Renormalisation

We saw that $Z = \langle 0 | S | 0 \rangle \not= 1$ representing corrections to our naive assumption coming from quantum fluctuations, virtual particle corrections.

What about other naive assumptions - $m, M$ masses - of interaction strength

Ex. SYM $\psi$ mass

$$M = \langle \psi(x) \phi, \phi^\dagger \psi(y) \rangle \propto \frac{\delta(p-q)}{p^2 - m^2_{phys}}$$

i.e. pole at $p^2 = m^2_{phys}$ defines physical mass

Consider full propagator for $\psi$ when $H_{int} \neq 0$

$$\Pi(x_1, x_2) = \frac{x_1 - x_2}{p^2 - M^2}$$

Connected diagrams only $\iff$ No vacuum diagrams

or in momentum space

$$\Pi(p) = \delta^4(p) e^{-i p(x_1-x_2)} \delta_+ (x_1-x_2)$$

$$= \frac{i}{p^2 - M^2} \quad \text{for Free propagator} \quad H_{int} = 0$$

$H_{int} \neq 0$: Expect $\Pi(p) \sim p^2 - M^2_{phys}$ for $p^2 \sim M^2_{phys}$ POLE = PHYSICAL MASS
\[ \Pi(p) = \Delta + \Delta \sum \Delta + \Delta \sum \Delta \sum \Delta + \ldots \]

\[ = \Delta \left( \frac{\sum_{n=0}^{\infty} (\Delta \Delta)^n}{1-\Delta} \right) \]

\[ = \Delta \left( \frac{1}{1-\Delta} \right) \left( \frac{\sum_{n=0}^{\infty} x^n}{1-x} \right) \]

\[ = \frac{1}{\Delta^{-1} - \Delta} \]

Def: IPI diagrams are truncated diagrams (no external legs) and external vertices can not be on two (or more) pieces if you cut one (internal) line.

\[ \Sigma = M(x_1, x_2, \ldots, x_n) \]

\text{e.g. S}7TH\text{ }

\[ 2\text{pt IPI } \Sigma = \]
But \( T(p) \sim \frac{k}{p^2-M_{\text{phys}}^2} \) at \( p^2 = M_{\text{phys}}^2 \).

\[ \Rightarrow \left[ T(p^2 = M_{\text{phys}}^2) \right]^{-1} = 0 \]

\[ \Rightarrow 0 = -i(N_{\text{phys}} - M^2) - Z(p^2 = M_{\text{phys}}^2) \]

Consider only diagram (A) \( \Sigma \to \)

(Large \( N \) approximation)

\[ \Rightarrow Z(p) = \Sigma \text{ and } \rho_n \]

\[ \Rightarrow \Sigma(p) = \frac{\rho}{A} = \frac{ig}{0-M^2+i\epsilon} + \frac{ig}{\int d^4k \frac{i}{k^2-M^2+i\epsilon}} \]

\[ = \frac{g^2}{M^2} \Delta(0) \sim \frac{g^2 A^2}{M^2} \]

Note \( \Sigma \) here is independent of momentum.

\[ \Rightarrow 0 = M_{\text{phys}}^2 - M^2 - i\Sigma \]

\[ \Rightarrow M_{\text{phys}}^2 = M^2 + i\Sigma \]

\[ \uparrow \quad \downarrow \quad \Rightarrow \]

Finally, \( \Sigma \to \infty \)

"Bare" Parameters in \( Z \) are infinite.
Suppose change H_{line}$ Now physical mass in propagator 

$Z_0 = \phi^+ \phi^+ - M_{phys}^\phi \phi^+ \phi + Z_{line}$

$L_{line} = - g \phi^+ \phi - (M^2 - M_{phys}^2) \phi^+ \phi$

**COUNTER TERM**

-Treat as $O(g^2)$

$H_{line} = - (\partial^2 \phi^+ \phi^+ - \partial^2) \phi^+ \phi$

$\Rightarrow$ New vertex (Two legs)

New propagator

$\Delta(\phi) = \frac{i}{\rho^2 - M_{phys}^2 + i\varepsilon}$

Full Propagator now $\Pi(\rho) = (\Delta^{-1}_{phys} - Z^{\text{NEW}})$

Where now $Z^{\text{NEW}} = 0$

$Z^{\text{NEW}} = Z_A + \sum M^2 = \sum_{\rho^2 - M_{phys}^2 - Z^{\text{NEW}}}$

However we want $(\Pi(\rho^2 - M_{phys}^2))^{-1} = 0$

$\Rightarrow Z^{\text{NEW}} (\rho^2 - M_{phys}^2) = 0$

$\Rightarrow \sum M^2 = -Z_A$

$\Rightarrow M^2 = M_{phys}^2 - Z_A = \infty$ !

Now $\Pi(\rho)$ is finite to $O(g^2)$.

All similar calculations now finite (need similar trick with $g$ !)