5.7 Renormalisation

We saw that $Z = \qquad \langle 0|S|0\rangle \neq 1$ representing corrections to our naive assumption coming from quantum fluctuations, virtual particle corrections.

What about other naive assumptions - m, M masses
- g interaction strength

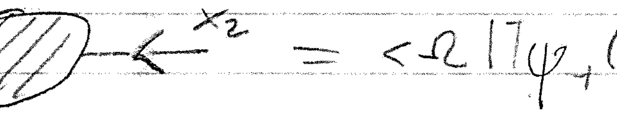
Ex SYTh ψ mass

CUT " Finally have no propagator

$$\mathcal{M} = \langle \psi(q), f | \psi(p), i \rangle \propto \frac{\delta^4(p-q)}{p^2 - m_{phys}^2}$$

i.e. pole at $p^2 = m_{phys}^2$ defines physical mass

Consider full propagator for ψ when $H_{int} \neq 0$

Π $G(x_1, x_2) \equiv$  $= \langle \Omega | T\psi_H(x_1)\, \psi_H^\dagger(x_2) | \Omega \rangle$

Connected diagrams only $\Leftrightarrow$ No vacuum diagrams
$|\Omega\rangle$ NOT $|0\rangle$

or in momentum space

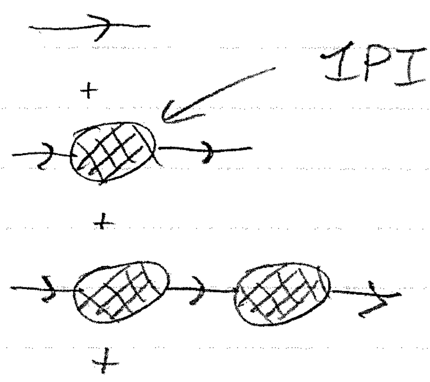
$$\Pi(p) = \int d^4x\, e^{-ip(x_1 - x_2)} G_\psi(x_1 - x_2)$$

$$= \frac{i}{p^2 - M^2} \quad \text{for Free propagator } H_{int} = 0$$

$H_{int} \neq 0$ Expect $\Pi(p) \sim \frac{1}{p^2 - M_{phys}^2}$ for $p^2 \sim M_{phys}^2$ POLE = PHYSICAL MASS

NOT ll so include $\Delta(p)$ here

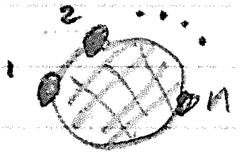
$\Pi(p) =$



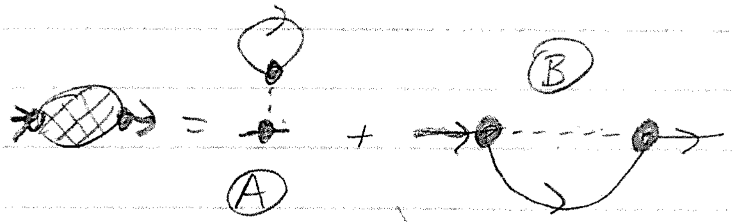
1PI = one-particle irreducible

etc

Def^n 1PI diagrams are ~~truncated~~ (amputated) diagrams (no external legs) external vertices can not be on two (or more) pieces if you cut one (internal) line

Symbol  $= \Gamma(x_1, x_2, \dots, x_n)$

e.g. SYTH

2pt 1PI $\Sigma =$  $+ O(g^4)$

$$\Pi(p) = \Delta + \Delta\Sigma\Delta + \Delta\Sigma\Delta\Sigma\Delta + \dots$$

$$= \Delta\left(\sum_{n=0}^{\infty}(\Sigma\Delta)^n\right)$$

$$= \frac{\Delta}{1-\Sigma\Delta} \qquad \left(\sum_{n=0}^{\infty}x^n = \frac{1}{1-x}\right)$$

$$\Pi(p) = \frac{1}{\Delta^{-1}-\Sigma}$$

But $\Pi(p) \sim \dfrac{i}{p^2 - M_{phys}^2}$ at $p^2 = M_{phys}^2$

c.f. sol^n of K.G.

$p^2 = M_{phys}^2$ def^n of particle mass

$$\Rightarrow \left[\Pi(p^2 = M_{phys}^2)\right]^{-1} = 0$$

$$\Rightarrow 0 = -i(M_{phys}^2 - M^2) - \Sigma(p^2 = M_{phys}^2)$$

Consider only diagram Ⓐ $\Sigma \simeq$ 

(Large N approximation)

$\Rightarrow \Sigma(p) = \Sigma$ ind of p^μ

$$\Rightarrow \Sigma(p) = \Sigma_A = -ig \cdot \frac{+i}{0 - M^2 + i\varepsilon} \cdot -ig \underbrace{\int d^4k \frac{i}{k^2 - M^2 + i\varepsilon}}$$

$$= +i \frac{g^2}{M^2} \Delta_4(0) \sim \frac{g^2 \Lambda^2}{M^2}$$

$$\sim \int^\Lambda dk \frac{k^3}{k^2} \sim \Lambda^2$$

Note Σ here is independent of momentum

$$\Rightarrow 0 = M_{phys}^2 - M^2 - i\Sigma_A$$

$$\Rightarrow \underset{\substack{\uparrow \\ \text{Finite}}}{M_{phys}^2} = \underset{\substack{\Downarrow}}{M^2} + \underset{\substack{\uparrow \\ \infty}}{i\Sigma_A}$$

"Bare" Parameters in $\mathcal{L}$ are infinite!

Suppose change H_{int}

← Now physical mass in propagator

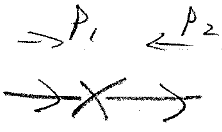
$$\mathcal{L}_0 = (\partial_\mu \psi)^\dagger (\partial^\mu \psi) - M_{phys}\psi^\dagger \psi + \mathcal{L}_{int}$$

$$\mathcal{L}_{int} = -g\,\psi^\dagger\psi\phi - \underbrace{(M^2 - M^2_{phys})}\psi^\dagger\psi$$

COUNTER TERM
— Treat as $O(g)$

$$H_{int} = -\int d^3x \; \mathcal{L}_{int}$$

⇒ New vertex
(Two legs)



$$= -i(\delta M^2)\,\delta^4(\Sigma p_e)$$

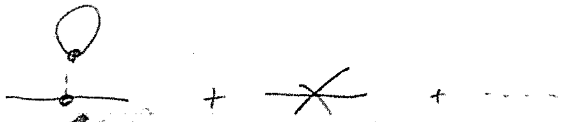
$\uparrow$
$O(g^2)$

& new propagator

$$\Delta_{phys}(p) = \frac{i}{p^2 - M^2_{phys} + i\epsilon}$$

Full Propagator now $\Pi(p) = (\Delta^{-1}_{phys} - \Sigma^{NEW})$
where now $= p^2 - M^2_{phys} - \Sigma^{NEW}$

$\Sigma^{NEW} =$



$= \Sigma_A + \delta M^2$

$p^2 - M^2_{phys} - \Sigma^{NEW}$

However we want $(\Pi(p^2 = M^2_{phys}))^{-1} = 0$

$\Rightarrow \Sigma^{NEW}(p^2 = M^2_{phys}) = 0$

$\Rightarrow \delta M^2 = -\Sigma_A$

$\Rightarrow M^2 = M^2_{phys} - \Sigma_A = \infty$!

Now $\Pi(p)$ is finite to $O(g^2)$
$\Sigma(p)$ " " " "

All similar calc now finite (need similar trick with g!)