Comments on Black Holes in Matrix Theory

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The recent suggestion that the entropy of Schwarzschild black holes can be computed in matrix theory using near-extremal D-brane thermodynamics is examined. It is found that the regime in which this approach is valid actually describes black strings stretched across the longitudinal direction, near the transition where black strings become unstable to the formation of black holes. It is argued that the appropriate dynamics on the other (black hole) side of the transition is that of the zero modes of the corresponding super Yang-Mills theory. A suggestive mean field theory argument is given for the entropy of black holes in all dimensions. Consequences of the analysis for matrix theory and the holographic principle are discussed.

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1. Introduction

String theory has recently provided a statistical description of many aspects of black hole thermodynamics (for reviews, see [1]). D-branes [2] have enabled a precise computation of the spectrum and interactions of a wide variety of near-extremal charged [3] and rotating [4] holes. In addition, the black hole correspondence principle [5] gives a general relation (though not as precise) between the entropy of essentially any black hole and that of weakly coupled strings and D-branes. However, in all cases, the quantum states are described in weakly coupled string theory, and the connection to large black holes is obtained by increasing the string coupling and taking a low energy semiclassical limit. In order to obtain a description of the quantum states of a black hole directly in the black hole regime one presumably needs a nonperturbative formulation of the theory. Matrix theory [6] purports to be such a formulation in the discrete light cone gauge [7]. In its eleven dimensional incarnation, the quantum mechanics of N D-zerobranes with coupling R appears to reproduce eleven dimensional supergravity[1], with one direction compactified on a circle of radius R and the total momentum $P = N/R$. We will refer to N as the number of matrix 'partons'. Compactifying further dimensions leads to a super Yang-Mills (SYM) theory. The SYM hamiltonian yields the invariant mass of the system via $H = M^2 R/N$. This prescription can be motivated [10,11] via a chain of dualities. Given that the matrix theory prescription (in the dimensions where it is known) is so closely tied to physics of D-branes in the large charge limit, it is not surprising that the D-brane results on black holes can be carried over quite successfully [12,13], although the reinterpretation of the calculations from this perspective is quite illuminating. In particular, one is led to the idea [13] that the statistical mechanics of (generalized) SYM theory could reproduce black hole thermodynamics.

In two recent interesting papers, Banks, Fischler, Klebanov, and Susskind [14,15] have argued that the entropy of Schwarzschild black holes can be computed from the SYM theory of near-extremal branes, at the special point where the number of matrix partons N equals the entropy S. There are three main claims:

1) To fit a large black hole of size $r_0 \gg R$ into the longitudinal box, one must boost it so that its longitudinal size is contracted:

$$R > \Delta X_{BH} \sim \frac{MR}{N} r_0 \qquad (1.1)$$

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[1] Recent calculations [8,9] indicate that there are subtleties in establishing this connection.

2) Therefore $N > N_{min} \sim Mr_0 \sim S$; $N \sim S$ is the minimal number of matrix constituents required to describe the states of black holes. At this lower bound, the black hole 'just fits inside the box'; moreover, this value is 'optimal' in the sense of the renormalization group, in that increasing N introduces a needlessly large number of degrees of freedom, most of which are in their ground state.

3) One can apply SYM statistical mechanics to calculate the thermodynamic properties of the system at this threshold. For $D = 8$ it was argued in [14] that the threshold $N \sim S$ was just on the borderline of validity of this approach, since the effective temperature was so low that the typical wavelength was comparable to the size of the system (in the appropriate holonomy sector corresponding to multiply wrapped branes).

A closer inspection yields a number of puzzles. First of all, we will show that black holes do not contract longitudinally when boosted. If they did, it is likely that there would be a problem with black hole thermodynamics. This is because the transverse size is presumably invariant, so a Lorentz contraction would cause the horizon area to decrease. But this area is related to the black hole entropy which is a physical quantity and should not change under the boosts. We will see that the horizon remains fixed and spherical with radius r_0 because the boost is effectively undone by the infinite gravitational redshift there. How then can a large black hole fit inside the longitudinal box? Secondly, the gluons and other excitations being counted in the SYM thermodynamics carry field momentum, and in the matrix dictionary (c.f. [16]), the field momentum is

$$\mathcal{P}_i = \int T_{0i} \propto \frac{1}{\Sigma_i} = \frac{RL_i}{\ell_{\mathrm{pl}}^3} \tag{1.2}$$

where L_i are the lengths of the compact dimensions, Σ_i are the lengths of the dual torus, and ℓ_{pl} is the eleven dimensional Planck length. So these SYM momentum modes on the dual torus represent longitudinally stretched membranes in the original spacetime, which are translationally invariant. They cannot represent a localized object like a black hole.

We resolve these puzzles below. While a boosted black hole does not contract, it does expand the geometry near the horizon. In section 2 below, we show that this effect is such that, precisely at $N = S$, the geometry expands to the extent that the longitudinal box size *at the horizon* is precisely r_0. Thus it is not the black hole that shrinks to fit in the box, rather the box which expands to accommodate the black hole! Moreover, it is clear that when the black hole fills the box, it is on the verge of becoming a *black string* stretched

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across the box. Indeed, for zero momentum, it is known that a black string becomes unstable when R exceeds its Schwarzschild radius [17]. We give an entropic argument that the same phenomenon happens for the boosted black holes and black strings. The analysis of [14,15] actually describes black strings, close to the transition point; we will see that this corresponds to N slightly smaller than S. As N is increased, the temperature of the SYM ensemble decreases, and for $N > S$ the system is frozen into the dynamics on the space of zero modes. Indeed, these zero modes on the dual torus represent objects which are not longitudinally stretched in the original spacetime, and are therefore the appropriate degrees of freedom to describe black holes rather than black strings.

In section 3, we reconsider the transition point $N \sim S$, approaching it from slightly larger values of N where zero mode quantum mechanics should prevail. A simple mean field theory analysis in the spirit of [6] yields the black hole size r_0 and entropy S as a function of the mass M, *uniformly for any dimension D*.

This result leads us in section 4 to a reexamination of the 'holographic principle' [19]. In its mildest form (which we will call the 'weak holographic principle'), this principle states that the dynamics of the theory depends only on data defined on a $D-2$ dimensional spatial surface. This may well be true in the matrix model, given the intricate relation between the dynamics of the transverse and longitudinal degrees of freedom embodied in the matrices themselves. The more virulent form of the conjecture (which we shall call the 'strong holographic principle') asserts that the physical size of objects increases with boosting. This idea is motivated by Bekenstein's proposal that objects should respect a bound of one bit of information per Planck area, and was in fact one of the prime motivations for matrix theory. It is not a necessary consequence of the weak holographic principle, nor is it necessary in order to account for black hole entropy in matrix theory. We construct examples of objects in matrix theory – ensembles of discretized membranes, to be specific – which scale canonically with longitudinal boosts, and therefore appear to provide a counterexample to the claims of [6].

We conclude in section 5 with some speculations about how our results extend to the regime $N \gg S$, and discuss some general consequences of our analysis for matrix theory and black holes. Since our goal is to understand how matrix theory reproduces the familiar scaling of black hole entropy and size with mass, we will often ignore numerical factors of order one, such as the solid angle of spheres, *etc.*

2. Boosting black holes and black strings

Let us begin by considering a D-dimensional spacetime with one direction compactified on a circle of radius R at infinity. A Schwarzschild black hole in this spacetime corresponds to an infinite periodic chain of D-dimensional black holes in the original uncompactified space, and is not translationally invariant along the circle. A black string is the product of a circle and a $(D-1)$-dimensional Schwarzschild black hole. It is easy to show that for equal masses, the black hole has greater entropy than the black string whenever R is greater than the Schwarzschild radius of the black hole r_0. In fact, the black string is known to be unstable in this regime [17]. For $R < r_0$, the chain of black holes becomes indistinguishable from the black string, which is now stable.

So far we have assumed that the momentum is zero. We now ask how things change when we apply a boost along the circle[2]. For the black string, one is boosting along a symmetry direction, so the metric is the same up to the usual coordinate substitution $d\hat{t} \pm d\hat{x}_{D-1} = e^{\pm\beta}(dt \pm dx_{D-1})$. The relevant piece of the metric transforms as

$$-[1-(\rho_0/\rho)^{D-4}]d\hat{t}^2+d\hat{x}_{D-1}^2 = -dt^2+dx_{D-1}^2+(\rho_0/\rho)^{D-4}[\cosh\beta \; dt+\sinh\beta \; dx_{D-1}]^2 \quad (2.1)$$

with the transverse part of the metric unchanged. If we periodically identify the coordinate x_{D-1}, the proper distance along this circle grows from $2\pi R$ at infinity to $2\pi R\cosh\beta$ at the horizon $\rho = \rho_0$. In a sense, the momentum exerts a 'pressure' on the geometry causing it to expand near the horizon.

Now consider the D-dimensional black hole. If this were an ordinary object, one might expect it to Lorentz contract when boosted. But as mentioned in the introduction, this would lead to problems with black hole thermodynamics. Fortunately, black holes do not Lorentz contract. This follows from the fact that every cross section of the event horizon has the same area. So in every reference frame, the area of the horizon is the same. To see an explicit example, consider the four dimensional Schwarzschild metric in isotropic coordinates

$$ds^2 = -\left(\frac{r-r_0}{r+r_0}\right)^2 d\hat{t}^2 + \left(1+\frac{r_0}{r}\right)^4 [d\hat{z}^2 + d\rho^2 + \rho^2 d\phi^2] \quad (2.2)$$

where $r^2 = \hat{z}^2 + \rho^2$. These coordinates cover both asymptotically flat regions, with the horizon at $r = r_0$. We now want to apply a boost along the $\hat{z}$ direction. Since this is

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[2] The compactification is different after the boost. Strictly speaking one applies the boost to the uncompactified spacetime, and then identifies points along the new spatial direction.

not a symmetry direction, there is some ambiguity about how one defines the boost in the interior of the spacetime. A natural choice is to simply set $d\hat{t} \pm d\hat{z} = e^{\pm \alpha}(dt \pm dz)$. The new horizon geometry defined by $t = \text{constant}$, $r = r_0$, is the surface in

$$ds^2 = 16[\cosh^2 \alpha dz^2 + d\rho^2 + \rho^2 d\phi^2] \tag{2.3}$$

given by

$$r_0^2 = r^2 = (\cosh\alpha \ z + \sinh\alpha \ t)^2 + \rho^2 \tag{2.4}$$

Differentiating this equation (with t constant) yields

$$(\cosh\alpha dz)^2 = \frac{\rho^2 d\rho^2}{r_0^2 - \rho^2} \tag{2.5}$$

so the induced metric on the horizon is

$$ds^2 = 16 \left(\frac{r_0^2 d\rho^2}{r_0^2 - \rho^2} + \rho^2 d\phi^2 \right) \tag{2.6}$$

which is completely independent of the boost and describes a round sphere. So the event horizon does not Lorentz contract. This is different from boosting a sphere in flat space-time, since then the $\cosh^2 \alpha$ factor in (2.3) is absent. (It combines with a $-\sinh^2 \alpha$ factor coming from $g_{\hat{t}\hat{t}}$. It is the infinite gravitational redshift at the horizon which removes this cancelling term.)

Having established that the black hole does not Lorentz contract, we now ask if the proper length of the circle expands near the horizon. It is clear from (2.3) that the answer is yes. In fact, at $t = 0$ the horizon is given by $r_0^2 = \cosh^2 \alpha \ z^2 + \rho^2$, so in terms of the coordinates (z, ρ), there is an apparent Lorentz contraction. It is the expansion of the metric near $r = r_0$ which ensures that the horizon remains spherical. Thus the net effect is similar to a real Lorentz contraction: One can fit large black holes into small compactified spaces by boosting.

We now turn to eleven-dimensional supergravity compactified to $D = 11 - d$ dimensions on a torus T^d of volume L^d. Consider a black hole of mass $M = \ell_{\text{pl}}^{-9} L^d r_0^{D-3}$ which is given a large boost in one of the D directions which is then compactified on a circle of radius R. (The factor $\ell_{\text{pl}}^9 / L^d \equiv G_D$ is just the D-dimensional Newton constant.) The boosted black hole has energy and momentum

$$\begin{aligned} E_{hole} &= M \cosh\alpha = \ell_{\text{pl}}^{-9} L^d r_0^{D-3} \cosh\alpha \\ P_{hole} &= M \sinh\alpha = \ell_{\text{pl}}^{-9} L^d r_0^{D-3} \sinh\alpha \end{aligned} \tag{2.7}$$

along the longitudinal direction. The boost does not change the number of internal states of the hole, which remains

$$S_{hole} \sim \ell_{\text{pl}}^{-9} L^d r_0^{D-2} \ . \tag{2.8}$$

Now choose the boost so that $P = S/R$. This fixes $e^\alpha \sim r_0/R$, and since the asymptotic longitudinal box size is R, at the horizon the size is r_0 – indeed the box just expands to accommodate the black hole for this magic value of the boost.

Now let us compare this with the behavior of a boosted black string stretched across the longitudinal direction. Its energy, momentum, and entropy are [18]

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\begin{aligned}
E_{string} &\sim \ell_{\text{pl}}^{-9} L^d R \rho_0^{D-4} [a + \cosh 2\beta] \\
P_{string} &\sim \ell_{\text{pl}}^{-9} L^d R \rho_0^{D-4} \sinh 2\beta \\
S_{string} &\sim \ell_{\text{pl}}^{-9} L^d R \rho_0^{D-3} \cosh \beta \ ,
\end{aligned}
\tag{2.9}
$$

where a is a constant of order one. The black string will be stable provided the length of the horizon is less than its Schwarzschild radius ρ_0. Since this length increases by e^β (for large β) as we boost, the instability begins when $e^\beta \sim \rho_0/R$. This implies $P \sim S/R$. So we see from both black hole and black string considerations, that the condition $N = S$ (where $P = N/R$) marks the transition between these two configurations. Clearly, when $P = S/R$, if the black hole and black string have the same momentum, then they have the same entropy as well.

One might be puzzled by the different scaling of the energy and momentum in (2.7) and (2.9) under a boost. The difference arises due to what is implicitly held fixed. In (2.7), one starts with an infinite chain of black holes of mass M in the spacetime with the longitudinal direction uncompactified. If the initial separation (asymptotically) is R, after the boost, the energy of each black hole is $M\cosh\alpha$ and the new separation will be Lorentz contracted $R/\cosh\alpha$. If we insist that the separation after the boost is R, there are two options. One can periodically identify after $O(\cosh\alpha)$ black holes are included. This produces another factor of $\cosh\alpha$ in (2.7) and (2.8) so that the energy, momentum, and entropy now scale like the black string (2.9). Alternatively, one can increase the initial separation between the black holes to $R\cosh\alpha$, which insures that the separation after the boost will be R. This was implicitly assumed in (2.7) and (2.8).

In terms of matrix theory, the natural variables to hold fixed are R and the invariant mass M.[3] The light cone energy is $E_{LC} = E - P$ and $P = N/R$, so $M^2 = E_{LC}N/R$.

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[3] We assume that R is the radius of a spacelike circle, as in [6]. However, since we are always

Eliminating the parameters r_0, α, ρ_0, and β in favor of $P = N/R$ and $E_{LC} = E - P$ yields

$$S_{hole} \sim (\ell_{\rm pl}^9/L^d)^{\frac{1}{D-3}} M^{\frac{D-2}{D-3}}$$

$$S_{string} \sim N^{1/2} \, (\ell_{\rm pl}^9/L^d)^{\frac{1}{D-4}} \left(\frac{E_{LC}}{R}\right)^{\frac{D-2}{2(D-4)}} \tag{2.10}$$

$$\sim N^{-\frac{1}{D-4}} \, (\ell_{\rm pl}^9/L^d)^{\frac{1}{D-4}} M^{\frac{D-2}{D-4}} \, .$$

If $S_{hole} = S_{string}$ for some choice of mass M and boost N, then clearly increasing N causes the black hole to have greater entropy and decreasing N causes the black string to have greater entropy.

This behavior is perfectly compatible with our expectations from SYM statistical mechanics. There, for fixed R and M, $E_{LC} = M^2 R/N$ decreases as $1/N$, *i.e.* the ensemble becomes colder with increasing N. At high temperature, corresponding to small N, the SYM theory is roughly a gas of interacting supergluons. As mentioned in the introduction, these modes correspond to longitudinal membranes (and fivebranes if $d \geq 4$) in the original spacetime, and thus should describe states of a black string. This is the situation described for instance in [13]. As we increase N, this description remains valid until the temperature of the gas drops to the point that the thermal wavelength is comparable to the effective size of the dual torus on which the SYM is defined. Then the gluonic degrees of freedom freeze out, leaving quantum mechanics on the space of zero modes. The statistical mechanics is that of this quantum mechanical system (including the various global fluxes on the internal torus). These states describe objects which are not longitudinally wrapped in the original spacetime, and which can thus be localized in the longitudinal direction. This system describes black hole states. From the above analysis, we see that the transition occurs when $N \sim S$.

The calculations of [14,15] attempt to explain the black hole entropy by approaching the black hole – black string transition from the 'wrong' side, using the equality of the density of states at the transition to infer the entropy on the other side. The procedure is similar in spirit to the black hole correspondence principle, in which one infers the black hole spectrum from the string spectrum by matching their densities of states at the transition from one to the other, and using the known string spectrum. Here however, one is on much shakier ground; much less is known about how to compute the SYM entropies

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in the regime of very large boosts, the distinction (in this Lorentz frame) between spacelike and lightlike compactification in the longitudinal direction is not expected to be important.

from first principles (although the case of 3+1 SYM corresponding to $D = 8$ is on a somewhat firmer footing).

3. A direct approach to black hole entropy

One might hope to arrive at the black hole entropy more directly, by an analysis of the zero-mode quantum mechanics that begins to dominate just above the transition. Indeed, a mean field analysis [6] appears to capture the essential physics. When the matrix partons are sufficiently far apart, the 'fast' off-diagonal matrix element dynamics can be integrated out. Treating the partons in mean-field approximation, the one-loop effective Lagrangian for the zero modes (of matrix theory compactified on T^d, with $D = 11 - d$) has the structure

$$\mathcal{L}_{\text{eff}} = \frac{Nv^2}{R} + \frac{N^2 \ell_{\text{pl}}^9 v^4}{R^3 L^d\, r^{D-4}} \ . \tag{3.1}$$

The parton mass is $1/R$ due to the origin of matrix theory in ten-dimensional D-zerobrane physics. Recall that the factor $\ell_{\text{pl}}^9/L^d \equiv G_D$ is just the D-dimensional Newton constant, so the second term can be interpreted as the gravitational self-energy of the partons due to their relative motion.

The dynamics determined by this Lagrangian is rather complicated, but mean field theory arguments indicate that there are solutions where the partons remain in a bounded region of space of radius r_0 for an extended period of time. The virial theorem then tells us that the two terms in the effective Lagrangian are of the same order. We will assume that the partons saturate the uncertainty bound,

$$\frac{r_0 v}{R} \sim 1 \ . \tag{3.2}$$

(Since (3.1) is derived under the assumption of nonrelativistic motion, $v \ll 1$, the size of the bound state must be much larger than the longitudinal box size as measured at infinity.) These assumptions determine a relation between N and the size of the bound state:

$$N \sim (\ell_{\text{pl}}^{-9} L^d) r_0^{D-2} \ . \tag{3.3}$$

The typical energy scale is then

$$E_{LC} = \frac{M^2 R}{N} \sim (\ell_{\text{pl}}^{-9} L^d R) r_0^{D-4} \ , \tag{3.4}$$

leading to a typical size of the bound state in terms of the mass:

$$M \sim (\ell_{\mathrm{pl}}^{-9} L^d) r_0^{D-3} \ . \tag{3.5}$$

Since $\ell_{\mathrm{pl}}^{-9} L^d = 1/G_D$, we recognize the relation between the mass and Schwarzschild radius of a black hole! Now consider the mass-entropy relation for D-dimensional black holes; using (3.3), (3.5), we have

$$S \sim (\ell_{\mathrm{pl}}^{-9} L^d)^{\frac{1}{D-3}} M^{\frac{D-2}{D-3}} \sim \ell_{\mathrm{pl}}^{-9} L^d r_0^{D-2} \sim N \ . \tag{3.6}$$

This is already clear from (3.3) – the number of partons is the surface area of the bound state in Planck units. In other words, the black hole entropy is the number of partons up to coefficients of order unity. One can argue that the entropy in the partons is also of order N if they are effectively distinguishable[4], since each parton has several polarization states. Notice that this argument works uniformly in all dimensions D, and does not require independent conjectures about the SYM thermodynamics. The basic assumptions are simply (1) mean field theory (3.1) is applicable; and (2) the system is in a minimal uncertainty bound state.

As we saw in the last section, since $S \sim N$, one is again at the transition between black holes and black strings. However since the above analysis only concerns the quantum mechanics of the zero modes, it approaches the black hole/black string transition from the black hole side, rather than the black string side as in [14,15].

4. Zero mode dynamics and the holographic principle

What can we expect for the zero mode quantum mechanics as we increase N to move away from the transition point? The answer depends crucially on what happens to the bound state's characteristics, in particular its transverse size, as we boost it by increasing N. The strong holographic principle would predict that the transverse size r increases with N, leaving us with two scales – the holographic size r_h and the Schwarzschild radius r_0. This seems awkward, since slow scattering experiments will presumably depend only on r_0 and not on r_h.

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[4] Recently, the importance of using Boltzmann statistics was stressed in [20].

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A set of classical solutions studied by Hoppe [21] reveals a more canonical boost behavior, at least at the classical level. These solutions are matrix discretizations of those found in [22]. Consider the ansatz

$$X^i(t) = x(t)r^i{}_j(t)\mathcal{M}^j \ , \tag{4.1}$$

with $x(t)$ an overall pulsation; $r^i{}_j(t) = exp[\varphi(t)\Omega]$ a rotation of constant angular momentum $L = R^{-1}x^2(t)\dot{\varphi}(t)$; and $\mathcal{M}^j$ a fixed matrix

$$\vec{\mathcal{M}} = \tfrac{1}{2\sqrt{2}}\left(U + U^{-1}, -i(U - U^{-1}), V + V^{-1}, -i(V - V^{-1}), 0, \ldots, 0\right) \tag{4.2}$$

in terms of the 't Hooft matrices U, V, satisfying $UV = \omega VU$, $\omega = exp[2\pi i/N]$. One may take the rotation to have $\vec{\mathcal{M}}$ as an eigenvector, $\Omega^2\vec{\mathcal{M}} = -\mu\vec{\mathcal{M}}$. The algebra of the 't Hooft matrices gives

$$\sum_j [[\mathcal{M}_i, \mathcal{M}_j], \mathcal{M}_j] = \lambda\mathcal{M}_i \quad , \qquad \lambda = 2\sin^2(\pi/N) \ , \tag{4.3}$$

and the solution to the classical equations of motion of the matrices X^i boils down to that of the overall pulsation

$$\frac{\ddot{x}}{R} + \lambda Rx^3 - \frac{\mu RL^2}{x^3} = 0 \ . \tag{4.4}$$

(In this section we measure x in Planck units.) At large N, one has $\lambda \sim N^{-2}$, and the conserved energy is simply

$$E_{LC} \sim N\left[\frac{\dot{x}^2}{R} + \frac{R}{N^2}x^4 + \frac{\mu RL^2}{x^2}\right] \ . \tag{4.5}$$

Since $E_{LC} \propto 1/N$, the relevant scales are $x \sim 1$, $t \sim N/R$, and $L \sim 1/N$. In other words, the transverse size remains constant, and the motion slows down as the system is boosted – canonical boost behavior. Of course, the true test of the system vis à vis the strong holographic principle is what happens when quantum fluctuations are turned on. Naively, these ought simply to lead to the gravitational interactions between the various bits of membrane. There will, of course, also be zero-point fluctuations which grow without bound as the cutoff N is removed; however, these do not usually affect the size of objects as seen in scattering experiments with slow heavy probes. For example, macroscopic strings appear local down to the string scale [23], and D-branes in slow relative motion may be localized down to the Planck scale and beyond [24]. In the large N limit, the quantum membrane considered here should resemble a D-twobrane.

A membrane is not a black hole, of course; from (4.5), its size scales as $M^2 R/N \sim x^4 R/N$; the mass is quadratic in the radius in any dimension (as opposed to $M_{BH} \sim r_0^{D-3}$) simply because the mass is the membrane area in Planck units. However, one could imagine assembling a black hole from a sufficient number of little nuggets of membrane, say k of them, each looking semiclassically like (4.1), and collapsing in on one another under gravitational attraction. In other words, the full $kN \times kN$ matrix X^i would decompose into blocks of size $N \times N$ of the form (4.1); the gravitational attraction between different blocks comes from integrating out the off-diagonal blocks. The entire system appears to obey canonical scaling under boosts $N \to e^{\alpha} N$.

5. Discussion and Speculations

Let us now consider what happens to the black hole states in matrix theory in the limit $N \gg S$. The following remarks will necessarily be rather speculative, since reliable calculations are not yet available in this regime. If the transverse size remains constant under boosts, as suggested by the preceding analysis, then the partons become denser as N increases. It seems likely that strongly interacting clusters will form. Within each cluster, the Born-Oppenheimer approximation will no longer be valid. This is because a given matrix parton is close enough to the other partons in the cluster so that the nonabelian degrees of freedom can no longer be consistently integrated out. The coherent interaction within a cluster should be more 'membrane-like' than 'graviton-like', since the commutator term in the matrix Hamiltonian *is* the membrane area element. The interaction between clusters might still be treatable in the Born-Oppenheimer approximation.

The typical size of a cluster can be estimated using Hawking radiation. In the rest frame of the black hole, the Hawking radiation has characteristic wavelength of order the Schwarzschild radius r_0. A boost to the transition point $P = S/R$ is such that the longitudinal component of this radiation is Lorentz contracted to the box size R [14]. An additional boost to $P = N/R$ ($N \gg S$) will make the characteristic longitudinal momentum of a Hawking quantum $p_{||} = N/SR$. In matrix theory, this corresponds to a (threshold) bound state of N/S partons. Partons in the black hole must therefore be strongly correlated over domains containing approximately this many partons. This observation leads one to expect that there will be roughly S clusters, each with approximately N/S partons.

For a fixed mass black hole, the energy $E_{LC} = M^2 R/N$ decreases as N increases and the system becomes colder. Since the partons are becoming denser, colder, and more

strongly interacting, one can think of this phase as a 'parton liquid' (in contrast to the 'gas' phase of the Born-Oppenheimer approximation that governs well-separated partons). If the transverse size remains constant under boosts, as suggested above, the typical virial velocities decrease as $1/N$. For large N, this appears to violate the uncertainty bound (3.2). However, the bound on the velocity of a cluster of N/S partons is decreased by S/N (since the mass is larger), so there is no contradiction. Of course, individual partons' velocities cannot violate the uncertainty principle. In order not to contribute too much to the overall energy, the partons within each cluster or domain must be very nearly in their ground state (so their energy approximately cancels between bosons and fermions due to supersymmetry). This is in accord with the assertion of [14], that most of the partons must be in their ground state for $N \gg S$.

The total energy E_{LC} will be distributed among kinetic energy of the clusters, gravitational potential energy (after integrating out fast nonabelian modes), and 'membrane stretching energy' from the slow nonabelian modes of nearby (and strongly correlated) partons. Computing the properties of the black hole in this regime will require understanding how the system apportions its energy budget among these, and perhaps other, aspects of the dynamics.

We have argued that matrix theory can describe some essential properties of Schwarzschild black holes. This may seem surprising in light of recent indications that matrix theory has difficulty reproducing eleven dimensional supergravity [8,9]. We believe that the black hole results indicate that matrix theory does capture the essential degrees of freedom of the theory. It is possible that some detailed aspects of the matrix dynamics may need to be modified, but the gross features are not likely to be affected.

We believe our analysis contains other lessons about matrix theory as well. It shows that the localized states of matrix theory are encoded in the zero mode dynamics of the generalized SYM theory that defines matrix theory in a particular compactification. All the nonzero modes which are the source of ultraviolet difficulties in the quantum theory (and an apparent stumbling block in defining the theory in compactifications to low dimensions) are longitudinally stretched objects which must decouple in the limit of interest $N, R \to \infty$. Since the great success of matrix theory seems to stem from its ability to exhibit all the dualities of M-theory, one might wonder whether there is a truncation of the dynamics to the zero mode sector which respects the dualities while throwing away all the troublesome aspects of the nonzero mode dynamics. What would change with dimension would be the

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combinatorics of the bound states of various fluxes.

The zero mode sector of matrix theory at finite N is (so far) a theory of the electrically charged objects of M-theory, containing a finite number of gravitons and/or discretized membranes. A complete theory must include the fluxes corresponding to the magnetic objects as well – the fivebrane and six-brane (Kaluza-Klein 'monopole'). Since these objects are solitonic in nature, it is unlikely that they will be present in the finite N theory; rather, they are 'condensates' of a nonperturbative number of partons. This may explain the difficulties encountered to date with matrix theory on T^5 with transverse fivebranes, and the apparent lack of a candidate for the theory at finite N on T^6 and beyond. While the scaling analysis of section 3 seems to work in any dimension, it is likely that a proper understanding (especially for $N \gg S$) will have to incorporate these fluxes in the dynamics for $D \leq 6$, where the continuum $N \to \infty$ theory may be needed.

Even without an understanding of such magnetic fluxes, there remains a fascinating condensed matter problem to determine the thermodynamics of the 'parton liquid' whose properties appear to govern black hole thermodynamics in quantum gravity. Reproducing properties of black holes when $N \gg S$ will teach us a great deal about the Lorentz covariance properties of matrix theory, and should be somewhat simpler than the threshold bound state problem for gravitons.

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