections in α' to the action. For $p > 3$ we have $g_s \to \infty$ which implies that we should use a dual description to analyze the decoupling issue.

When we take the limit (1) we are interested in finite energy configurations in the field theory. This corresponds to finite Higgs expectation values. We are, therefore, considering the limit

$$U \equiv \frac{r}{\alpha'} = \text{fixed}, \qquad \alpha' \to 0. \tag{2}$$

In terms of the field theory U is the expectation value of the Higgs. Note that in this limit $\frac{r}{l_s} \to 0$ which means that we study the system at substringy distances. At a given energy scale, U, the effective dimensionless coupling constant in the corresponding super-Yang-Mills theory is $g_{eff}^2 \approx g_{YM}^2 N U^{p-3}$. Thus, perturbative calculations in super-Yang-Mills can be trusted in the region

$$g_{eff}^2 \ll 1 \quad \Rightarrow \quad \begin{cases} U \gg (g_{YM}^2 N)^{1/(3-p)}, & p < 3 \\ U \ll 1/(g_{YM}^2 N)^{1/(p-3)}, & p > 3 \end{cases} \tag{3}$$

The type II supergravity solution describing N coincident extremal Dp-branes is (in the string frame) [14]

$$ds^2 = f_p^{-1/2}(-dt^2 + dx_1^2 + ... + dx_p^2) + f_p^{1/2}(dx_{p+1}^2 + ... + dx_9^2),$$
$$e^{-2(\phi - \phi_\infty)} = f_p^{(p-3)/2}, \tag{4}$$
$$A_{0...p} = -\frac{1}{2}(f_p^{-1} - 1),$$

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[1] We use conventions in which $g_s \to 1/g_s$ under S-duality for type IIB. The IIA conventions are such that the radius of the eleventh circle is $R_{11} = g_s \sqrt{\alpha'}$. The eleven dimensional Planck length is defined as $l_p = g^{1/3}\sqrt{\alpha'}$.

where f_p is a harmonic function of the transverse coordinates $x_{p+1}, ..., x_9$

$$\alpha'^2 f_p = \alpha'^2 + \frac{d_p g_{YM}^2 N}{U^{7-p}} \; , \qquad\qquad d_p = 2^{7-2p} \pi^{\frac{9-3p}{2}} \Gamma(\frac{7-p}{2}) \; . \tag{5}$$

In the field theory limit of eqs.(1), (2) the solution is

$$ds^2 = \alpha' \left(\frac{U^{(7-p)/2}}{g_{YM}\sqrt{d_p N}} dx_\parallel^2 + \frac{g_{YM}\sqrt{d_p N}}{U^{(7-p)/2}} dU^2 + g_{YM}\sqrt{d_p N} U^{(p-3)/2} d\Omega_{8-p}^2 \right) ,$$

$$e^\phi = (2\pi)^{2-p} g_{YM}^2 \left(\frac{g_{YM}^2 d_p N}{U^{7-p}} \right)^{\frac{3-p}{4}} \sim \frac{g_{eff}^{(7-p)/2}}{N} . \tag{6}$$

Note that the effective string coupling, e^ϕ, is finite in the decoupling limit. In terms of g_{eff} the curvature associated with the metric (6) is

$$\alpha' R \approx \frac{1}{g_{eff}} \sim \sqrt{\frac{U^{3-p}}{g_{YM}^2 N}}. \tag{7}$$

From the field theory point of view U is an energy scale. Thus, going to the UV in the field theory means taking the limit $U \to \infty$. In this limit we see from eq.(6) that for $p < 3$ the effective string coupling vanishes and the theory becomes UV free. For $p > 3$ the coupling increases and we have to go to a dual description before we can investigate it reliably. This property of the supergravity solution is closely related to the fact that for $p > 3$ the super-Yang-Mills theories are non-renormalizable and hence, at short distances, new degrees of freedom appear.

We further note that, as in [3], we have α' in front of the metric, which might lead to the incorrect conclusion that one should only consider the zero modes of the fields. However, a field theory quantum of energy ω will have proper energy $w_{proper} = \omega\sqrt{g^{tt}} = \frac{1}{\sqrt{\alpha'}} \omega \frac{g_{YM}\sqrt{d_n N}}{U^{(7-p)/2}}$ which remains finite in string units. Therefore we can consider excitations which have proper enegies comparable to the string mass.

Consider the case that we break $U(N+1) \to U(N) \times U(1)$ by a Higgs expectation value U. In the supergravity description we get a Dp-brane sitting at the corresponding position U. The mass of a string stretched between the N branes and the probe is $m = U/2\pi$ from the gauge theory point of view. This is a BPS state whose mass does not depend on the coupling. In the supergravity side this state is represented by a string stretched between the probe and the horizon (at $r = 0$). If we calculate its "gauge theory" energy from supergravity we find that it is again U, since in curved space this energy contains a factor of $\sqrt{g_{rr}}$ (due to the fact that we should consider the proper distance) which is canceled by the $\sqrt{g_{tt}}$ factor needed to convert from local proper energies to gauge theory energies (canonically conjugate to t).

We will also consider near extremal configurations which correspond to the decoupled field theories at finite temperature. On the supergravity side we start from a near extremal black p-brane solution and we take the limit (1) keeping the energy density on the brane finite. In this limit only the metric is modified

$$ds^2 = \alpha' \left\{ \frac{U^{(7-p)/2}}{g_{YM}\sqrt{d_p N}}[-(1 - \frac{U_0^{7-p}}{U^{7-p}})dt^2 + dy_{||}^2]+ \right.$$

$$\left. \frac{g_{YM}\sqrt{d_p N}}{U^{(7-p)/2}(1 - \frac{U_0^{7-p}}{U^{7-p}})}dU^2 + g_{YM}\sqrt{d_p N}U^{(p-3)/2}d\Omega_{8-p}^2 \right\} \tag{8}$$

The dilaton is the same as in (6) and

$$U_0^{7-p} = a_p g_{YM}^4 \epsilon, \qquad a_p = \frac{\Gamma(\frac{9-p}{2})2^{11-2p}\pi^{\frac{13-3p}{2}}}{(9-p)} \tag{9}$$

Here ϵ is the energy density of the brane above extremality and corresponds to the energy density of the Yang-Mills theory. With these formulas one can calculate the entropy per unit volume and we find

$$s = \frac{S}{V} = \left(\frac{\Gamma(\frac{9-p}{2})^2 2^{43-7p}\pi^{13-3p}}{(7-p)^{7-p}(9-p)^{9-p}} \right)^{\frac{1}{2(7-p)}} g_{YM}^{\frac{p-3}{7-p}}\sqrt{N}\epsilon^{\frac{9-p}{2(7-p)}} \tag{10}$$

The temperature follows from the first law of thermodynamics.

In order to trust the type II supergravity solution (6) we need both the curvature (7) and the dilaton (6) to be small. This implies

$$1 \ll g_{eff}^2 \ll N^{\frac{4}{7-p}} . \tag{11}$$

We see that, as expected, the perturbative super-Yang-Mills and supergravity descriptions do not overlap (see eq.(3)). We will later see that the supergravity description can be extended to the region $N^{\frac{4}{7-p}} < g_{eff}^2$ but in terms of a dual theory. In particular, we see that we can trust the entropy computation (10) as long as the energy density above extremality is such that $g_{eff}(U_0)$ obeys (11) with U_0 as in (9)

The isometry group of the metric (6) is $ISO(1,p) \times SO(9-p)$ (for $p \neq 3$). From the super-Yang-Mills point of view the $ISO(1,p)$ symmetry is the Poincare symmetry and $SO(9-p)$ is the R-symmetry. It is an R-symmetry since spinors on the world-volume of the Dp-branes transform also as spinors in the directions transverse to the brane, and thus under $SO(9-p)$, whereas the brane scalars transform in the vector representation of $SO(9-p)$.

3   2+1 super-Yang-Mills and D2-branes

We start by considering a collection of N D2-branes in the super-Yang-Mills limit,

$$U = \frac{r}{\alpha'} = \text{fixed}, \qquad g_{YM}^2 = \frac{g_s}{\sqrt{\alpha'}} = \text{fixed}, \qquad \alpha' \to 0, \tag{12}$$

where $g_s = e^{\phi_\infty}$, $g_{Y.M}$ is the Yang-Mills coupling constant which has dimensions of $(\text{energy})^{1/2}$ and U is the expectation value of the Higgs. After taking the limit (12) we decouple the bulk from the theory on the D2 branes which turns out to be a U(N) super-Yang-Mills in 2+1 dimensions, with 16 supersymmetries. At a given energy scale, U, the dimensionless effective coupling of the gauge theory is $g_{eff}^2 \sim g_{YM}^2 N/U$ and, hence, perturbative super-Yang-Mills can be trusted in the UV region where g_{eff} is small

$$g_{YM}^2 N \ll U. \tag{13}$$

The supergravity solution of N D2-branes [14] yields in this limit

$$ds^2 = \alpha' \left(\frac{U^{5/2}}{g_{YM}\sqrt{6\pi^2 N}} dx_\parallel^2 + \frac{g_{YM}\sqrt{6\pi^2 N}}{U^{5/2}} dU^2 + g_{YM}\sqrt{6\pi^2 N/U} d\Omega_6^2 \right)$$
$$e^\phi = \left(\frac{g_{YM}^{10} 6\pi^2 N}{U^5} \right)^{1/4}. \tag{14}$$

The type II supergravity description can be trusted when the curvature (7) in string units and the effective string coupling are small

$$g_{YM}^2 N^{1/5} \ll U \ll g_{YM}^2 N. \tag{15}$$

We see that a necessary condition is to have $N \gg 1$. In the region $g_{eff} \approx 1$ we have a transition between the perturbative super-Yang-Mills description and the supergravity description.

In the region $U < g_{YM}^2 N^{1/5}$ the dilaton becomes large. In other words the local value of the radius of the eleventh dimension, $R_{11}(U)$, becomes larger than the Planck scale since $R_{11} = e^{2\phi/3} l_p$. Even though the string theory is becoming strongly coupled we will be able to trust the supergravity solution if the curvature is small enough in eleven dimensional Planck units. The relation between the eleven dimensional metric and the ten dimensional type IIA string metric, dilaton and gauge field is

$$ds_{11}^2 = e^{4\phi/3}(dx_{11} + A^\mu dx_\mu)^2 + e^{-2\phi/3} ds_{10}^2 \tag{16}$$

which implies that the curvature in 11D Planck units is

$$l_p^2 R \sim e^{2\phi/3} \frac{1}{g_{eff}} \sim \frac{1}{N^{1/3}} \left(\frac{g_{YM}^2}{U} \right)^{1/3}. \tag{17}$$
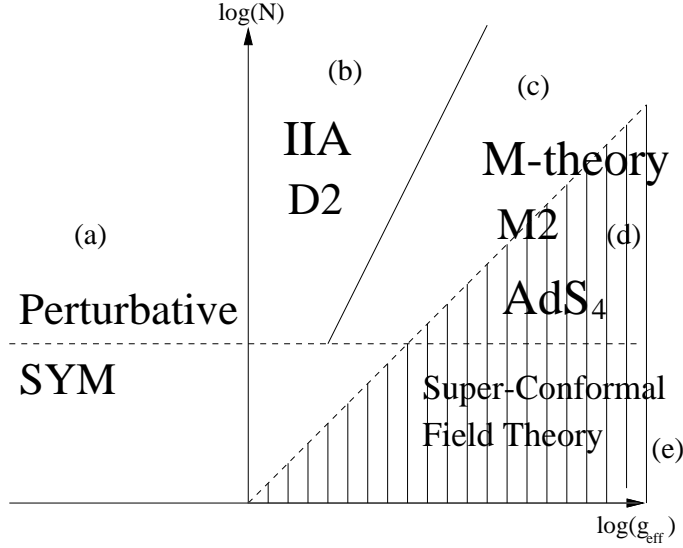
Figure 1: The D2-brane map: The horizontal dashed line separates between the small N region and the large N region. The other dashed line separates the IR region from the rest. The UV description is via perturbative super-Yang-Mills (a). In the IR the theory flows to a super-conformal region (the marked region) with SO(8) R-symmetry [15]. For large N we have a region described by IIA supergravity (b), a region described by the periodic array of M2-branes solution in eleven dimensions (c) and finally in the IR we have M-theory on the $AdS_4 \times S^7$ background (d).

For large N, in the region $g_{YM}^2 < U$, the curvature in 11D Planck units is small. We show below that in the region $U < g_{YM}^2$ we should use a different solution corresponding to M2-branes localized on the circle associated with the 11th dimension. The curvature of N M2-branes in the field theory limit is $l_p^2 R \sim \frac{1}{N^{1/3}}$ [3]. Note that the curvature does not depend on U since the theory is conformal in the IR limit. We conclude, therefore, that for large N the supergravity description is valid in the region $U \ll g_{YM}^2 N^{1/5}$.

The 11 dimensional solution that we get by uplifting the D2 brane solution (14) using eq.(16) is not exactly the M2 brane solution[2]. The uplifted solution is the M2 brane solution averaged over one of the transverse directions. Let us be more explicit. The M2 brane solution is characterized by an harmonic function H and is given by

$$ds_{11}^2 = H^{-2/3}dx_\parallel^2 + H^{1/3}dx_\perp^2 \tag{18}$$

and there is also a fourform field strength given in terms of H. When we take $H \sim N/x^6$ we have a solution where the M2 branes are localized in the eight transverse non-compact dimensions. If one of the dimensions is compact (let us say the 11th dimension) we can take $H \sim N/r^5$ where now r denotes the radial distance in the seven transverse non-compact dimensions. This is the solution we get from uplifting (14). We will later

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[2] "Uplifting" means that we find the eleven dimensional metric and fourform field strength (independent of x^{11}) which give the solution (14) upon Kaluza-Klein reduction.

see that this solution is unstable when we raise the temperature a little bit. The more physical solution is the one in which we take the M2 branes to be localized in the compact dimension so that the harmonic function is

$$H = \sum_{n=-\infty}^{\infty} \frac{2^5 \pi^2 N l_p^6}{(r^2 + (x_{11} - x_{11}^0 + 2\pi n R_{11})^2)^3} \tag{19}$$

with $x_{11} \sim x_{11} + 2\pi R_{11}$. For distances much larger than R_{11} we can Poisson resum this expression to

$$l_p^3 H = \frac{6\pi^2 g_{YM}^2 N}{U^5} + \sum_{m=1}^{\infty} N e^{-mU/g_{YM}^2} \cos(m x_{11}^0/R_{11}) \mathcal{O}(U^{-5}) \tag{20}$$

where we have used that $R_{11} = g_{YM}^2 \alpha'$. For $g_{YM}^2 \ll U$ we can, therefore, use the uplifted solution to describe the physics while for smaller values of U we should use (19). Note that for such small energy scales it becomes necessary to specify the expectation value of $\phi^{11} = g_{YM}^2 x_{11}^0/R_{11}$ which is the new scalar coming from dualizing the vector in 2+1 dimensions. In fact it was shown in [16] that the v^4 term in the effective action of a D2-brane probe receives instanton contributions which produce the whole series (20). For very low energies

$$U \ll g_{YM}^2, \tag{21}$$

we are very close to the M2 branes and we can neglect the "images" in (19). Thus, the solution will resemble that of M2 branes in non-compact space and we have the conformal field theory with SO(8) symmetry, which is the case described in [3]. We note that the physical size of the eleventh circle at the point of the transition between the localized and the delocalized solution, $U \sim g_{YM}^2$, is much larger than the Planck length, $R_{11}^{phys}|_{U=g_{YM}} \sim l_p N^{1/6}$. Hence, we can trust the supergravity solution whenever we have some non-trivial dependence of the solution on x_{11} and we have a smooth transition to the IIA supergravity regime when the physical size of R_{11} becomes small. In other words, starting from the IR and flowing to the UV the eleven dimensional supergravity solution becomes independent of x_{11} before $R_{11}^{phys}(U)$ becomes smaller than the eleven dimensional Planck length.

3.1 Near extremality

We would like to consider now finite temperature configurations in the super-Yang-Mills theory. We always take the energy above extremality and the temperature to be finite in the decoupling limit (12). The supergravity solution corresponding to a near extremal D2 brane in IIA string theory has one more harmonic function $h = 1 - U_0^5/U^5$ as in (8).

The parameter U_0 is finite in the decoupling limit and is given by

$$U_0^5 = \frac{240\pi^4}{7}g_{YM}^4\epsilon, \qquad (22)$$

where ϵ is the energy density. This is the energy density of the field theory, and it corresponds to the energy density of the brane above extremality. This solution describes the physics appropriately as long as $U_0 \gg g_{YM}^2$ (although it might be necessary to uplift the solution to eleven dimensions). When $U_0 \ll g_{YM}^2$ the uplifted solution (which is a valid solution of the equations of motion) becomes unstable [17]. This is just a classical instability associated with non-extremal black p-branes. There is an easy way to understand it. Let us first remember how the delocalized solution is generated. We start with M-theory on $T^2 \times S^1$ and we take the radius of S^1 to be much larger than the Planck length. We consider a Schwarzschild black hole solution in the 7+1 non-compact dimensions. Then we apply a boost along one of the directions of the T^2 (a symmetry of the supergravity equations) which generates some Kaluza-Klein momentum charge and then we U-dualize it into M2 branes wrapped on T^2. This procedure gives a solution which does not depend on the coordinate along S^1, which we call the "eleventh" dimension. This can actually be done for *any* supergravity solution of M-theory on T^2, regardless of whether it is localized on the extra S^1 or not. More explicitly, if we start from an uncharged static solution in 8+1 dimensions $ds_{1+8}^2 = g_{00}dt^2 + g_{ij}dx^i dx^j$ (where g_{00}, g_{ij} depend on x_i), then the solution with M2 brane charge obtained after performing this process, "uplifted" to eleven dimensions will be

$$ds_{11}^2 = (\cosh^2\alpha + \sinh^2\alpha g_{00})^{-2/3}[g_{00}dt^2 + dy_1^2 + dy_2^2] + (\cosh^2\alpha + \sinh^2\alpha g_{00})^{1/3}g_{ij}dx^i dx^j$$
$$(23)$$

where y_i are the two spatial coordinates along the brane and the periodic coordinate is among the x_i. Therefore, properties of the uncharged solution will translate into properties of the near extremal M2 brane configuration. Consider the uncharged Schwarzschild solution in 7+1 non-compact dimensions described above. It is translational invariant along S^1. It is, therefore, a black string. This black string is unstable when the Schwarzschild radius of the string becomes smaller than the radius of S^1 [17]. It seems plausible to think that the solution decays into a solution which is localized along the circle, which looks like a black hole in 8+1 dimensions (though we have not shown this explicitly)[17]. This is supported by the observation that the entropy of the 8+1 black holes is bigger than the entropy of the black string when their Schwarzschild radii are smaller than the radius of the circle. Of course, in order for the supergravity analysis to be valid all these radii should be much bigger than the Planck length. To describe more precisely this transition one should find supergravity solutions which are localized in the compact dimensions. These solutions would look like an infinite array of Schwarzschild black holes (they do not

collapse on each other because of the periodicity conditions). Some solutions of this type were found in 3+1 dimensions with one compact spatial dimension [18]. After we do the boosts and U-duality transformations to produce M2 brane charge we see that the point where the Schwarzschild radius is comparable to the radius of the S^1 corresponds, in the new solution, to the point where

$$U_0 \sim g_{YM}^2 \ . \tag{24}$$

In summary, for $U_0 > g_{YM}^2$ we have a translational invariant solution (along x_{11}) while for $U_0 < g_{YM}^2$ we have a localized solution. When $U_0 \ll g_{YM}^2$ we can find an approximate near extremal solution by considering a linear superposition of Schwarzschild black holes and performing the above procedure. The thermodynamics for this solution will be, by construction, the same as the one for the near extremal M2 brane in non-compact 11 dimensions. This is what we expect for 2+1 SYM at low energy, i.e. that the results should be those of the corresponding IR superconformal field theory.

Conclusions:

We are always considering 2+1 dimensional super-Yang-Mills. This theory has a large N dual which is the supergravity solution described above. This supergravity solution has various regions. The supergravity description requires that $U \ll g_{YM}^2 N$ (when $g_{YM}^2 N \ll U$ perturbative Yang-Mills is a good description). In the region $N^{1/5} < U/g_{YM}^2 < N$ we have a IIA string theory description, we expect to have strings, etc. as in [3]. When $U/g_{YM}^2 < N^{1/5}$ we should use an eleven dimensional supergravity solution. The transition from the ten dimensional to the eleven dimensional solution is smooth from the point of view of supergravity and the gradient of the dilaton remains always small (for large N). When $U \ll g_{YM}^2$ the geometry becomes that of $AdS_4 \times S^7$ which is the one that corresponds to the low energy conformal field theory with SO(8) R-symmetry.

Similarly when we consider a near extremal configuration we see that for very low temperatures the behavior is that of the M2 brane conformal field theory. One could, in principle, follow the transition between the near extremal M2 brane and the near extremal D2 brane behavior if one knew more precisely the localized supergravity solution.

4   1+1 super-Yang-Mills and D1-branes

Next we turn our attention to a collection of N D1-branes corresponding to the case $p = 1$ in section 2. The decoupling limit takes now the form

$$U = \frac{r}{\alpha'} = \text{fixed}, \quad g_{YM}^2 = \frac{1}{2\pi}\frac{g_s}{\alpha'} = \text{fixed}, \quad \alpha' \to 0. \tag{25}$$
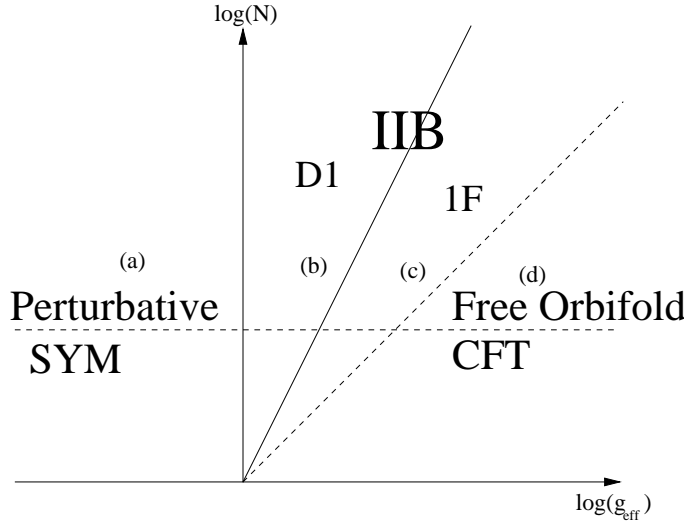
Figure 2: The D1-brane map: The horizontal dashed line separates between the small N region and the large N region. The other dashed line separates between the IR region and the rest. For any N the UV region is described by perturbative SYM (a) and in the IR by a free orbifold CFT (d). When N is large there is an intermediate region (b,c) which is described by a IIB supergravity solution interpolating between the D1 brane solution (b) and the F-string solution (c).

The supergravity solution in this limit yields

$$ds^2 = \alpha' \left(\frac{U^3}{g_{YM}\sqrt{2^6\pi^3 N}} dx_{\parallel}^2 + \frac{g_{YM}\sqrt{2^6\pi^3 N}}{U^3} dU^2 + g_{YM}\frac{\sqrt{2^6\pi^3 N}}{U} d\Omega_6^2 \right),$$
$$e^\phi = \left(\frac{g_{YM}^6 2^8 \pi^5 N}{U^6} \right)^{1/2}. \tag{26}$$

Super-Yang-Mills in $1+1$ dimensions is super-renormalizable and it can be trusted at high energies $g_{YM}\sqrt{N} \ll U$. The curvature in string units (7) is small for $U \ll g_{YM}\sqrt{N}$. From eq.(26) we see that the expansion in string coupling is valid in the region $g_{YM}N^{1/6} \ll U$. Therefore, the type IIB supergravity solution (26) can be trusted in the region $g_{YM}N^{1/6} \ll U \ll g_{YM}\sqrt{N}$. In the region $U \ll g_{YM}N^{1/6}$ the string coupling is large. To get a more accurate description we need to apply S-duality, which takes $\phi \to -\phi$ and hence $g_s \to \tilde{g}_s = 1/g_s$. Since the ten dimensional Newton constant, $G_N^{10} = 8\pi^6 g_s^2 \alpha'^4$, is invariant S-duality also takes $\alpha' \to \tilde{\alpha}' = g_s\alpha'$. Note that in the decoupling limit we considered $\tilde{\alpha}' \to 0$. S-duality, therefore, maps (26) to the small r region of the fundamental string solution [19]

$$ds^2 = \tilde{\alpha}' \left(\frac{U^6}{g_{YM}^4 2^7 \pi^4 N} dx_{\parallel}^2 + \frac{1}{2\pi g_{YM}^2} dU^2 + \frac{U^2}{2\pi g_{YM}^2} d\Omega^2 \right),$$
$$e^\phi = \left(\frac{g_{YM}^6 2^8 \pi^5 N}{U^6} \right)^{-1/2}. \tag{27}$$

In the IR limit ($U \to 0$) the string coupling vanishes. The curvature in the new string units is

$$\tilde{\alpha}' R \sim \frac{g_{YM}^2}{U^2}, \tag{28}$$

and does not depend on N. The reason is that the dependence of the metric on N drops out after the coordinate change $x_{||} \to x_{||}/\sqrt{N}$. Note that there is a curvature singularity in the IR limit. This means that the supergravity description breaks down[3] for small U. In fact the IR limit of super-Yang-Mills is a trivial orbifold $((R^8)^N/S_N)$ conformal field theory. Furthermore, the first irrelevant operator that appears in this theory was found in [7]. We could now consider the theory at finite temperature and ask when the orbifold CFT is a valid description. The orbifold is characterized by N fields and the first correction is given by the twist operator $\frac{1}{g_{YM}} V_{ij}$ where i, j label the two fields on which the twist is acting [7]. The power of g_{YM} follows from dimensional analysis. We compute the partition function for the orbifold and we see at which temperature the correction due to the twist operator becomes large. We find that the free field theory will be a good approximation if the temperature satisfies $T \ll g_{YM}/N^{1/2}$. This in turn translates into a parameter U_0 of the near extremal solution (8) which is $U_0 \ll g_{YM}$ [4]. It implies that the point where the supergravity solution breaks down is related to the point where the free conformal field theory takes over. Notice that it would have been impossible to have a free field theory dual to a supergravity system.

Conclusions:

The 1+1 dimensional super-Yang-Mills under consideration flows for any N both in the UV and IR to free field theories. The large N dual of these theories is a supergravity solution that is valid in the intermediate region $g_{YM} \ll U \ll g_{YM}\sqrt{N}$. At both ends of this limit the curvature grows and the solution breaks down. Furthermore for $g_{YM} \ll U \ll g_{YM}N^{1/6}$ the proper description is through the fundamental string solution while for $g_{YM}N^{1/6} \ll U \ll g_{YM}\sqrt{N}$ we should use the D-string supergravity solution.

5   D0-branes and super quantum mechanics

In this section we consider the super quantum mechanical theory associated with N D0-branes in the limit

$$U = \frac{r}{\alpha'} = \text{fixed}, \quad g_{YM}^2 = \frac{1}{4\pi^2}\frac{g_s}{\alpha'^{3/2}} = \text{fixed}, \quad \alpha' \to 0. \tag{29}$$

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[3] This breakdown of the fundamental string solution was also studied by Sen [20].
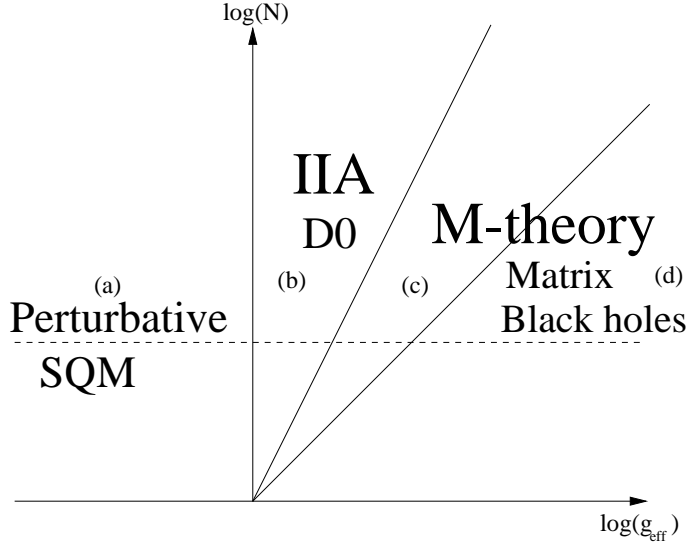
[4] The same conclusion was reached in [21].

Figure 3: The D0-brane map: The horizontal dashed line separates between the small N region and the large N region. For any N the UV description is via perturbation theory in super quantum mechanics (a). For large N we have a region (b) which is described by the IIA D0 brane solution, which for smaller energies becomes a gravitational wave background in eleven dimensions (c). Finally, at very low energies (d) we enter into the matrix black hole region.

Notice that $U/g_{YM}^{2/3} \sim r/l_p$ as in [2]. Perturbation theory in the quantum mechanics can be trusted at high energies $g_{eff} \ll 1$ which gives $U > g_{YM}^{2/3} N^{1/3}$.

The supergravity solution in the decoupling limit gives

$$ds^2 = \alpha' \left(-\frac{U^{7/2}}{4\pi^2 g_{YM}\sqrt{15\pi N}}dt^2 + \frac{4\pi^2 g_{YM}\sqrt{15\pi N}}{U^{7/2}}dU^2 + \frac{4\pi^2 g_{YM}\sqrt{15\pi N}}{U^{3/2}}d\Omega^2 \right),$$
$$e^\phi = 4\pi^2 g_{YM}^2 \left(\frac{240\pi^5 g_{YM}^2 N}{U^7} \right)^{3/4}. \tag{30}$$

For this solution the effective string coupling and the curvature in string units are small in the region

$$g_{YM}^{2/3} N^{1/7} \ll U \ll g_{YM}^{2/3} N^{1/3} \tag{31}$$

Thus, in this region one can trust the type IIA description.

Now we would like to study the low energy region. To this end we need to uplift the solution to eleven dimension. This solution can be generated by starting with an uncharged black string along x_{11} and then boosting it along x_{11} while taking the limit

$$\gamma \to \infty, \quad \gamma\mu = \frac{N}{2\pi R_{11}^2} = \text{fixed}, \tag{32}$$

where μ is the mass per unit length of the black string in its rest frame. As we show below this plane wave description cannot be trusted for $U < g_{YM}^{2/3} N^{1/9}$. This region is closely related to the matrix model black holes [26, 27].

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5.1 Matrix black holes

The eleven dimensional solution which corresponds to the uplifted near extremal D0 brane solution is translational invariant along the circle. This solution is unstable for small enough energies above extremality. In the D2-M2 case we estimated the energy scale at which this instability was happening by starting from the neutral black string solution and tracing through the steps in the solution generating technique. In this case we do the same. The solution is gotten by performing a boost along an uncharged black string. As explained in [27] the instability appears when

$$U_0 \sim g_{YM}^{2/3} N^{1/9} \tag{33}$$

which is the "correspondence" point of [26, 27]. Of course this localization makes sense only in the large N limit where we keep a fraction $M \ll N$ of the total momentum as gravitons so that the total system of gravitons plus black hole is in a momentum eigenstate.

For $U_0 \ll g_{YM}^{2/3} N^{1/9}$ we have a Schwarschild black hole boosted along the eleventh direction. It is interesting to note that the Schwarschild radius of the black hole in Planck units is given by

$$\left(\frac{r_s}{l_p}\right)^8 \sim \frac{E^{1/2} N^{1/2}}{g_{YM}^{1/3}} \sim \frac{U_0^{7/2} N^{1/2}}{g_{YM}^{7/3}} \tag{34}$$

So that we trust the gravity description if $U_0 \gg g_{YM}^{2/3} N^{-1/7}$. For lower values of the energy we expect that the system should start to behave more as a single graviton. Notice that if we keep the ratio r_s/l_p fixed then the energy above extremality goes as $E \sim 1/N$ as we expect [25].

6 D4-brane, 4+1 SYM and the (0,2) 6-d SCFT on a circle

Let us consider now a system of N D4 branes in the limit

$$U = \frac{r}{\alpha'} = \text{fixed}, \qquad g_{YM}^2 = (2\pi)^2 g_s \sqrt{\alpha'} = \text{fixed}, \qquad \alpha' \to 0. \tag{35}$$

This system is better described by considering a system of M5-branes wrapped on the eleventh dimensional circle in M-theory and taking the limit $l_p \to 0$ while keeping $R_{11} = g_s \sqrt{\alpha'} = g_{YM}^2/(2\pi)^2$ fixed. So we have the (0,2) six dimensional conformal field theory on a circle, which, at low energies, reduces to 4+1 dimensional super-Yang-Mills [28].
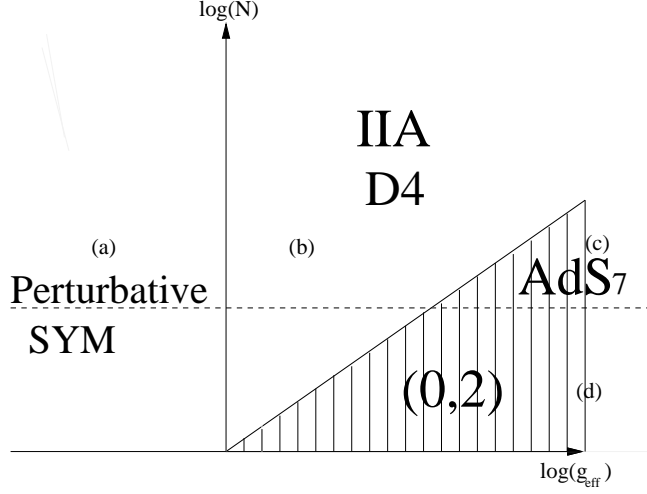
Figure 4: The D4-brane map: The horizontal dashed line separates between the small N region and the large N region. The UV region is described by a super-conformal theory on a circle (the marked region) (c,d), which is dual for large N to M-theory on a background $AdS_4 \times S^7$ (c) (with an identification). In the IR the theory is described by "perturbative" super-Yang-Mills (a). For large N we have the intermediate region (b) described by the IIA D4 brane solution.

The supergravity description involves a type IIA supergravity region and an M-theory region. The IIA solution is

$$ds^2 = \alpha' \left(\frac{2\sqrt{\pi}U^{3/2}}{g_{YM}\sqrt{N}}dx_{||}^2 + \frac{g_{YM}\sqrt{N}}{2\sqrt{\pi}U^{3/2}}dU^2 + \frac{g_{YM}\sqrt{NU}}{2\sqrt{\pi}}d\Omega^2 \right),$$

$$e^\phi = \left(\frac{U^3 g_{YM}^6}{2^{10}\pi^9 N} \right)^{1/4}. \tag{36}$$

The "perturbative" super-Yang-Mills[5] description can be trusted in the IR region where the effective coupling is small, $g_{eff}^2 = g_{YM}^2 NU \ll 1$. In the region where $N^{-1} \ll g_{YM}^2 U$ the curvature (7) is small in string units. The dilaton is small for $g_{YM}^2 U \ll N^{1/3}$. Therefore the type IIA supergravity solution can be trusted in the region $N^{-1} \ll g_{YM}^2 U \ll N^{1/3}$. In the region $N^{1/3} \ll g_{YM}^2 U$ the dilaton is large and the description is via 11D supergravity. Using eq.(16) we find the 11D solution of N NS5-branes wrapped along the x_{11} direction,

$$ds^2 = l_p^2 \left(\frac{\tilde{U}^2}{(\pi N)^{1/3}}dx_{||}^2 + 4(\pi N)^{2/3}\frac{d\tilde{U}^2}{\tilde{U}^2} + (\pi N)^{2/3}d\Omega_4^2 \right), \tag{37}$$

where $\tilde{U}^2 = (2\pi)^2 U/g_{YM}^2 = U/R_{11}$. This describes a M-theory background of $AdS_7 \times S^4$ [3] with an identification along a circle. The radius of the Anti-de Sitter space and the radius of the sphere are large (in eleven dimensional Planck units) for large N. Hence,

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[5] We have put "perturbative" in quotes because the theory is non-renormalizable. In principle we could use perturbation theory to calculate diagrams which are finite. Examples of finite diagrams are the v^4 terms [25] (and, of course, all tree level diagrams).

the solution can be trusted for large N, as long as the physical length of the circle that we are identifying is large enough.

Note that the g_{YM} dependence drops out in (37). This is a result of the theory being conformal at the UV and g_{YM}^2 having dimensions of length.

Conclusions:

In the present case we are dealing with the (0,2) theory compactified on a circle, which becomes 4+1 dimensional Yang-Mills at low energies. This theory is free in the IR. For large N we have a dual supergravity description. It involves a type IIA supergravity solution in the region $N^{-1} \ll g_{YM}^2 U \ll N^{1/3}$ and M-theory on $AdS_7 \times S^4$ with identifications in the region $N^{1/3} \ll g_{YM}^2 U$.

7 D5-branes and IIB NS fivebranes

For the system of N D5-branes the relevant decoupling limit is

$$U = \frac{r}{\alpha'} = \text{fixed}, \quad g_{YM}^2 = (2\pi)^3 g_s \alpha' = \text{fixed}, \quad \alpha' \to 0. \tag{38}$$

In this limit the supergravity solution gives

$$ds^2 = \alpha' \left(\frac{(2\pi)^{3/2} U}{g_{YM}\sqrt{N}} dx_{||}^2 + \frac{g_{YM}\sqrt{N}}{(2\pi)^{3/2} U} dU^2 + \frac{g_{YM}\sqrt{N} U}{(2\pi)^{3/2}} d\Omega_6^2 \right),$$
$$e^\phi = \frac{g_{YM} U}{(2\pi)^{3/2}\sqrt{N}}. \tag{39}$$

The super-Yang-Mills can be trusted in the IR region $g_{YM} U \ll \frac{1}{\sqrt{N}}$. In the region $\frac{1}{\sqrt{N}} \ll g_{YM} U \ll \sqrt{N}$ the string coupling and the curvature in string units are small so one can trust the type D5-brane supergravity solution. For $\sqrt{N} \ll g_{YM} U$ the string coupling is large and we have to go to the S-dual system of N NS5-branes where

$$\tilde{\alpha}' = g_s \alpha' = g_{YM}^2/(2\pi)^3 \tag{40}$$

and, therefore, remains finite in the limit (38). The solution for the NS5-branes is

$$ds^2 = dx_{||}^2 + \tilde{\alpha}' \left(\frac{N}{U^2} dU^2 + N d\Omega^2 \right),$$
$$e^\phi = \left(\frac{(2\pi)^3 N}{g_{YM}^2 U^2} \right)^{1/2}. \tag{41}$$

The curvature in string units is $\tilde{\alpha}' R \sim \frac{1}{N}$, and, therefore, it is small in string units for large N and hence the supergravity description is valid for large N [29]. It is, however,
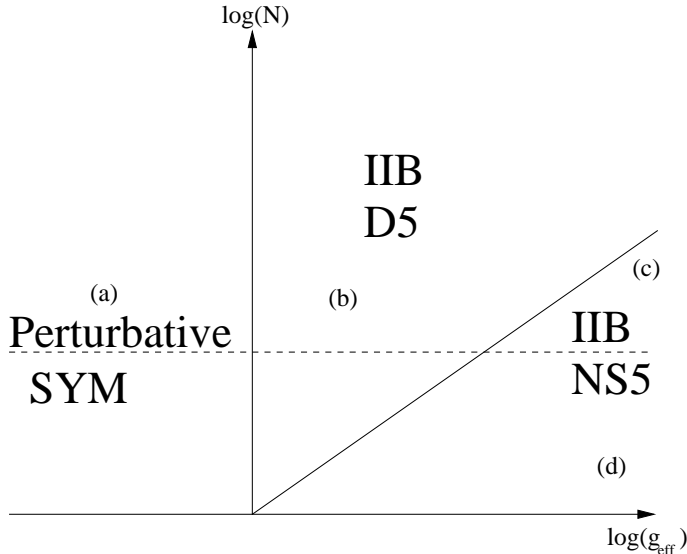
Figure 5: The D5-brane map: The horizontal dashed line separates between the small N region and the large N region. In the IR the theory are described by "perturbative" SYM (a). For the UV description is via type IIB on NS background (c,d) which has supergravity description for large N (c). For large N there is an intermediate region described by the D5 brane background (b).

possible to have an exact conformal field theory description of this background as a classical solution of string theory (which is appropriate in this region where the dilaton is becoming small) [30]. This case is different from other field theory cases because massive particles can propagate all the way to infinity along the throat. The throat is infinite in the limit (38).

Conclusions:

We have the theory defined as the $g_s \to 0$ limit of NS IIB fivebranes [31]. This theory is characterized by a scale $g_{YM}^2 \sim \tilde{\alpha}'$. For large N we can analyze properties of the theory using supergravity. This theory flows in the IR, $g_{YM}U \ll N^{-1/2}$, to super-Yang-Mills. For scales $N^{-1/2} \ll g_{YM}U \ll N^{1/2}$ we should use the D5 brane supergravity solution. Finally for $N^{1/2} \ll g_{YM}U$ we should use the NS fivebrane solution. This solution involves an infinite throat region (for large U) which does not decouple from the physics [29] and should therefore be included in the description.

8   IIA NS fivebranes

This system was studied in [29] for high temperatures. In this section we add some comments which are relevant for the description at lower temperatures (and large N). We observe that in the region where the dilaton is large we can go to an eleven dimensional

description where nothing singular happens (for large N), and we flow in the IR to $AdS_7 \times S^4$ which is the large N dual of the (0,2) conformal field theory, so that things work as expected. We are interested in the system of N NS fivebranes in the limit

$$g_s \to 0, \ \alpha' = \text{fixed}, \ U = \frac{r}{g_s \alpha'} = \text{fixed} \qquad (42)$$

Notice the additional factor of g_s in the definition of U, this ensures that the tension of D2-branes stretched between different fivebranes is constant. The supergravity solution is the same as (41). This system was analyzed in [29] for large temperature. For low energies it is more appropriate to go to eleven dimensions and, therefore, consider M5 branes transverse to a compact circle. An instability of the type described for the M2 branes will lead us to consider M5 branes localized on the circle. (The instability would be present only for non-zero temperature). The supergravity solution is then

$$ds^2 = H^{-1/3}dx_\parallel^2 + H^{2/3}dx_\perp$$
$$H = \sum_{n=-\infty}^{\infty} \frac{\pi N l_p^3}{(r^2 + (x_{11} + 2\pi R_{11}n)^2)^{3/2}} \sim \frac{N\alpha'}{r^2} + N \sum_{m \geq 1} e^{-mr/R_{11}} \mathcal{O}(r^{-2}) \qquad (43)$$

where $r/R_{11} = U\sqrt{\alpha'}$. Hence, for $U \ll \frac{1}{\sqrt{\alpha'}}$ we are very close to one of the centers in (43) and we flow into the (0,2) conformal field theory. Again as in the case of the D2 brane we see that we first go into the eleven dimensional description and then into the localized description (if N is sufficiently large). These localized solutions were analyzed in detail in [32], but for the present discussion it is very important that N is large, otherwise we cannot trust the supergravity solutions. Notice that even though the radius of the eleventh dimension seems to go to zero very far away from the NS brane, it becomes large close to the NS brane. It is so large that we are arguing that we should use a supergravity solution that explicitly depends on the eleventh dimension. If we heat up the system, considering it at a finite temperature, we expect to have a localized solution for small energy densities ($\epsilon \ll 1/\alpha'^3$), with the thermodynamic behavior of the (0,2) theory. An approximate solution can be found as for the D2-M2 brane case. For larger energy densities we expect to have a solution that is translational invariant along the eleventh dimension.

9   D6-brane and 6+1 super-Yang-Mills

The last system we analyze in this paper is the system on N D6-branes. The candidate decoupling limit in this case seems to be

$$U = \frac{r}{\alpha'} = \text{fixed}, \qquad g_{YM}^2 = (2\pi)^4 g_s \alpha'^{3/2} = \text{fixed}, \qquad \alpha' \to 0. \qquad (44)$$
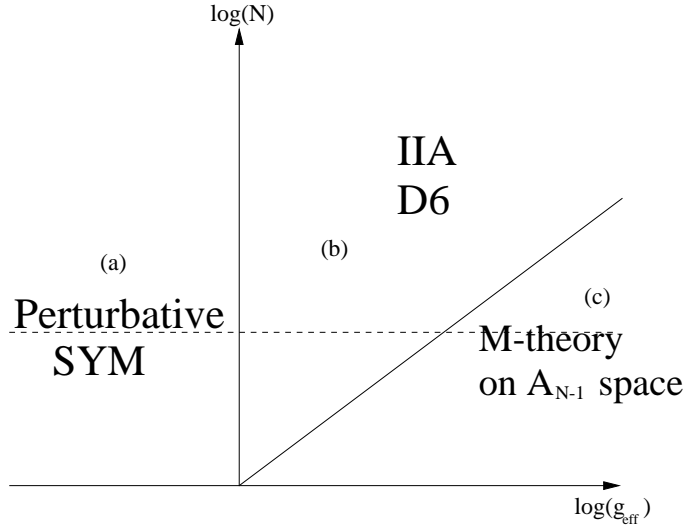
Figure 6: The D6-brane map: The horizontal dashed line separates between the small N region and the large N region. The IR is described by SYM (a). For any N the UV region is described by M-theory on an ALE space with A_{N-1} singularity (c). In the large N limit there is a region described by the IIA D6 brane solution (b).

which is better analyzed by going to M-theory on a circle with N Kaluza-Klein monopoles. The limit is now simply $l_p = \text{fixed}$, $R_{11} \to \infty$ which leaves an ALE singularity with $l_p = g_{YM}^{2/3}/(2\pi)^{4/3}$. The IIA supergravity solution in the limit (44) gives

$$ds^2 = \alpha' \left(\frac{(2\pi)^2}{g_{YM}} \sqrt{\frac{2U}{N}} dx_{||}^2 + \frac{g_{YM}}{(2\pi)^2} \sqrt{\frac{N}{2U}} dU^2 + \frac{g_{YM}}{(2\pi)^2 \sqrt{2}} \sqrt{N} U^{3/2} d\Omega^2 \right),$$

$$e^\phi = \frac{g_{YM}^2}{2\pi} \left(2 \frac{U}{g_{YM}^2 N} \right)^{3/4}. \tag{45}$$

The super-Yang-Mills effective coupling is $g_{eff}^2 = g_{YM}^2 N U^3$. Thus, super-Yang-Mills is a good approximation at low energies, $U \ll \frac{1}{g_{YM}^{2/3} N^{1/3}}$. The curvature (7) is small in string units in the region $\frac{1}{g_{YM}^{2/3} N^{1/3}} \ll U$. The dilaton is small in the region $U \ll \frac{N}{g_{YM}^{2/3}}$. Thus the type IIA supergravity solution can be trusted in the region $\frac{1}{g_{YM}^{2/3} N^{1/3}} \ll U \ll \frac{N}{g_{YM}^{2/3}}$. In the region $\frac{N}{g_{YM}^{2/3}} \ll U$ we get from eq.(45) that $R_{11}^{phys}(U) \gg l_p$ hence we should lift up the type IIA solution to eleven dimensions. Using eq.(16) we get for the 11D metric

$$ds^2 = dx_{||}^2 + \frac{l_p^3 N}{2U} dU^2 + \frac{l_p^3 NU}{2} (d\theta^2 + \sin^2\theta d\varphi^2) + \frac{2Ul_p^3}{N} [d\phi + \frac{N}{2}(\cos\theta - 1)d\varphi]^2 \tag{46}$$

where $\phi \equiv x_{11}/R_{11}$ has period $\phi \sim \phi + 2\pi$. Defining the new variables $y^2 = 2Nl_p^3 U$, $\tilde\theta = \theta/2$, $\tilde\varphi = \varphi + \phi/N$, $\tilde\phi = \phi/N$ we obtain the metric

$$ds^2 = dx_{||}^2 + dy^2 + y^2(d\tilde\theta^2 + \sin^2\tilde\theta d\tilde\varphi^2 + \cos^2\tilde\theta d\tilde\phi^2), \tag{47}$$

where $0 \le \tilde\theta \le \pi/2$ and $0 \le \tilde\varphi, \tilde\phi \le 2\pi$ with the identification $(\tilde\varphi, \tilde\phi) \sim (\tilde\varphi, \tilde\phi) + (2\pi/N, 2\pi/N)$. This identification leads to an ALE space with an A_{N-1} singularity [33].

Note that we are saying that an ALE singularity in M-theory has, for large N, a region which is properly described by a type IIA solution.

The Riemann curvature tensor of (47) vanishes and the identifications involve circles of large proper length if $y \gg l_p N$ (i.e. $g_{YM}^{2/3} U \gg N$). This means that unlike the cases analyzed so far the 11D supergravity solution can be trusted in the UV *for any N*. Since the large U solution is just flat eleven dimensions, we do not expect to find any seven dimensional field theory in the UV which flows, in the IR, to super-Yang-Mills in $6 + 1$. In particular, this implies that DLCQ of M-theory on T^6 is as complicated as M-theory [11, 12].

Another way to state the difference between this case and the previous ones is to observe that, in the present case, massive geodesics can escape all the way to infinity. There is a second asymptotic region which is eleven dimensional and is described by M-theory itself. For other branes (except for NS 5 branes) all massive geodesics either fall back into the small U region (IR) or the supergravity solution is invalid for large U and is replaced by perturbative super-Yang-Mills (for $p < 3$). In the Anti-deSitter cases the only geodesics that reach infinity are the massless geodesics. In the quantum description one sees that only s-waves can propagate to infinity.

9.1   Non-extremal

In order to analyze this decoupling problem more closely we consider a near extremal configuration. We consider a system of D6 branes at finite temperature in the limit (44) and analyze the corresponding supergravity solution. For small energy densities above extremality we are in the super-Yang-Mills regime. For intermediate energy densities we have a description involving the type IIA supergravity solution (8). However, if the energy densities above extremality are large, $\epsilon \gg N l_p^{-7}$, we should use the eleven dimensional description. As we saw the 11D supergravity can be trusted for any N in the UV. Starting with the near extremal D6 brane solution (8) and "uplifting" it, as we did for the extremal one, we find a metric which corresponds to the metric of an uncharged Schwarzschild black hole sitting at the ALE singularity. More explicitly we get the metric

$$ds^2 = -(1 - \frac{y_0^2}{y^2})dt^2 + \frac{dy^2}{(1 - \frac{y_0^2}{y^2})} + y^2(d\tilde{\theta}^2 + \sin^2 \tilde{\theta} d\tilde{\varphi}^2 + \cos^2 \tilde{\theta} d\tilde{\phi}^2) + dx_i^2 \quad (48)$$

where $i = 1, .., 6$ and the angles have the same identifications that they had before. The parameter y_0 is related to the energy above extremality via the formula $y_0^2 = 2N l_p^3 U_0$ with U_0 given in terms of the energy density as in (9). The Hawking temperature is $T_H \sim 1/\sqrt{N l_P^9 \epsilon}$. We find, therefore, that we have Hawking radiation into the asymptotic

region where we have bulk eleven dimensional supergravity. Thus we conclude as in [12, 11] that there is no decoupled limit for the D6 brane theory.

The main difference between the discussion in this sub-section and [29] is that here the supergravity description can be trusted for any N while the supergravity solution found in [29] is valid only for large N (and large energy density as in our case).

Conclusions:

For any N the UV region is described by M-theory on a flat background (an A_{N-1} singularity). Note that there is no field theory description in the UV for any N. This theory flows in the IR to super-Yang-Mills. For large N there is an intermediate region which can be described by the IIA D-sixbrane solution.

Acknowledgments

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