§5.5

**Full Vacuum**

10> is the vacuum of the free theory $\Rightarrow \hat{a}_0 10> = 0$

1.2> is the vacuum of the full (interacting) theory $\Rightarrow \hat{H} 1.2. > = E_0$ lowest energy, wrong $E_0$

**NOT THE SAME**! 1.2 > is full of virtual particles 10 > is empty $\forall t > 0$

We want initial / final states build on 1.2 > NOT 10>

$\Rightarrow$ We want $\mu < 0.1.5.1.2 >$ NOT $\mu < 0.1.5.1.0 >$

Lemma:

For any state $1.4, \xi >

then $\xi < 4, \xi | 0.1.0 > \leq \int_{1.4} e^{1.4 | 0.1.0 >}

\& we find $\lim_{\xi \to 0.5} < 4, \xi | 0.1.0 > \Rightarrow < 4.1.2 > < 2.1.0 >$

$\Rightarrow \lim_{\xi \to 0.5} < 0.1.5.4, \xi > \Rightarrow < 0.1.5.1.0 > < 2.1.4.5 >$

2.1.4.23

**Meaning**

For long time separations only the overlap between 10> & 1.2 > is significant in higher energy states insensitivity cancels out

So for all significant contribution from higher energy states insensitivity cancels out

we can 1.2. \equiv \frac{1.1.0 >}{2.1.3.}

$\Rightarrow < \sqrt{T(\text{Anytime})} 1.2 > = \frac{< 1.0 | T \text{(Anytime)} 1.5.1.0 >}{< 0.1.5.1.0 >}$

(All I pic.)
\[ \text{Proof} \]

Consider \( \langle 4, \bar{1}, 0, \bar{t} \rangle \)

\[ \Rightarrow = \langle 4, 1 \mid e^{-i(\delta E + H_{\text{f}})} \mid 0 \rangle \quad \text{with } H = H_0 + H_{\text{exc}} \]

as wlog choosing \( H_0 \mid 0 \rangle = 0 \) (ignoring details)

Insert complete set of energy states using (1.2) as vacua and with \( \sum_n \) as excited states \( H_{\text{exc}} \)

\[ = \langle 4 \mid \sum_n e^{-iE_n(t - \bar{t})} \left[ \sum_n < n | + \sum_n | n \rangle \langle n | \right] \mid 0 \rangle \]

\[ = \langle 4 \mid \sum_n < \bar{n} | < n \langle 0 \mid \]

As \( E_n > 0 \) or \( \text{Riemann-Lebesgue theorem for} \]

\[ \lim_{t, \bar{t} \to \pm \infty} \int_a^b f(x) e^{ix} \, dx \quad \text{is well behaved finite} \]

this term clamp away

\[ \sum_n \]

when energy continuous

First oscillatory \( (E_n > 0) \) terms die away relative to lowest energy term.

\[ \Rightarrow \lim_{t, \bar{t} \to \pm \infty} \langle 4, \bar{1}, 0, \bar{t} \rangle \to \langle 4 | 2 \rangle \langle 2 | 0 \rangle \]

(likewise for \( \bar{t} \to \pm \infty \))

\[ \lim_{t, \bar{t} \to \pm \infty} \langle 0, \bar{t}, 1, 4, \bar{t} \rangle \to \langle 0 | 1 \rangle \langle 1 | 2 \rangle \langle 2 | 4 \rangle \]
We really want

\[ M = \langle \Omega | S | \Omega \rangle = \langle \Omega | (\text{annihilation}) \cdot S (\text{creation}) | \Omega \rangle \]

\[ G_c = \langle \Omega | (T(\text{fields})) \cdot S (\text{Fields}) | \Omega \rangle \]

Connected 

Full Interacting Vacuum

c.f. \( G = \langle 0 | T(\text{fields}) | 10 \rangle \) is what our F. Rules calculate.

Lemma

\[ \langle \Omega | T(\text{fields}) \cdot S | \Omega \rangle = \frac{1}{Z} G(\bar{\eta} \eta) \]

Proof

\[ \text{RHS} = \frac{\langle 0 | \Omega \rangle \langle \Omega | T(\text{fields}) \cdot S | \Omega \rangle \langle \Omega | 210 \rangle}{\langle 0 | 21 \rangle \langle 21 | S | 12 \rangle \langle 12 | 210 \rangle} \]

\[ \langle 0 | 21 \rangle \langle 21 | S | 12 \rangle \langle 12 | 210 \rangle = 1 \text{ Vacuum state stable (} E_0 = 0 \text{)} \]

\[ = \langle \Omega | T(\text{fields}) | \Omega \rangle \]
Matrix Elements

We really want to hold on to the physical vacuum
\( M = \langle f | S | i \rangle \)

\[ = \langle f | (\text{final}) S (\text{initial}) | i \rangle \]

\[ = \sum_{\text{Diagrams}} \langle f | T (\text{fields}) | i \rangle \]

\[ M = \frac{1}{2} \sum_{\text{Diagrams}} \langle f | T (\text{fields}) | i \rangle \]

\[ \uparrow \]

\[ \langle 0 | S | 10 \rangle \]

Rules calculated above

What is \( Z = \langle 0 | S | 10 \rangle \)?

If \( \text{Hint} = 0 \Rightarrow Z = \langle 0 | S | 10 \rangle = 1 \)

If \( \text{Hint} \neq 0 \Rightarrow \text{What is } Z? \)

Lemma: \( Z \) is given by the sum of all "vacuum diagrams" diagrams with no external legs.

Example: \( \sum \)

\[ Z = 1 + \quad + \quad + \quad + \ldots \]

\[ = 1 + O(g^2) + g^4(\infty) \]

\[ = \infty \]
Corollary

Physical vacuum expectation values of time ordered products are given by sums of diagrams without any vacuum diagrams.

Proof (Sketch of)

Consider any diagram without vacuum diagrams, i.e., every part connected to at least one external leg.

\[ G = \sum_{\text{all vertices}} [O(V) \text{ term in}] \]

For \( G \), we also get a another diagram with each of the \( G \) along with any disconnected diagram.

\[ G = \sum \left( 1 + \text{all vacuum diagrams} \right) + \cdots \]

Combinatorics: \( \frac{\binom{V+U}{V} \cdot \binom{V+U}{V}}{V! \cdot U!} = \frac{1}{V+U} \cdot \frac{(V+U)!}{V! \cdot U!} \)

Such diagrams give a result equal to the value of \( \sum \) above multiplied by vacuum contributions.

\( \Rightarrow \) The vacuum diagrams are completely cancelled by \( \frac{1}{2} \) contribution