Tong §3.7 p75+

NOT covering 2017

§5.5

Full Vacuum

$|0\rangle$ is the vacuum of the free theory $\Rightarrow \hat{a}_p|0\rangle = 0$

$|\Omega\rangle$ is the PHYSICAL vacuum of the full (interacting) theory $\Rightarrow H|\Omega\rangle = E_0$ lowest energy, w.l.o.g. $E_0 = 0$

NOT THE SAME! $|\Omega\rangle$ is full of virtual particles
$|0\rangle$ is empty NOT time ind

We want initial/final states built on $|\Omega\rangle$ NOT $|0\rangle$
$\Rightarrow$ We want $\mathcal{U} = \langle\Omega|S|\Omega\rangle$ NOT $\tilde{\mathcal{U}} = \langle 0|S|0\rangle$

Lemma

Tong §3.7 p75

For any state $|\psi, t\rangle_S$ $\hat{a}|0\rangle$

then ${}_S\langle\psi, t|0, t_i\rangle_S = {}_I\langle\psi, t|U(t, t_i)|0\rangle_I$

& we find $\lim_{t_i \to -\infty} {}_S\langle\psi, t|0, t_i\rangle_S \to \langle\psi, t|\Omega\rangle\langle\Omega|0\rangle$

& $\lim_{t_f \to +\infty} {}_S\langle 0, t_f|\psi, t\rangle_S \to \langle 0|\Omega\rangle\langle\Omega|\psi, t\rangle$

e.o L23 12/12/14

Meaning

For long time separations only the overlap between $|0\rangle$ & $|\Omega\rangle$ is significant, contribution from higher energy states insignificant - damped out / cancels

c.o L22 2/12/16

time notation :(

So for $\mathcal{U}$ we can substitute $|\Omega\rangle \equiv \frac{1}{Z^{\frac{1}{2}}}|0\rangle$ where $Z = \langle 0|S|0\rangle$

$$\Rightarrow \langle\Omega|T(\text{Anything})|\Omega\rangle = \frac{\langle 0|T(\text{Anything})S|0\rangle}{\langle 0|S|0\rangle}$$

(All I pic.)

Tong p 76 *Proof*

Consider $_S\langle \psi, t | 0, t_i \rangle_S =$

$\Rightarrow \; = \langle \psi | \, \mathcal{U}(t, t_i) \, | 0 \rangle$ in I. pic

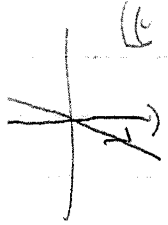
$= \langle \psi | \underbrace{T e^{-i\int_{t_i}^{t} dt\, H_{full}}}_{U_{Full}} | 0 \rangle$ with $H = H_0 + H_{int}$ $\quad H_{FULL} = H_0 + H_{int}$

as wlog choosing $H_0 |0\rangle = 0$ (w.l.o.g. ignoring c-numbers)

Insert complete set of energy eigstates using $|\Omega\rangle$ as vacuum with $\{|n\rangle\}$ as excited states $H|n\rangle = E_n|n\rangle$

$$= \langle \psi | T \exp\{-i\int_{t_i}^{t} dt\, H\} \left[|\Omega\rangle\langle\Omega| + \sum_n |n\rangle\langle n| \right] |0\rangle$$

$$= \langle \psi | \Omega \rangle \langle \Omega | 0 \rangle$$



$\boxed{\text{As } H|\Omega\rangle = 0}$ $\quad + \underbrace{\sum_n e^{-iE_n(t-t_i)} \langle \psi | n \rangle \langle n | 0 \rangle}$

As $E_n > 0$ THEN send $t_i \to -\infty(1-i\varepsilon)$ & this term damps away

OR Riemann-Lebesgue theorem for "well" behaved func

$$\lim_{\mu=0} \int_a^b dx\, f(x)\, e^{i\mu x} = 0$$

$\sum_n$ when energy continuous.

Fast Oscillatory ($E_n > 0$) terms die away relative to lowest energy term.

$$\Rightarrow \lim_{t_i \to -\infty} {}_S\langle \psi, t | 0, t_i \rangle \longrightarrow \langle \psi | \Omega \rangle \langle \Omega | 0 \rangle$$

(likewise for $t_f \to +\infty$)

$$\lim_{t_f \to \infty} \langle 0, t_f | \psi, t \rangle \longrightarrow \langle 0 | \Omega \rangle \langle \Omega | \psi \rangle$$

USE

Matrix Elements, Green Functions & Vaccua

We really want

$$\mathcal{M} = \langle f | S | i \rangle$$
$$= \langle \Omega | (\text{annihilation})_f \, S \, (\text{creation})_i | \Omega \rangle$$
$$G_c = \langle \Omega | T(\text{fields})_f \, S \, (\text{fields})_i | \Omega \rangle$$

Connected ... Full Interacting Vaccum

c.f. $G = \langle 0 | T(\text{fields}) \, S \, | 0 \rangle$ is what our F. Rules calculate

Lemma

Tong (3.95) p76

$$\langle \Omega | T(\text{fields}\, S) | \Omega \rangle = \frac{\langle 0 | T(\text{fields}\, S) | 0 \rangle}{\langle 0 | S | 0 \rangle}$$

$$G_c(\{y\}) = \frac{1}{z} G(\{y\}) \quad \text{where } z = \langle 0 | S | 0 \rangle$$

Proof

$$\text{RHS} = \frac{\langle 0 | \Omega \rangle \langle \Omega | \, T(\text{fields}) \, | \Omega \rangle \langle \Omega | 0 \rangle}{\langle 0 | \Omega \rangle \underbrace{\langle \Omega | S | \Omega \rangle}_{{}_S\langle \Omega, t_f | \Omega, t_i \rangle_S = 1} \langle \Omega | 0 \rangle}$$

WRITE OUT } Vaccum state stable ($E_0 = 0$)

be 2015 3(i)(ii) ANS

$$= \langle \Omega | T(\text{fields}) | \Omega \rangle$$

Matrix Elements CUT

CUT

We really want to build on true physical vacuum

$$\mathcal{M} = \langle f | S | i \rangle$$

$$= \langle \Omega | \begin{pmatrix}\text{final} \\ \text{fields}\end{pmatrix} S \begin{pmatrix}\text{initial} \\ \text{fields}\end{pmatrix} | \Omega \rangle$$

$$= \sum_{\substack{\text{ALL} \\ \text{DIAGRAMS}}} \langle \Omega | T(\text{fields}) | \Omega \rangle$$

$$\mathcal{M} = \underset{\substack{\uparrow \\ \langle 0|S|0\rangle}}{\frac{1}{Z}} \sum_{\substack{\text{ALL} \\ \text{DIAGRAMS}}} \underbrace{\langle 0 | T(\text{fields}) | 0 \rangle}_{\text{Rules calculated above}}$$

What is $Z = \langle 0|S|0\rangle$?

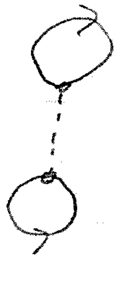
If $H_{int} = 0 \Rightarrow Z = \langle 0|S|0\rangle = 1$

If $H_{int} \neq 0 \Rightarrow$? What is Z?

Lemma Z is given by sum of all "vacuum diagrams"
diagrams with no external legs.

e.o.L23 10/11/15

Example SYTh

$$Z = 1 +$$  $$+ \begin{pmatrix}\text{etc.} \\ \text{another} \\ O(g^2)\end{pmatrix} + \dots$$

$$= 1 + \infty g^2 + g^4(\infty)$$

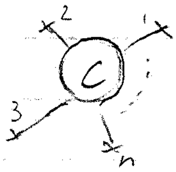
$$= \infty\ !$$

Corollary

Physical Vacuum expectation values of time ordered products are given by sums of diagrams WITHOUT any vacuum diagrams.

Proof (Sketch of)

Consider any diagram without vacuum diagrams
i.e. every part connected to at least one external leg

$G_c =$  $- V$ vertices $\in \langle 0|T\phi_1 \ldots \phi_n S|0\rangle = G$ [$O(V)$ term in S expansion]

$+ \ldots$

For G, we also get another diagram with each of the G_c along with any disconnected diagram.

Diagram $D_{AB} = \underbrace{D_A D_B}_{\text{Disconnected}}$
$\Rightarrow V_{AB} = \underbrace{V_A \cdot V_B}_{\text{Evidence}}$

$G =$  $\underbrace{\left(1 + \text{all vacuum diagrams} \right)}_{\langle 0|S|0\rangle}$ $+ \ldots$

$O(V+U)$ term in S expansion

PSS Qg!? SYTh

Combinatorics: If have (O- VAC) with V & U vertices
$\Rightarrow \dfrac{1}{(V+U)!} \cdot \dfrac{(V+U)!}{V!\,U!} = \dfrac{1}{V!} \cdot \dfrac{1}{U!}$

Such diagrams give a result equal to the value of O- alone multiplied by vacuum contributions

$\Rightarrow$ The vacuum diagrams are completely cancelled by $\frac{1}{Z}$ contribution