AdS-CFT

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Problem sheet 2: AdS

1. Consider the hyper surface,

$$U^2 + V^2 - X^i X^i = \ell^2 \tag{1}$$

with $i = 1, \ldots d$ embedded in $(d+2)$-dimensional Minkowski spacetime,

$$ds^2 = -dU^2 - dV^2 + dX^i dX^i \tag{2}$$

Firstly show that we may parameterize it as;

$$\begin{aligned} U &= \frac{z}{2}\left(1 + \frac{1}{z^2}\left(\ell^2 + (x^a)^2 - t^2\right)\right) \\ V &= \frac{\ell}{z} t \\ X^a &= \frac{\ell}{z} x^a \\ X^d &= \frac{z}{2}\left(1 - \frac{1}{z^2}\left(\ell^2 - (x^a)^2 + t^2\right)\right) \end{aligned} \tag{3}$$

for $a = 1, \ldots, d-1$.

Secondly show the metric induced on the surface is,

$$ds^2 = \frac{\ell^2}{z^2}\left(\eta_{\mu\nu} dx^\mu dx^\nu + dz^2\right) = \frac{\ell^2}{z^2}\eta_{AB} dx^A dx^B \tag{4}$$

writing $x^\mu = (t, x^a)$ and $x^A = (x^\mu, z)$ - ie. that of Poincare-AdS.

2. Consider Poincare-AdS with coordinates as above. Compute its Christoffel symbol components to show;

$$\Gamma^z{}_{zz} = -\frac{1}{z}\ , \quad \Gamma^z{}_{\mu\nu} = +\frac{1}{z}\eta_{\mu\nu}\ , \quad \Gamma^\mu{}_{z\nu} = -\frac{1}{z}\delta^\mu_\nu \tag{5}$$

with all other components vanishing. Recall;

$$\Gamma^C{}_{AB} = \frac{1}{2}g^{CD}\left(\partial_A g_{BD} + \partial_B g_{AD} - \partial_D g_{AB}\right) \tag{6}$$

3. Consider a particle at constant spatial location. Compute the $(d+1)$-acceleration required to keep the particle location fixed - recall that for a timelike curve with tangent v^A with respect to the proper time parameter τ (so $v^A = dx^A/d\tau$) then the acceleration $a^A = v^B\nabla_B v^A$. Firstly show the acceleration is only in the z direction. You should see the particle wants to fall to larger z. Secondly show that the magnitude of the acceleration is independent of the point the particle is located at.

4. Consider again Poincare-AdS. Pick a point in the interior and on the boundary at the same time t and consider a spacelike curve in the constant t slice connecting these. Show the curve must have infinite distance.

5. The null geodesic curves of a metric g_{AB} remain the same for a conformally related metric $g'_{AB} = \Omega(x)g_{AB}$. Use this fact to compute all the null geodesic curves in Poincare-AdS. Show that the ones that connect a point in the interior and reach the conformal boundary do so at finite coordinate time t. Show that the affine parameter of the geodesic does go to infinity as it approaches the boundary.

6. Consider the scalar field with equation of motion,

$$\nabla^2\phi = m^2\phi \tag{7}$$

in the Poincare-AdS spacetime given in the previous question. Show that if we Fourier decompose in the $x^\mu = (t, x^a)$ directions, so we consider a mode,

$$\phi = e^{ik_\mu x^\mu} f(z) \tag{8}$$

that then we obtain,

$$f''(z) - \frac{d-1}{z}f'(z) - \left(k^2 + \frac{m^2\ell^2}{z^2}\right)f = 0\ . \tag{9}$$