2.1 Classical Field Theory

a) Classical Dynamics of Particles - Hamiltonian $H$

$$H = H \left[ \frac{\partial}{\partial t} \vec{x}(t), \vec{p}(t) \right] = T.E + P.E$$

For each particle $n=1,2,\ldots,N$ have one position $\vec{x}_n(t)$ and one momentum $\vec{p}_n(t)$ (in d-space dimensions $\in \mathbb{R}^d$)

e.o.m (Equation of Motion)

$$\frac{\partial \vec{p}_n}{\partial t} = -\vec{f}_n, \quad \frac{\partial \vec{x}_n}{\partial t} = \vec{v}_n$$

2(Nd) equations in time

Example: SHO (Simple Harmonic Oscillator) 1D

$$H = \sum_n \left( \frac{\vec{p}_n^2}{2m_n} + \frac{1}{2} m_n \omega^2 \vec{x}_n^2 \right)$$

Density $\tilde{H} = \hat{\rho} \cdot \hat{H}$

E.o.s find $\ddot{x}_n + \omega_n^2 x_n = 0$ as e.o.m

with solution $x_n(t) = A e^{-i\omega_n t} + B e^{i\omega_n t}$

$$= A \cos(\omega_n t) + B \sin(\omega_n t)$$
6) Classical Dynamics of Particles - Lagrangian $L$

$$L = \sum_{\text{n}} L_{\text{n}} = \sum_{\text{n}} \left[ \mathbf{x}(t) \cdot \mathbf{x}(t) \right] = K.E. - P.E.$$  

Now use $\mathbf{x}(t)$ and $\dot{x}(t)$ as independent quantities needed to specify solutions

E.O.M. - Lagrange's equations

$$\frac{\partial L}{\partial \mathbf{x}} - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\mathbf{x}}} \right) = 0$$

E.P.S. check S.M.O. e.o.m. obtained from

$$L = \sum_{\text{n}} \left( \frac{m}{2} \frac{\dot{x}_n^2}{n^2} - \frac{1}{2} m \omega_n^2 x_n^2 \right)$$

N.B. $H(p,q,t) = \sum_{n} p_n \dot{q}_n - L(q_n, \dot{q}_n, t)$  

$\dot{q}_n = \frac{\partial L}{\partial \dot{q}_n}$
LINEARITY

P52 Quadratic H or L \iff Linear e.o.m
O(x^2), O(p^2) \iff O(x) p.d.e.

Linear equation means two solutions of p.d.e.
e.g. \( x_1(t) \) \& \( x_2(t) \)
can be added to give a new solution

\[ x_{\text{new}}(t) = a \cdot x_1(t) + b \cdot x_2(t) \]

Interpretation

There is NO interaction between solutions
\( \Rightarrow \) Two waves pass through each other
and continue on exactly the same
as before, no losses, no changes.

Find More?

Non-linear e.o.m. can change this
e.g. period doubling, generation of
new waves of different frequencies.

Later with wave \( \leftrightarrow \) particle duality will
see non-linearity = interacting particles
linear e.o.m. = free or non-interacting
particles.