AdS-CFT

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Problem sheet 1: CFT

1 a). Recall that P_μ acts on a function as $P_\mu f = i\partial_\mu f$ and generates the infinitessimal translation $x^\mu \to x^\mu - a^\mu$ by $f \to e^{ia^\mu P_\mu} f$. Confirm that the dilatation transformation,

$$x^\mu \to \lambda x^\mu \tag{1}$$

has corresponding generator D acting on a function as,

$$Df(x) = i\left(x^\mu \partial_\mu\right) f \tag{2}$$

1 b). Confirm that for the special conformal transformation in Minkowski,

$$x^\mu \to \frac{x^\mu + b^\mu x^2}{1 + 2x^\mu b_\mu + b^2 x^2} \tag{3}$$

the corresponding generator K^μ has the action on a function,

$$K_\mu f(x) = i\left(x^2 \partial_\mu - 2x_\mu x^\nu \partial_\nu\right) f \tag{4}$$

1 c). Confirm the following two commutators hold for the action on functions;

$$[D, P_\mu] f = -iP_\mu f \ , \quad [D, K_\mu] f = iK_\mu f \tag{5}$$

2. Show that in a relativistic QFT the one point function of a scalar field $\phi(x)$ in vacuum is constant,

$$\langle \phi(x) \rangle = c \tag{6}$$

and cannot be x^μ dependent.

3. Show that in a CFT the one point function of a primary scalar field $\phi(x)$ in vacuum must vanish if the field has dimension $\Delta > 0$ (as typically required by unitarity).

4. Show that two scalar fields $\phi_1(x)$ and $\phi_2(x)$ with scale dimensions Δ_1 and Δ_2 respectively have a two point function,

$$\langle \phi_1(x)\phi_2(y) \rangle = \frac{c}{((x-y)^2)^{\frac{\Delta_1+\Delta_2}{2}}} \tag{7}$$

5. If instead the fields are both primary, show that the two point function vanishes unless they have the same scale dimension so, $\Delta_1 = \Delta_2$. Then of course we recover the usual result,

$$\langle \phi_1(x)\phi_2(y) \rangle = \frac{c}{|x-y|^{2\Delta}} \tag{8}$$

where Δ is the common scale dimension.