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Complexity and Networks

Level 3 Physics Course

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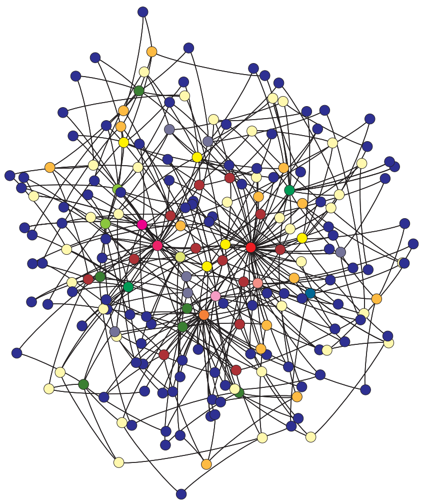
1 Overview

Stephen Hawking has predicted that the 21st Century will be "the century of complexity". A major theme in Complexity Science is seeing how interactions between many individual parts on the small (microscopic) scales can lead to the emergence of dramatic results on the large (macroscopic) scales. This could be the way that the we see rare, yet occasionally observed large fluctuations in complex systems: stock market crashes, earthquakes, extreme weather are all good examples. Simple Gaussian statistics do not describe the fluctuations in these non-equilibrium systems.



If we are to understand these emergent features, we must understand which parts of a system interact with one another. This is where networks play an important role. Physicists provided major new insights in the late 1990's showing how to describe the links between parts of a complex system when they were connected neither randomly (gas molecules) nor regularly (as in a regular lattice of atoms).

Complexity & Networks are areas which are truly cross-disciplinary involving mathematicians, physicists, computer scientists, engineers, biologists, and even the humanities. Over the last two decades the ideas of Complexity & Networks have become of central importance, and have already made a profound impact on our modern world. For example, the core idea behind Google's search algorithm relies on insights on how web pages are connected. Controlling the spread of the next flu epidemic will be based on models rooted in concepts from Complexity & Networks. The emergence of Complexity Science reflects the rapid growth in computational power and the role of information in the modern era.

This course dips physicists' toes into the basic ideas and tools for describing and analysing complex phenomena.

2 Practicalities

A detailed syllabus and timetable are still being developed. The key aspects of the course are as follows:-

- This is a **theoretical and numerical** course.
- Level 3 theory course.
- Term 2 (Spring term 2013-14).
- KC teaching half (Complexity), TSE teaching other half (Networks).
- Prerequisites: Level 3 "Statistical Mechanics" (now in term 1).
- 14 Lectures in addition to (around) twelve hours in computing lab.
- Maximum 72 students on course due to computer lab limitations. **First come first served**.
- Assessment: One project (hand in end of term 2), one exam (probably a short mastery style exam).
- Computational examples are likely to be given in the programming language `Python`.
  However work (examples, project etc) can be done in **any** language e.g. `C++`, `MatLab`.
  `Python` is easy to learn and allows for rapid prototyping. The code is easily understood by those with experience in other computer languages making the course examples intelligible by all. `Python` is open source and can be run on without any cost on any computer.

3 Draft Syllabus and setup

Below is a list of items from which a course could be drawn. Because a larger lab based component is envisaged, lengthy exact solutions may on occasions be dropped in favour of back-of-the-envelope style derivations. Students would learn the details by programming examples and may need to include theoretical derivations in their project.

3.1 Complexity — 1st half — Kim Christensen

- Criticality – basic properties, examples, emergence of macroscopic behaviour from microscopic rules; quick review of scaling and criticality in equilibrium systems.
- Review examples of non-equilibrium systems in nature displaying complex behaviour such as fluctuations of an observable that are not described by Gaussian statistics.
- Theoretically, we will address the phenomena of complex behaviour in non-equilibrium systems using the framework of simple numerical models.
- Describe qualitatively and quantitatively the concept of self-organised criticality, that is, systems displaying criticality without any external fine-tuning of a control parameter.
- Emphasise the fundamental conceptual difference between scaling in an equilibrium system at a phase transition and a non-equilibrium system displaying self-organised criticality.
- Describe qualitatively and quantitatively the concept of scaling and apply scaling arguments to non-equilibrium systems displaying finite-size scaling.
- Numerical exercise — investigate simple model displaying complexity behaviour.

3.2 Networks — 2nd half – Tim Evans

- Definition of network, different types (random, small-world, scale-free, complete, regular), real examples including web pages, social networks, citation networks.
- Basic properties — degree, clustering, shortest paths, degree distribution.
- Random Graphs — exact results for phase transitions (possibly via approximate derivations).
- Growing networks — Price (Barabasi-Albert) model, approximate or exact solutions, hubs, fat-tailed distributions.
- Analysis of Networks — algorithms for components and shortest paths, defining and measuring centrality measures: degree, in-degree (citation count), betweenness and currents, PageRank.
- Processes on graphs — random walks, Markov processes, ranking via PageRank, epidemiology.
- Numerical exercises — write code to implement one of the algorithms, use to test random graph theorems, use to do open ended investigation of a real network

4 Support and Assessment

- Problem sheets with practical and theoretical exercises to support lectures.
- Computing lab to develop theoretical ideas through applications
- Computing lab to support work on assessed project.
- Project assessed through a report.
- Exam, probably short, mastery style exam.

References

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[4] M.Newman, "Complex Networks", OUP 2011.

[5] David Easley and Jon Kleinberg, "Networks, Crowds, and Markets: Reasoning About a Highly Connected World" (Cambridge University Press, 2010 but free downloadable copy).

[6] Robert Hanneman and Mark Riddle, "Introduction to social network methods", 2005 (free download from web site).