Probing Matrix Black Holes

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Black holes in matrix theory may consist of interacting clusters (correlated domains) which saturate the uncertainty principle. We show that this assumption qualitatively accounts for the thermodynamic properties of both charged and neutral black holes, and reproduces the asymptotic geometry seen by probes.

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1. Introduction

The geometry of a spacetime, and in particular the causal structure, is naturally probed by an ideal massless particle. In M theory, the only universal object of this sort is the supergraviton itself. In matrix theory (or more generally in the dynamics of D-branes), curved space geometry is the effective description of probe dynamics in the abelianized, moduli space approximation; bending of the probe's path is an effect of the spatially dependent vacuum polarization caused by sources [1,2]. There can be no global horizons in this description (at least at finite N) since the starting point is nonabelian dynamics in Minkowskian spacetime. The idea of using D-branes to probe black holes has been explored in [2-6].

Recently, states in matrix theory [7] bearing the qualitative properties of Schwarzschild black holes have been constructed [8-13]. One would like to see how to reconstruct the black hole geometry from these states. The model developed in [11,13] describes Schwarzschild black holes as a collection of matrix partons (the eigenvalues of the matrices) bound together by the semiclassical dynamics of the 'off-diagonal' (unitary conjugation) degrees of freedom. The partons are treated as the principal dynamical objects. Combining the uncertainty principle, the virial theorem, and mean field theory, one finds that a bound state of N partons has the energy and entropy of a Schwarzschild black hole with longitudinal momentum $P = N/R$:

$$E_{LC} = M^2 R/N$$
$$M \sim r_0^{D-3}/G_D \tag{1.1}$$
$$S \sim r_0^{D-2}/G_D \sim N \ .$$

The special point $P = S/R$ puts the system at the transition point between black holes and black strings; in other words, $N = S$ is the minimum value for which the system approximates a black hole. To obtain the entropy, the partons must be treated as distinguishable objects, as was implicit in [11] and discussed in detail in [13]. The justification is as follows: Energetic arguments [11,13] indicate that soft excitations of the unitary degrees of freedom are not energetically costly relative to the overall energy of the resonant bound state; the presence of such a unitary mode background entwines the permutation of the eigenvalues with the wavefunction of the unitary modes, effectively destroying any statistics symmetry among the eigenvalues themselves. It is important for the validity of this argument that the time scale of motion of the unitary modes is at least as long as the crossing time r_0^2/R of the eigenvalues in the resonant bound state; otherwise we could integrate out the uni-

tary modes and obtain an effective dynamics for the eigenvalues which would treat them as identical particles. This bound is easily seen to be satisfied if one crudely approximates the unitary modes as a membrane of size $\sim r_0$ [13](the membrane is indeed an excitation of the unitary modes [14,7]). The membrane time scale is $T_{memb} \sim r_0^{D-3}\ell_{\rm pl}^3/G_D R$ [11], which exceeds the eigenvalue crossing time if $D > 5$ ($D = 5$ appears to be marginal). Also $E_{LC,{\rm memb}} \sim r_0^4 R/N \ll E_{LC,{\rm bh}}$, so these modes don't substantially affect the energetics.

When $N \sim S$, the black hole radius is close to the longitudinal box size [8,11]; one expects the statistical properties to suffer from finite size effects (for instance, the modes identified in [8,9] as responsible for black hole entropy). Our main result is a clearer picture of the composition of the black hole at $N \gg S$ as a collection of clusters or correlated domains of matrix partons,[1] as proposed in [11,12].[2] In this regime the boost is sufficiently large that the finite size corrections due to the longitudinal box should be irrelevant. We will show that this scenario qualitatively reproduces the asymptotic geometry and thermodynamic properties of both charged and neutral black holes.

In section 2, we argue for this picture of the highly boosted black hole as a collection of interacting clusters; the picture is essentially a boosted version of the mean field picture outlined above, with the partons replaced by clusters of partons. One difference will be that processes where clusters exchange partons will have to be included in the analysis. We show that the same picture can account for the entropy of charged black holes, by boosting the parton distribution along an internal circle. Section three derives the asymptotic geometry that should be seen by a supergraviton probe in the presence of a (charged) black hole, and shows how the probe effective action is consistent with our picture of the structure of the black hole. All our considerations are up to factors of order unity, to which our approximations are insensitive. In section 4 we interpret our results in the context of previous work on black holes in string/M-theory [15,16,17].

Incidentally, most of what is put forth here and in [11,12,13] has validity independent of the ultimate fate of matrix theory, since it applies to black holes extremely boosted along

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[1] We will use interchangeably the phrases 'cluster' and 'correlated domain' to describe the objects entering the effective dynamics; our model is at the moment too crude to decide how the degrees of freedom are ordered, *e.g.* whether the ordering takes place in position space (as in a ferromagnet at its Curie point), momentum space (as in a BCS superconductor); or in matrix space, or (as suggested by Lorentz symmetry) some more exotic combination of all of these coordinates.

[2] The structure of the interactions differs, however, from that deduced in [12].

x_{10} – independent of the size R of the longitudinal box, which could be sub-Planckian in size. On the other hand, our approach may prove useful in exploring macroscopic black holes in string theory using the matrix string construction of [18]; the unitary mode background ought to reduce to a matrix string background when a transverse circle is shrunk to sub-Planckian size. One may in this way make contact with the correspondence principle of [19].

2. Cluster decomposition of matrix black holes

Suppose the black hole consists of S clusters, each containing approximately N/S matrix partons. The characteristic property of a cluster is its coherence – interactions that probe it on size scales larger than the correlation length affect the whole cluster. Thus we will treat the clusters as the basic participants in the dynamics, rather than the individual matrix partons. The total longitudinal momentum of the system is $p_- = N/R$. The cluster 'mass' (its longitudinal momentum) is N/SR. If the spread of the wavefunction of the cluster's center of mass is r_0, the cluster's transverse velocity is SR/Nr_0 by the uncertainty principle. The virial theorem implies that the kinetic and potential energies of the clusters are of the same order. The kinetic energy is

$$
\begin{aligned}
E_{\text{kin}} &\sim (\#\text{clusters}) \cdot m_{\text{cl}} v_{\text{cl}}^2 \sim S \, \frac{N}{SR} \left(\frac{SR}{Nr_0} \right)^2 \\
&\sim \left(\frac{S}{r_0} \right)^2 \frac{R}{N} \,,
\end{aligned}
\tag{2.1}
$$

the correct scaling since $M \sim r_0^{D-3}/G_D$ and $S \sim r_0^{D-2}/G_D$. The leading term in the interaction energy is the Newtonian gravitational interaction between the transverse kinetic energies of the clusters (together with additional terms of the same order, as required by Galilean invariance). For $N \gg S$ one must take into account such interactions in which the clusters exchange longitudinal momentum. These parton exchange processes are essential for the restoration of locality in the longitudinal direction in the large N limit, but difficult to calculate in matrix theory. For the purposes of our order-of-magnitude estimate, we will approximate all such contributions to the interaction as having the same magnitude and phase as that coming from zero longitudinal momentum exchange. Thus the leading

interaction is roughly

$$E_{\text{pot}} \sim G_D \sum_{\delta p_-=0}^{N/S} \sum_{a,b=1}^{S} f(\delta p_-) \frac{(m_{\text{cl}} v_{\text{cl}}^2)_a (m_{\text{cl}} v_{\text{cl}}^2)_b}{R r_{ab}^{D-4}}$$
$$\sim G_D \frac{N}{S} S^2 \frac{S^2 R}{N^2 r_0^D} \tag{2.2}$$
$$\sim E_{kin} \frac{G_D S}{r_0^{D-2}} \; .$$

Thus the virial theorem is satisfied given the proper scaling of the entropy.

The matrix theory effective action is expected to contain an infinite series of terms of the form $(Nv^2)^{\ell+1}/r^{\ell(D-4)}$, as might be expected from the expansion of the Born-Infeld action for zerobranes [6,20,21] (see below). At the transition point $N = S$, all terms of this form are comparable in magnitude due to the uncertainty relation $v \sim R/r_0$ [12,13]. Since the system at $N \gg S$ is, apart from finite size effects, simply a boosting of the system at $N = S$, the same should hold true for the ensemble of clusters. For example, consider the spin-orbit term [22]. It contains a contribution which is the spin-orbit energy of a cluster, interacting with the kinetic energy of another cluster

$$V_{\text{spin-orb}} \sim G_D \sum_{\delta p_-} \sum_{a,b} f(\delta p_-) \left(\frac{(r^{[i} p^{j]}) \Sigma_{ij}}{m_{\text{cl}}} \right)_a (m_{\text{cl}} v_{\text{cl}}^2)_b \frac{1}{R r_{ab}^{D-2}} \; . \tag{2.3}$$

Evaluating this in the same way as (2.2), one finds that these two contributions to the potential are of the same order. It is trivial to see that the other spin-orbit terms required by Galilean invariance are comparable. In fact, replacing partons by clusters in the scaling analysis leads to the conclusion that all terms in the expansion of the Born-Infeld action are of comparable magnitude in the black hole. In reaching this conclusion, it is important to realize that all factors of $1/R$ in the effective action of an individual D0-brane are to be replaced by cluster longitudinal momenta m_{cl}, except for the one factor of $1/R$ which arises from averaging the spatial Green's function over the longitudinal direction.

This picture of the black hole as composed of (clusters of) partons interacting through effective forces due to the off-diagonal degrees of freedom of the matrices is not completely accurate. As mentioned in the introduction, there are 'membrane' modes that couple to the parton clusters whose time and energy scales make them relevant to the dynamics. Moreover, the virial theorem is a crucial ingredient of the argument; yet it actually says that the action of the fluctuations of off-diagonal modes (the 'potential' term) and the

diagonal modes (the uncertainty principle saturated 'kinetic term') are of the same order. Thus, the loop expansion which integrates them out is breaking down. Clearly the true state of affairs lies in a matrix wavefunction for the black hole where much of the matrix is excited in a rather complicated way. The characterization of the dynamics via interacting clusters of partons is at best an approximation that captures the rough bulk properties of the black hole state; it cannot possibly be expected to yield precise numerical coefficients.

The above picture of the microstates of a black hole as a collection of interacting parton clusters also explains much of the structure of (singly) charged black holes. The macroscopic properties of such holes are

$$
\begin{aligned}
M &\sim G_D^{-1} r_0^{D-3} (\text{ch}^2 \gamma + \tfrac{1}{D-3}) \\
Q &\sim G_D^{-1} r_0^{D-3} \text{sh}\gamma\text{ch}\gamma \\
S &\sim G_D^{-1} r_0^{D-2} \text{ch}\gamma \; .
\end{aligned}
\tag{2.4}
$$

As in the uncharged case treated in [11], the matrix Hamiltonian at small N describes black string states stretched across the longitudinal direction, and at sufficiently large N describes black holes localized in the longitudinal direction. For fixed values of the mass and charge, the transition between the two takes place at the value of longitudinal boost where the entropies are the same [23]. The quantities (2.4) in the boosted frame are of course unchanged, with $E_{LC} \sim Me^{-\alpha}$ and $P \sim Me^{\alpha}$; on the other hand, the black string has[3]

$$
\begin{aligned}
M &\sim G_D^{-1} R r_{\text{str}}^{D-4} (\text{ch}^2 \delta \text{ch}^2 \beta + \tfrac{1}{D-3}) \\
P &\sim G_D^{-1} R r_{\text{str}}^{D-4} \text{sh}\beta\text{ch}\beta\text{ch}^2\delta \\
Q &\sim G_D^{-1} R r_{\text{str}}^{D-4} \text{sh}\delta\text{ch}\delta\text{ch}\beta \\
S &\sim G_D^{-1} R r_{\text{str}}^{D-3} \text{ch}\delta\text{ch}\beta \; .
\end{aligned}
\tag{2.5}
$$

In contrast to the uncharged case, the matching of these quantities does not uniquely specify the parameters of the black string in terms of those of the black hole; further assumptions are required. Demanding that the metric coefficients match smoothly onto one another forces the horizon radii $r_0 = r_{\text{str}}$, boost rapidities $\alpha = \beta$, and charge parameters $\gamma = \delta$ all to be equal; the rapidity of the longitudinal boost at the transition is determined by $r_0 = Re^{\alpha}$, the same as in the uncharged case. Loosely speaking, for this value of the boost the charged black hole 'just fits inside the longitudinal box'.

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[3] We are grateful to H. Awata for collaborating in the calculations that established these relations.

It is important to note that the entropy at the transition is related to the longitudinal boost by $P = Sch\gamma/R$; in other words $N = Sch\gamma \equiv \hat{N}$ is greater than the number of 'bits of information' stored in the charged black hole. This is entirely reasonable, since as one increases the charge at fixed mass to move toward extremality, the longitudinal boost required to maintain the validity of IMF kinematics does not decrease, while the entropy of the hole does decrease. Thus even at the transition point $N = \hat{N}$, the hole is made up of clusters of partons, with $\hat{N}/S > 1$ partons per cluster.

The cluster hypothesis is also consistent with the entropy of charged black holes. The simplest way to obtain a nonextremal charged black hole is by boosting an uncharged black hole along an internal circle. Let us assume that there are S clusters each containing of order N/S partons, with transverse velocity $v_{\mathrm{cl}} \sim SR/Nr_0$ in the noncompact dimensions, and internal velocity $w_{\mathrm{cl}} \sim (SR/Nr_0)\mathrm{sh}\gamma$ to account for the charge. The cluster 'mass' is again $m_{\mathrm{cl}} \sim N/SR$. The entropy of the ensemble in the transverse rest frame of the clusters should be that of the uncharged black hole, $S \sim r_0^{D-2}/G_{D,\mathrm{rest}}$. However, the circle along which the partons are moving must have proper size $Lch\gamma$ in order that its size be L after boosting along that circle; thus $G_{D,\mathrm{rest}} = G_D/ch\gamma$ (where $G_D = \ell_{\mathrm{pl}}^9/L^d$ is the usual Newton constant in D dimensions; we take all the compactification radii to be $O(L)$ for simplicity). Thus the entropy is $S \sim r_0^{D-2}ch\gamma/G_D$, $c.f.$ (2.4). The kinetic energy of the clusters in the noncompact directions is

$$E_{\mathrm{kin}} \sim \tfrac{1}{2}\sum_{\mathrm{cl}} m_{\mathrm{cl}}v_{\mathrm{cl}}^2 \sim \frac{S^2R}{r_0^2 N} \ . \tag{2.6}$$

The virial theorem requires that the interaction energy of the clusters be of the same order as this transverse kinetic energy in the noncompact directions. Remarkably, following the logic of (2.2), one has

$$E_{\mathrm{pot}} \sim G_D \sum_{\delta p_- = 0}^{N/\hat{N}} \sum_{a,b=1}^{S} f(\delta p_-)\frac{(m_{\mathrm{cl}}v_{\mathrm{cl}}^2)_a (m_{\mathrm{cl}}v_{\mathrm{cl}}^2)_b}{Rr_{ab}^{D-4}}$$
$$\sim E_{\mathrm{kin}}\frac{S^2 G_D}{\hat{N}r_0^{D-2}} \ ; \tag{2.7}$$

as argued above, the last factor is of order one. Note that we have assumed that the maximum longitudinal momentum transfer between clusters is $N/\hat{N}$ rather than the larger quantity N/S, since the former is the ratio of the longitudinal box size to the size of the hole in the highly boosted frame and represents the effective resolution in the longitudinal

direction. Finally, the total energy is

$$E_{LC} \sim E_{\text{kin}}(1 + \text{sh}^2\gamma) + E_{\text{pot}}$$

$$\sim \frac{S^2 R}{r_0^2 N}[\text{ch}^2\gamma + a] , \tag{2.8}$$

where a is a number of order unity; we again find qualitative agreement with (2.4).

3. Probe dynamics

The action of a probe zerobrane (supergraviton) in a background gravitational field may be derived either from the constrained Hamiltonian dynamics of a massless particle, as in [3]; or via the massless limit of the Routhian, as in [20].[4] The latter route is somewhat simpler in the present context. Let $x^+ = \tau$ be the probe time coordinate; then the Routhian is simply $S = -p_-\dot{x}^-$, where $\dot{x}^-$ is determined from the light cone condition $ds^2 = 0$

$$0 = g_{--}(\dot{x}^-)^2 + 2g_{+-}\dot{x}^- + g_{++} + g_{ij}v^i v^j + 2g_{i-}\dot{x}^- v^i + 2g_{i+}v^i ; \tag{3.1}$$

here v^i is the probe transverse velocity. The probe action is thus (up to a total derivative)

$$S_{\text{pr}} = p_- \int d\tau \frac{1}{g_{--}}\left[(g_{+-} + g_{-i}v^i) - \left((g_{+-} + g_{-i}v^i)^2 - g_{--}(g_{++} + g_{ij}v^i v^j + 2g_{+i}v^i)\right)^{1/2}\right]. \tag{3.2}$$

The leading order large distance terms in the metric scale as

$$g_{\mu\nu} \sim \eta_{\mu\nu} + a_{\mu\nu}\left(\frac{r_0}{r}\right)^{D-3} . \tag{3.3}$$

Expanding (3.2) to this order gives

$$S = p_- \int \left[\tfrac{1}{2}v^2 + \tfrac{1}{2}\left(\frac{r_0}{r}\right)^{D-3}\left[a_{++} + a_{ij}v^i v^j + a_{+i}v^i - (a_{+-} + a_{-i}v^i)v^2 + \tfrac{1}{4}a_{--}v^4\right] + \ldots\right] . \tag{3.4}$$

The relation between the parameter r_0 and the ADM mass is

$$r_0^{D-3} = \frac{4\pi G_D M}{(D-2)\omega_{D-2}} , \tag{3.5}$$

where ω_{D-2} is the solid angle in D spacetime dimensions. Also we will need to sum over images along the periodically identified longitudinal direction. Let us denote

$$\rho^2 = r^2 - \tfrac{1}{4}(x^+ e^{-\alpha} + x^- e^{\alpha})^2 \tag{3.6}$$

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[4] We concentrate on the bosonic terms only.

as the distance along the transverse noncompact dimensions. We have included a boost of the source by a rapidity α in the longitudinal direction in order to facilitate passage to the infinite momentum frame. At long distances $\rho \gg Re^{\alpha}$ (where x^- has period $2\pi R$), the leading nontrivial term in the metric is just the smearing of (3.3) across x^-, leading to

$$\frac{1}{r^{D-3}} \longrightarrow \frac{e^{-\alpha}}{\pi R \rho^{D-4}} \int_{-\infty}^{\infty} dt (1+t^2)^{(3-D)/2} = \frac{e^{-\alpha}}{R\rho^{D-4}} \frac{(D-3)\omega_{D-2}}{\pi(D-4)\omega_{D-3}} \ . \tag{3.7}$$

All told, in the presence of a source which has been boosted in the compact longitudinal direction, one finds the following probe effective action

$$S_{\mathrm{pr}} = p_- \int d\tau \left[\tfrac{1}{2}v^2 + \frac{Ar_0^{D-3}e^{-\alpha}}{R\rho^{D-4}} \left(a_{++} + a_{ij}v^iv^j + a_{+i}v^i - (a_{+-} + a_{-i}v^i)v^2 + \tfrac{1}{4}a_{--}v^4 \right) + \dots \right], \tag{3.8}$$

where $A = \frac{(D-3)\omega_{D-2}}{2\pi(D-4)\omega_{D-3}}$.

Consider a metric of the asymptotic form

$$ds^2 = dx \cdot dx + \left(\frac{r_0}{r}\right)^{D-3} \sum_{\mu} c_{\mu}(dx^{\mu})^2 \ . \tag{3.9}$$

Boosting first by a rapidity γ along the x^9 direction, then by a large rapidity α along the x^{10} direction to pass to the infinite momentum frame, one finds

$$\begin{aligned}
ds^2 = dx^+ dx^- + dx_i^2 + \left(\frac{r_0}{r}\right)^{D-3} \Big[&\tfrac{1}{4}\big[(c_0+c_{10}) + (c_0+c_9)\mathrm{sh}^2\gamma\big](e^{-2\alpha}(dx^+)^2 + e^{2\alpha}(dx^-)^2) \\
&+ \tfrac{1}{2}\big[(c_0-c_{10}) + (c_0+c_9)\mathrm{sh}^2\gamma\big]dx^+dx^- \\
&+ \big[c_9 + (c_0+c_9)\mathrm{sh}^2\gamma\big]dx_9^2 + c_i dx_i^2 \\
&+ (c_0+c_9)\mathrm{sh}\gamma\,\mathrm{ch}\gamma\,dx_9(e^{-\alpha}dx^+ + e^{\alpha}dx^-)\Big] \ .
\end{aligned} \tag{3.10}$$

This is the asymptotic IMF metric of a charged black hole. Suppose the probe velocity in the x^9 direction is w_{pr}, and $\vec{v}_{\mathrm{pr}}$ in the noncompact directions. Then the probe effective action (3.8) is (after suitably averaging over the longitudinal direction)

$$\begin{aligned}
S_{\mathrm{pr}} = \tfrac{1}{2}p_- \int d\tau \Big[v_{\mathrm{pr}}^2 + w_{\mathrm{pr}}^2 + \left(\frac{\rho_0}{\rho}\right)^{D-4} \Big(&\tfrac{1}{4}[(c_0+c_{10}) + (c_0+c_9)\mathrm{sh}^2\gamma]e^{-2\alpha} \\
&+ \tfrac{1}{2}[(c_0+c_9)\mathrm{sh}\gamma\,\mathrm{ch}\gamma e^{-\alpha}]w_{\mathrm{pr}} \\
&+ [(c_9 + \tfrac{1}{2}c_0 - \tfrac{1}{2}c_{10}) + \tfrac{3}{2}(c_0+c_9)\mathrm{sh}^2\gamma]w_{\mathrm{pr}}^2 \\
&+ [(c_i + \tfrac{1}{2}c_0 - \tfrac{1}{2}c_{10}) + \tfrac{1}{2}(c_0+c_9)\mathrm{sh}^2\gamma]v_{\mathrm{pr},i}^2 \\
&- \tfrac{1}{2}[(c_0+c_9)\mathrm{sh}\gamma\,\mathrm{ch}\gamma e^{\alpha}]w_{\mathrm{pr}}(v_{\mathrm{pr}}^2 + w_{\mathrm{pr}}^2) \\
&+ \tfrac{1}{16}[(c_0+c_{10}) + (c_0+c_9)\mathrm{sh}^2\gamma]e^{2\alpha}(v_{\mathrm{pr}}^2 + w_{\mathrm{pr}}^2)^2 \Big) + \dots \Big],
\end{aligned} \tag{3.11}$$

with $\rho_0^{D-4} \sim r_0^{D-3} e^{-\alpha}/R$ due to (3.7).

To proceed further, we must choose a coordinate system; for illustrative purposes, let us work in Schwarzschild coordinates, where the unboosted metric is

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega_{D-2}^2 + \sum_{i=D}^{9} dx_i^2 , \qquad (3.12)$$

with $f = 1 - (r_0/r)^{D-3}$; in the asymptotic metric (3.9), $c_0 = c_r = 1$, while $c_9 = c_\Omega = 0$.

We would like to compare this expectation for the probe dynamics with the effective action of matrix theory

$$S_{\text{eff}} \sim \sum_a \frac{v_a^2}{R} + \frac{\ell_{\text{pl}}^9}{L^d} \sum_{a,b} \frac{(v_a - v_b)^4}{R^3 r^{D-4}} \qquad (3.13)$$

with N partons comprising the black hole and one set apart as the probe. The effective action, and therefore the effective metric, seen by the probe in matrix theory involves an average over the distribution of parton positions and velocities inside the black hole. Of course, a more precise treatment would keep the full matrix dynamics of the source, as in [24]; however, the abelianized approximation will suffice for our considerations.

The parton positions relative to the black hole center will only affect the subleading terms in the metric; on the other hand, due to the direction dependence of the velocity coupling in (3.13), the velocity distribution directly affects the leading term. For example, if the transverse velocity distribution of the partons in the black hole is isotropic, $\langle v^i v^j \rangle = \frac{1}{D-2} \delta^{ij} \langle v^2 \rangle$, then the leading term in the long distance metric (3.3) is in isotropic coordinates $a_{ij} \propto \delta_{ij}$; on the other hand, if the partons are all in an S-wave spatial wavefunction, then the angular component of their velocity vanishes at lowest order and one is in Schwarzschild-type coordinates $a_{rr} \neq 0$, $a_{\Omega\Omega} = 0$.

Let us assume that all the black hole parton clusters are in S-wave states in the noncompact directions, so that $c_\Omega = 0$; and that they have the average velocity w_{bh} along x_9. Consider the term linear in the probe internal velocity w_{pr}; it is

$$\frac{r_0^{D-3}}{R\rho^{D-4}} \text{sh}\gamma \text{ch}\gamma \, e^{-2\alpha}(p_{-,\text{pr}} w_{\text{pr}}) \sim \frac{G_D M e^{-\alpha}}{R\rho^{D-4}} \left(e^{-\alpha} \frac{\text{sh}\gamma}{\text{ch}\gamma} \right) (p_{-,\text{pr}} w_{\text{pr}}) \qquad (3.14)$$

up to coefficients of order one. On the other hand, the effective interaction between the probe internal velocity and the matrix black hole constituent clusters is

$$G_D \frac{m_{\text{cl}}(v_{\text{bh}}^2 + w_{\text{bh}}^2) w_{\text{bh}}}{R\rho^{D-4}} \left(\frac{N_{\text{pr}}}{R} w_{\text{pr}} \right) . \qquad (3.15)$$

Since $m_{\rm cl}(v_{\rm bh}^2 + w_{\rm bh}^2) \sim E_{LC}^{(\rm bh)} \sim Me^{-\alpha}$, comparing with (3.14) shows that

$$w_{\rm bh} \sim \frac{{\rm sh}\gamma}{{\rm ch}\gamma} e^{-\alpha} \ . \tag{3.16}$$

This is precisely the relation between boost velocity and rapidity, with the factor of $exp[-\alpha]$ arising from the further time dilation due to the orthogonal boost involved in passing to the infinite momentum frame. Note also that

$$v_{\rm bh} \sim \left(\frac{SR}{Nr_0}\right) \sim \left(\frac{M}{{\rm ch}\gamma P}\right) \sim \frac{e^{-\alpha}}{{\rm ch}\gamma} \ , \tag{3.17}$$

so that the total cluster kinetic energy is

$$(\#{\rm clusters}) \cdot m_{\rm cl}(v_{\rm bh}^2 + w_{\rm bh}^2) \sim S \cdot \frac{N}{SR} \cdot \frac{1 + {\rm sh}^2\gamma}{{\rm ch}\gamma^2} e^{-2\alpha} \sim \frac{M^2}{P} \ , \tag{3.18}$$

comparable to the total IMF energy of the black hole (whereas the interaction energy is only of order the kinetic energy of noncompact motion; see above).

Next consider the $v_{\rm pr}^2$ and $w_{\rm pr}^2$ terms in the probe effective action. In the one-loop effective action of matrix theory, these will again come from the relevant terms in the v^4/ρ^{D-4} interaction by averaging over the internal motions of the black hole; we find

$$\left\langle [(v_{\rm pr} - v_{\rm bh})^2 + (w_{\rm pr} - w_{\rm bh})^2]^2 \right\rangle \sim \ldots + 2w_{\rm pr}^2 \left\langle v_{\rm bh}^2 + 3w_{\rm bh}^2 \right\rangle + 2v_{\rm pr}^2 \left\langle w_{\rm bh}^2 + 3v_{\rm bh}^2 \right\rangle + \ldots \tag{3.19}$$

The form of the $v_{\rm pr}^2$ term depends more sensitively on assumptions about the motion of the clusters in the noncompact directions (in (3.19), we have assumed there is only radial motion in the noncompact directions). The corresponding terms in the matrix effective action to this order are

$$(\#{\rm clusters}) \frac{G_D}{R\rho^{D-4}} \left[m_{\rm cl} \left\langle v_{\rm bh}^2 + 3w_{\rm bh}^2 \right\rangle m_{\rm pr} w_{\rm pr}^2 + m_{\rm cl} \left\langle w_{\rm bh}^2 + 3v_{\rm bh}^2 \right\rangle m_{\rm pr} v_{\rho,{\rm pr}}^2 \right]$$

$$\sim S \frac{G_D}{R\rho^{D-4}} \left(\frac{N}{SR}\right) \frac{e^{-2\alpha}}{{\rm ch}^2\gamma} \left[m_{\rm pr} w_{\rm pr}^2 \left(1 + 3{\rm sh}^2\gamma\right) + m_{\rm pr} v_{\rho,{\rm pr}}^2 \left(3 + {\rm sh}^2\gamma\right) \right]$$

$$\sim \frac{r_0^{D-3} e^{-\alpha}}{R\rho^{D-4}} \left[m_{\rm pr} w_{\rm pr}^2 \left(1 + 3{\rm sh}^2\gamma\right) + m_{\rm pr} v_{\rho,{\rm pr}}^2 \left(3 + {\rm sh}^2\gamma\right) \right] \ . \tag{3.20}$$

Here we have used the approximation $r_0^{D-3} \sim G_D M/{\rm ch}^2\gamma$, valid up to coefficients of order one. We wish to compare to (3.11), with $c_\Omega = c_9 = c_{10} = 0$ to lowest order, and $c_0 = c_\rho = 1$ in the chosen coordinate system. The relevant terms in (3.11) are

$$\tfrac{1}{4} \frac{r_0^{D-3} e^{-\alpha}}{R\rho^{D-4}} \left[(1 + 3{\rm sh}^2\gamma) p_{-,{\rm pr}} w_{\rm pr}^2 + (3 + {\rm sh}^2\gamma) p_{-,{\rm pr}} v_{\rho,{\rm pr}}^2 \right] \ ; \tag{3.21}$$

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thus the cluster hypothesis reproduces the velocity-squared terms in the probe effective action up to coefficients of order one. In a similar fashion, one finds the remaining terms as well.

The leading terms in the probe effective action are admittedly a rather weak test of the composition of the black hole; any collection of objects of the same total mass and charge contained in a finite region will have roughly the same properties. It is the fact that the cluster picture reproduces the long-distance geometry, while at the same time explaining the entropy of both neutral and charged black holes, that gives us confidence in its validity.

An effect we have neglected in our analysis of the probe dynamics are the three-body and higher interactions among the matrix constituents; these have been discussed in [25]. In the present context, these interactions are responsible for the response of the probe to the gravitational binding energy of the source parton clusters. Since these terms are of comparable magnitude to the probe's response to the cluster kinetic energies, they are expected to make at most a correction of order one to the coefficients in the effective action (3.11).

4. Discussion

The picture of the black hole in matrix theory as a resonant bound state of clusters, many of whose properties resemble those of threshold bound states, fits rather well with the overall view of black hole thermodynamics in string theory [15]. Near-extremal black holes in string theory can often be regarded as a gas of massless excitations [16,26,27]. For instance, in the case of near-extremal three- and four-charge black holes in five and four dimensions, respectively, precise numerical understanding has been achieved by representing the black hole microstates in terms of waves along the mutual intersection locus of a collection of branes [15,26,30]. The massless gas is the representation of the near-horizon physics as seen by an asymptotic observer who probes the black hole at long wavelengths. The description is confined to physics outside the horizon. At no point does the infalling matter appear to lose contact with the asymptotic observer, nor is there a point in the description of a probe's evolution that might conceivably represent it hitting a singularity of the background spacetime. In string theory limits, the corresponding D-brane/string gas describes the degrees of freedom on a 'stretched horizon' where infalling string matter is thermalized and reradiated as Hawking quanta. Semiclassical D-brane calculations are

performed in a background Minkowskian geometry, so that temporal and spatial intervals are those that would be measured by an asymptotic observer. The effects of spacetime geometry are the residue of fluctuations of the D-brane and string degrees of freedom [1,2,30]. Thus, the temperature of the D-brane gas is approximately the Hawking temperature rather than some blue-shifted 'local temperature' that might be experienced by stationary observers very near the horizon (including the gas quanta themselves).

The collection of zerobrane clusters in the above description of highly boosted black holes is rather similar. The black hole thermodynamics is described by a fluid, whose properties are neither wholly zerobrane nor wholly membrane, but rather some of each. The temperature of the transverse virial motion of the correlation domains is approximately the (longitudinally boosted) Hawking temperature. The size of the system is the thermal wavelength (the horizon radius), which is the thickness of the stretched horizon redshifted to infinity. Hawking radiation is, roughly speaking, the 'solar wind' of this zerobrane/membrane (i.e. matrix) 'star'. One might expect that, as an escaping Hawking quantum climbs out of the gravitational well, it experiences some redshift in its wavelength. This cannot be more than order one if our picture is to be self-consistent. Thus the zerobrane cluster fluid would appear to be a description of the horizon physics as measured by clocks and meter sticks at infinity. Infalling matter would not appear to fall behind a horizon or encounter a singularity. One difference is that the description would not appear to be limited to low energies, as may be the case in the D-brane gas [30].

Another major distinction between our picture of the matrix black hole and the D-brane gas picture of near-extremal black holes, is that the latter has a macroscopically 'rigid' backbone of branes (those bound together to form the extremal configurations); the nonextremal excitations are then draped on this scaffolding. This allows one to find 'dilute gas' regimes where the density of nonextremal excitations is small, and their interactions weak. In the generic, highly nonextremal situation, there is no dilute gas limit; the kinetic and interaction energies of the constituents are of the same order.

The following picture of the state space of matrix theory at fixed, large N emerges from our study. The ground state is the threshold bound state graviton of momentum $P = N/R$, with 256 polarization states and vanishing light cone energy $E_{LC} = 0$. To achieve this, the spin and orbital wavefunctions of the constituent partons must be highly correlated; the spins must behave antiferromagnetically, and the zero-point motions of bosonic and fermionic degrees of freedom must delicately balance to zero. In the language we have

been using to describe the black hole, we would say that the ground state consists of a single correlated domain or cluster of partons. According to the relation between entropy and area, a low-energy observer would assign Planckian dimensions to the ground state supergraviton (for *any N*).[5] As we pump energy into the system, it becomes increasingly disordered. Instead of being correlated across its entire volume, there will be separate domains which are individually ordered much as in the ground state, with little correlation between domains. As with a liquid, there is no permutation symmetry among correlation domains, since each lives in a different environment (of off-diagonal modes). Ascribing a finite number of states to each domain (as in the case of the single domain of the ground state), the number of domains should be of order the entropy. The size of these resonant bound states is governed by the ability of the constituent domains to resolve one another via their interactions: r_0 in the transverse directions due to uncertainty principle; and $RS/N \sim e^{-\alpha} r_0$ in longitudinal direction, again due to the Fourier resolution of the clusters.

One might wonder, what distinguishes this picture from a collection of interacting wavepackets of gravitons in general relativity? For instance, $S \sim r_0^{D-2}/G_D$ gravitons of wavelength $\sim r_0$ would have roughly the right kinetic energy, and would satisfy the virial theorem if the static gravitational interaction were used. The difference is that the graviton gas in general relativity is at its Schwarzschild radius, where it is unstable to collapse toward shorter wavelengths (as viewed from infinity); on the other hand, in matrix theory the clusters are stable at the scale r_0 because gravitational forces turn off at that scale – gravity comes from integrating out 'membrane' degrees of freedom, an approximation which breaks down at this point. Including these degrees of freedom in the dynamics stabilizes the system. A second crucial distinction is that the new degrees of freedom 'distinguish' the clusters, whereas gravitons always have a permutation symmetry; this allows the system to have an enormous entropy.

We have been describing the matrix black hole as a collection of gravitons in a diffuse, membrane-like background. Reversing the background and the foreground, one might also

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[5] It is not clear what physical significance to assign to the growth in the size of the parton cloud in a graviton with N due to zero-point motions; it is their average position that matters, as in the membrane example given in section 4 of [11], and in the mean field analysis above. It has not been necessary in our analysis to utilize any sort of 'holographic spreading' of an object with boost [28]. The system seems more governed by standard quantum mechanics, and by the duality between membranes and gravitons, than by some sort of 'holographic principle'.

visualize the state as a 'Hagedorn phase' of the membrane, as was proposed in the second of [3] in the context of six-dimensional matrix black strings. This connects the matrix black hole to the weak-coupling Hagedorn strings which arise through the string/black hole correspondence principle [19]. The Hagedorn string has zero effective tension, so there is no communication between different regions of the string, just as there is effectively no communication between domains of the Hagedorn membrane that could establish a permutation symmetry among the domains. In the correspondence principle, the Hagedorn string arises when the spacetime curvature expected from general relativity is of order the string scale. The surprising feature of the Hagedorn membrane is that it does not need a curvature of order the Planck scale to make its appearance; the fingering instability allows the membrane to extend its tendrils to the weak-curvature region at the Schwarzschild radius at little cost in energy.

In order to complete the picture of black hole dynamics in M-theory, it is important to recover the description of the evolution experienced by freely falling observers passing through the classical event horizon. The horizon degrees of freedom implicitly contain this information, spread throughout the full matrix wavefunction of the matrix black hole in subtle correlations. Along the lines discussed in [3], one would like to carry out the matrix transformation that isolates the probe dynamics from the geometrical background by carrying out the sequence of boosts that keeps it in its proper rest frame. A coherent macroscopic object such as the spinning membrane [11,29] is a good candidate for a probe – its classical rotation acting as a proper clock, its radius a proper measuring rod.

Since the boost between the proper rest frame of the infalling probe and that of asymptotic observers becomes infinite at the horizon, it is not entirely obvious that the finite N matrix theory will allow an accurate description of classical infall. However, in the classical limit, the probe is kept at a fixed size relative to the black hole, while the Planck length is taken to zero. This limit forces $N, R \rightarrow \infty$. The asymptotic observer sees the clock's motion freeze as it approaches the classical horizon. The proper motion has the clock execute several more ticks before its obliteration on the 'singularity'. It would be very interesting if one could extract this classical clock variable from the diffusion of the probe across the full matrix wavefunction of the resulting black hole, and thereby reconstruct the interior geometry from the degrees of freedom already present in the matrix description. As we have argued before [3], this may be the ultimate meaning of black hole complementarity: Degrees of freedom that describe supergravity outside the black hole do not commute with

the membrane-like degrees of freedom into which the probe wavefunction diffuses as it penetrates the black hole wavefunction. In this regard, it is interesting that at the black hole scale r_0, the off-diagonal matrix elements appear to be new degrees of freedom not present in the low-energy description of supergravity (where low-energy is as measured by asymptotic observers). It is these new degrees of freedom that transform our notion of causality in a theory of extended objects [31,32]. The effective notion of causal structure is induced from the behavior of massless probes. Signals propagate differently in the matrix black hole; zerobranes interact strongly with membrane-like degrees of freedom, and there may be no localized operational definition of causal structure.[6] There is no separation of ingoing and outgoing null rays, as one might have expected in weakly perturbed general relativity.

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[6] Alternatively, one may say that in matrix theory the underlying causal structure is that of Minkowski space. There is an apparent causal structure induced by matter fluctuations which can cause complicated effects in signal propagation (*c.f.* [33], section 7.11), but no acausality.

References

[1] M. Douglas, D. Kabat, P. Pouliot, and S. Shenker, hep-th/9608024; Nucl. Phys. **B485** (1997) 85.

[2] M.R. Douglas, J. Polchinski and A. Strominger, hep-th/9703031.

[3] M. Li, E. Martinec, hep-th/9703211; hep-th/9704134; hep-th/9709114.

[4] R. Dijkgraaf, E. Verlinde, and H. Verlinde, hep-th/9704018.

[5] J. Maldacena, hep-th/9705053; hep-th/9709099.

[6] I. Chepelev and A. Tseytlin, hep-th/9709087.

[7] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, hep-th/9610043; L. Susskind, hep-th/9704080.

[8] T. Banks, W. Fischler, I. Klebanov and L. Susskind, hep-th/9709091.

[9] I. Klebanov and L. Susskind, hep-th/9709108.

[10] E. Halyo, hep-th/9709225.

[11] G. Horowitz and E. Martinec, hep-th/9710217.

[12] M. Li, hep-th/9710226.

[13] T. Banks, W. Fischler, I. Klebanov and L. Susskind, hep-th/9711005.

[14] B. de Wit, J. Hoppe, and H. Nicolai, Nucl. Phys. **B305** (1988) 545.

[15] J. Maldacena, hep-th/9607235, hep-th/9705078; G. Horowitz, contribution to the Symposium on Black Holes and Relativistic Stars (dedicated to the memory of S. Chandrasekhar), Chicago, IL, 14-15 Dec 1996, hep-th/9704072; A. Peet, hep-th/9712253.

[16] A. Strominger and C. Vafa, hep-th/9601029, Phys. Lett. **379B** (1996) 99; C. Callan and J. Maldacena, hep-th/9602043, Nucl. Phys. **B472** (1996) 591; G. Horowitz and A. Strominger, hep-th/9602051, Phys. Rev. Lett. **77** (1996) 2368.

[17] J. Breckenridge, R. Myers, A. Peet and C. Vafa, hep-th/9602065, Phys. Lett. **391B** (1997) 93; J. Breckenridge, D. Lowe, R. Myers, A. Peet, A. Strominger and C. Vafa, hep-th/9603078, Phys. Lett. **381B** (1996) 423.

[18] L. Motl, hep-th/9701025; T. Banks and N. Seiberg, hep-th/9702187; R. Dijkgraaf, E. Verlinde and H. Verlinde, hep-th/9703030.

[19] G. Horowitz and J. Polchinski, Phys. Rev. **D56** (1997) 2180; hep-th/9707170.

[20] K. Becker, M. Becker, J. Polchinski and A. Tseytlin, hep-th/9706072.

[21] H. Liu and A. Tseytlin, hep-th/9712063.

[22] J. Harvey, hep-th/9706039; P. Kraus, hep-th/9709199.

[23] R. Gregory and R. Laflamme, hep-th/9301052; Phys. Rev. Lett. **70** (1993) 2837.

[24] D. Kabat and W. Taylor, hep-th/9711078; hep-th/9712185.

[25] M. Dine and A. Rajaraman, hep-th/9710174.

[26] C. Callan and J. Maldacena, hep-th/9602051, Nucl. Phys. **B472** (1996) 591; S. Das and S. Mathur, hep-th/9606185, Nucl. Phys. B478 (1996) 561; A. Dhar, G. Mandal, and S. Wadia, hep-th/9605234, Phys. Lett. **388B** (1996) 51.

[27] S. F. Hassan, S. R. Wadia, hep-th/9703163, Phys. Lett. **402B** (1997) 43; hep-th/9712213.

[28] L. Susskind, hep-th/9307168, Phys. Rev. Lett. **71** (1993) 2367; hep-th/9308139, Phys. Rev. **D49** (1994) 6606.

[29] S.-J. Rey, hep-th/9711081.

[30] J. Maldacena and A. Strominger, hep-th/9609026, Phys. Rev. **D55** (1997) 861.

[31] E. Martinec, hep-th/9304037, Class. Quant. Grav. **10** (1993) L187; hep-th/9311129; D. Lowe, hep-th/9312107, Phys. Lett. **326B** (1994) 223.

[32] D. Lowe, J. Polchinski, L. Susskind, L. Thorlacius, and J. Uglum, hep-th/9506138, Phys. Rev. **D52** (1995) 6997.

[33] J.D. Jackson, *Classical Electrodynamics*, 2nd ed.; J. Wiley and sons (1975).