Imperial College London

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Tim Evans
Theoretical Physics

Networks for the Minoan Aegean

T.S.Evans, C.Knappett, R.J.Rivers
Imperial Exeter Imperial

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Minoa

– reconstruction using original tools and techniques, as far as they are known, in order to make the best guess at the methods, design and capabilities of Minoan ships

Approaches to modelling

Several approaches all take CITIES (sites for us) as the core unit.

Agent Based Modelling

Cities as agents following basic rules

SIMPOP2

Increasing Detail

Network Optimisation

ariadne

Coarse Graining Increasing

Equations

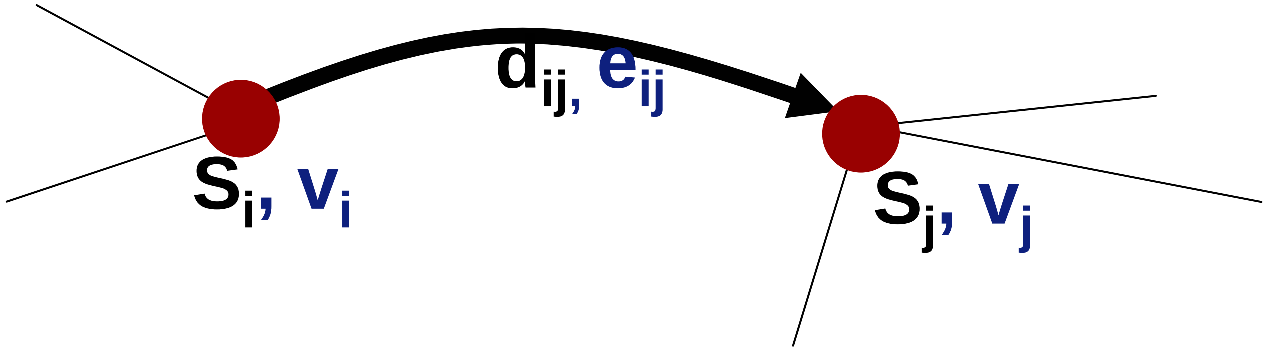
(Geoff, Luis, Jose)

Network Description

- d_{ij} Fixed distance between sites
  may be physical but may include penalties for land travel etc.
- S_i Fixed site size = maximum local resources
- v_i Variable site occupation fraction
  so if $v_i > 1$ then site needs external resources
  $\Longrightarrow$ Site **Weight** ($S_i\, v_i$) = Site `*population*'
- e_{ij} Fractional Edge values $0 \leq e_{ij} \leq 1$
  $\Longrightarrow$ Edge **Weights** ($S_i\, v_i\; e_{ij}$)
  = '*Trade*' (interaction) going from site i to site j

How we describe our networks

- Site Strength = $\Sigma_j (S_i\ v_i\ \ e_{ij})$
  = Total Trade Going Out



We find the values of site occupation (v_i) and trade levels (e_{ij}) that give us the most efficient use of resources (lowest energy) for given input of site size (S_i) and distances (d_{ij})

Optimisation of what?

`Energy', resources

$$H =$$

Isolated sites have optimal size $v_i = 0.5$

$$-\kappa \sum_i S_i v_i (1 - v_i)$$

Trade (interactions) bring benefits

$$-\lambda \sum_{i,j} V(d_{ij} / D).S_i v_i .e_{ij} .S_j v_j$$

Increasing 'population' has a cost

$$+ j \sum_i S_i v_i$$

Each trade link has a cost

$$+ \mu \sum_{i,j} S_i v_i e_{ij}$$

Efficiency?

- Need not be space filling in any sense.
- Need not be lowest number of links needed to connect all sites (Minimal Spanning Tree).
- ‘Deliberate Waste’ -
  may well favour redundancy to reduce path lengths, to increase possible interactions, to increase resilience to change.

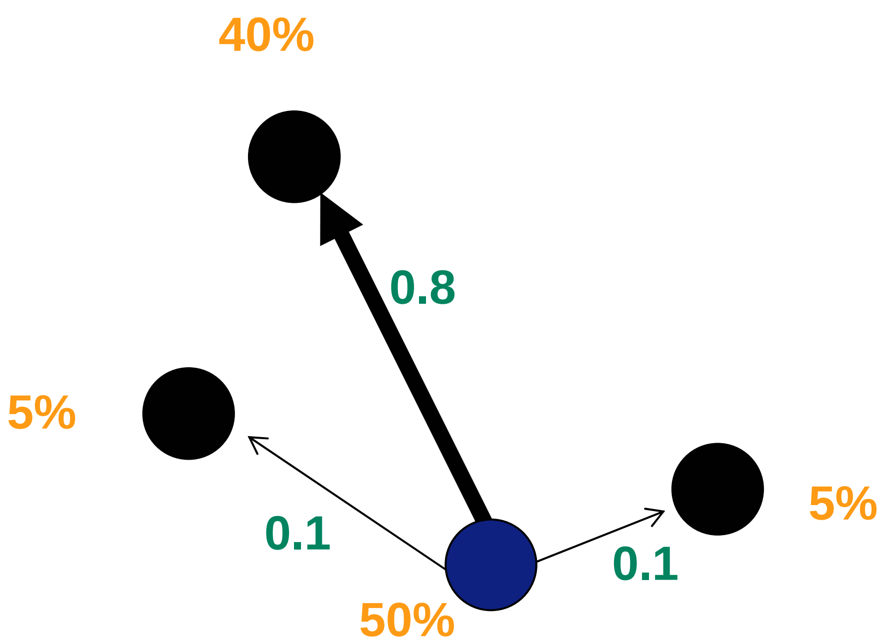
Analysis

- Now working with 34 sites (not 19 as before)
- Can not assign parameter values in model from physical data so make comparisons between different data sets
  e.g. vary one parameter, hold rest fixed.
  This represents slow evolution where system remains in effective equilibrium.
- For any given set of (reasonable) values:
  a) can analyse intrinsic parameters
  b) can perform further `game' to analyse properties e.g. emulate trade in physical object

Ranking Method

- This is equivalent to asking the percentage of time spent at each node by a random walker on the network.

  The walker chooses to follow a link with probability proportional to its strength. (Other choices possible).

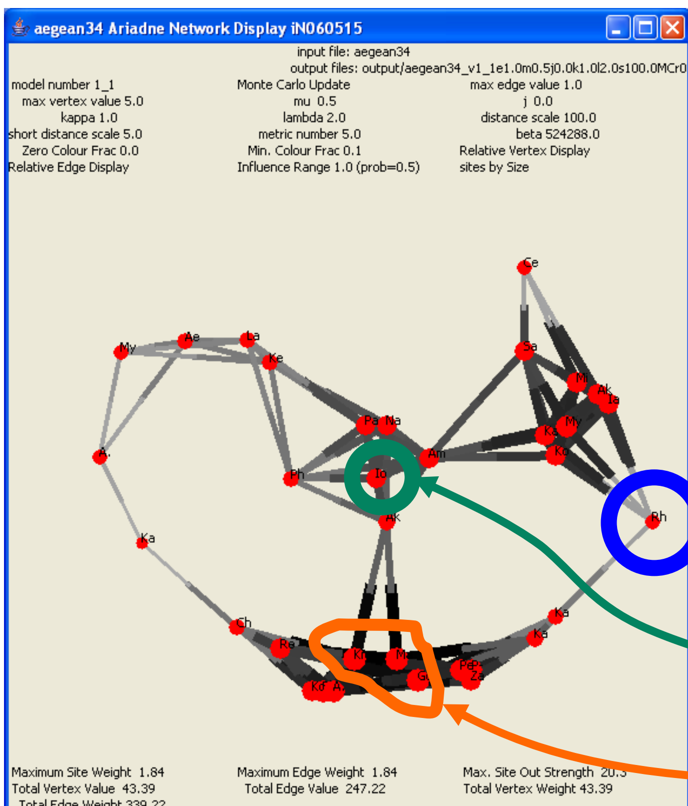


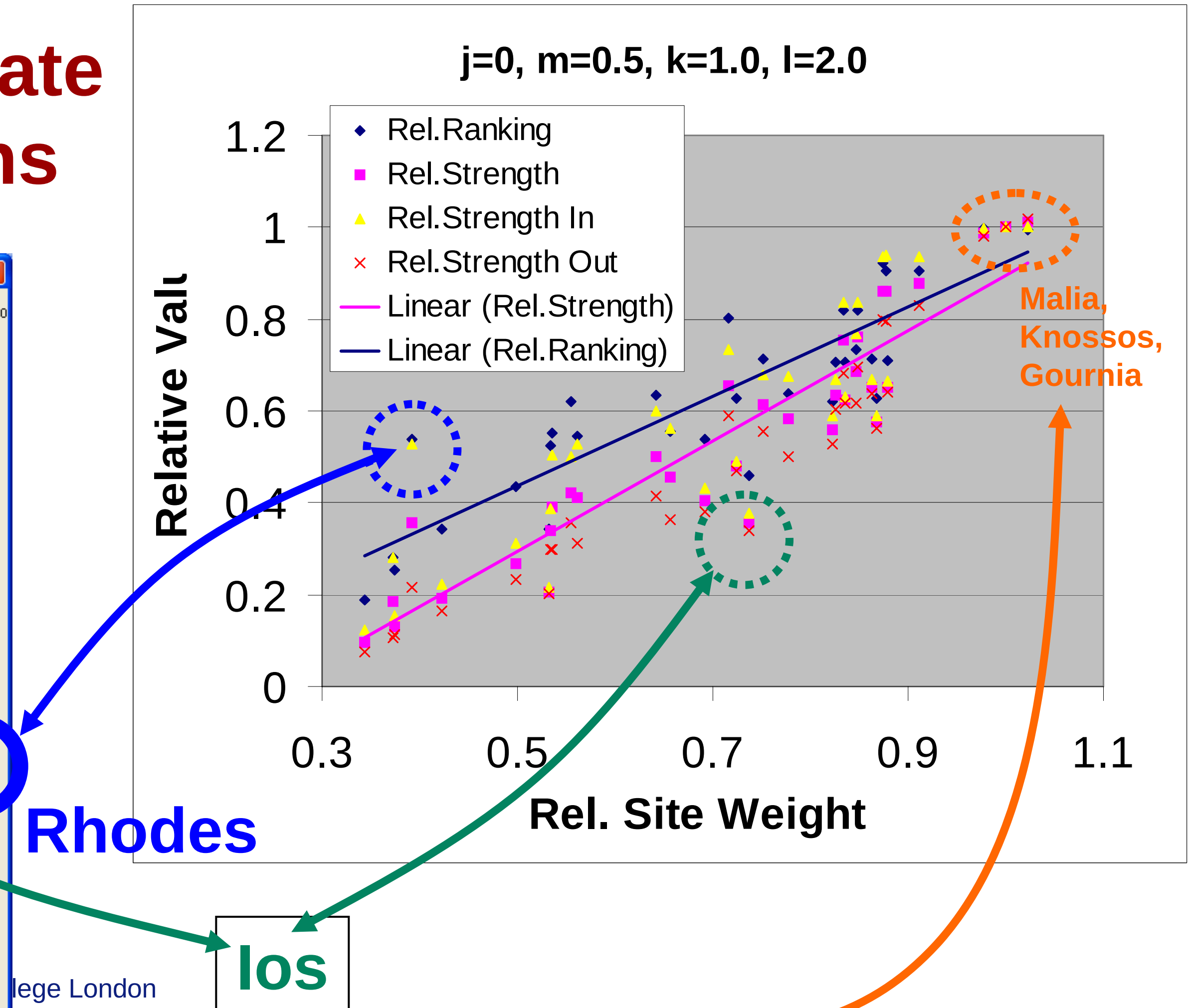
As used by Google UK and Hage and Harary 1991

Quantity vs. Size plots

Rank (a la Google) or total trade in or out of a site (Strength in, out, both) basically scale with site size.

⇒ Deviations indicate special situations

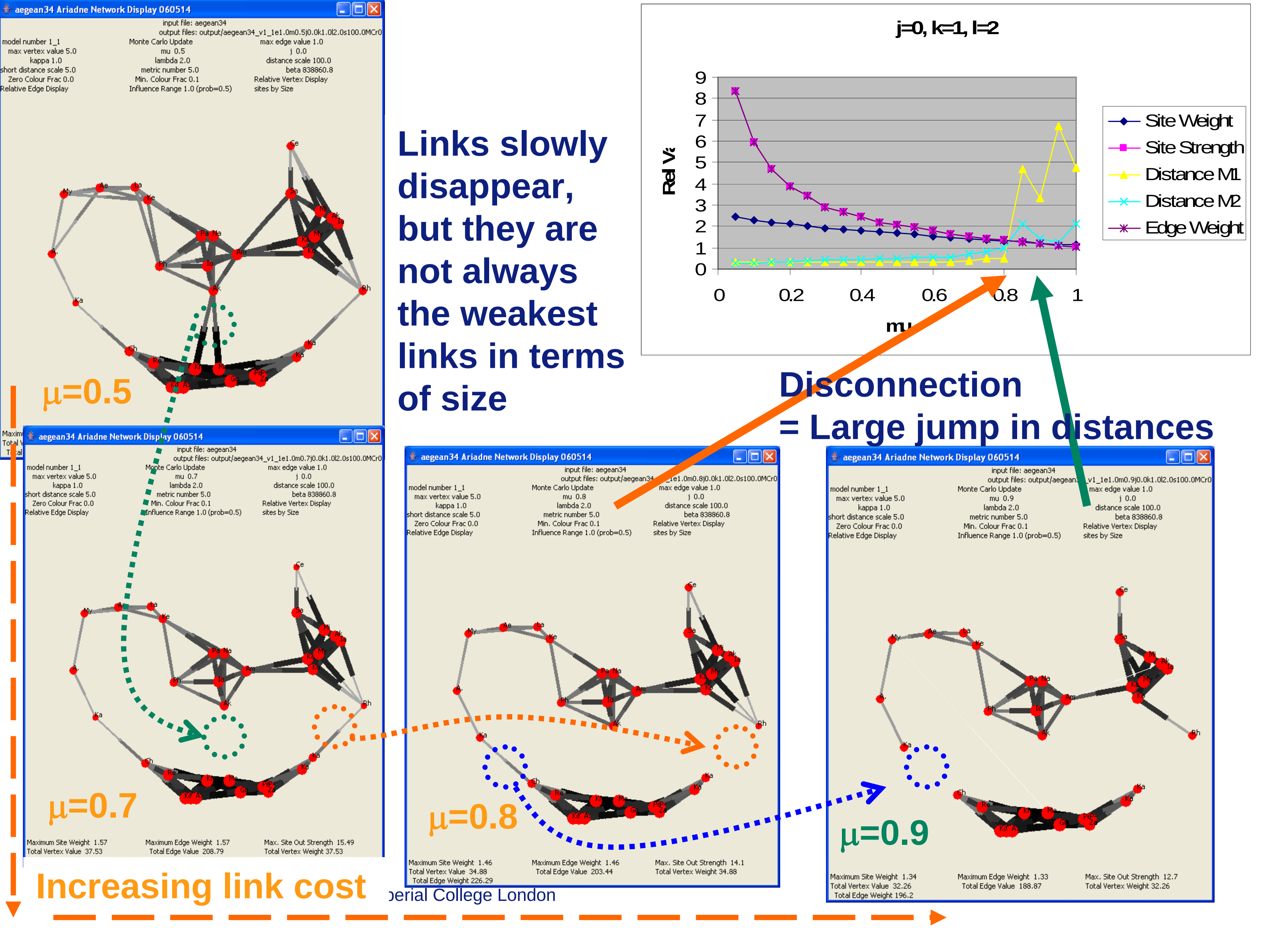




Vary cost of adding a link (μ) (j=0, κ=1.0, λ=2.0, d=100km)

- As the cost the cost per link μ increases, the most “fragile” links disappear
- Seem to be the longer length links
- These are *not* the `weak’ links (low value e_{ij}, thin, light coloured).

- For these values j=0, κ=1.0, λ=2.0, the biggest and smallest sites differ in size by a factor of 3.


Links slowly disappear, but they are not always the weakest links in terms of size
Disconnection = Large jump in distances
j=0, k=1, l=2
Rel Va
mu
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5
6
7
8
9
Site Weight
Site Strength
Distance M1
Distance M2
Edge Weight
aegean34 Ariadne Network Display 060514
μ=0.5
μ=0.7
μ=0.8
μ=0.9
Increasing link cost
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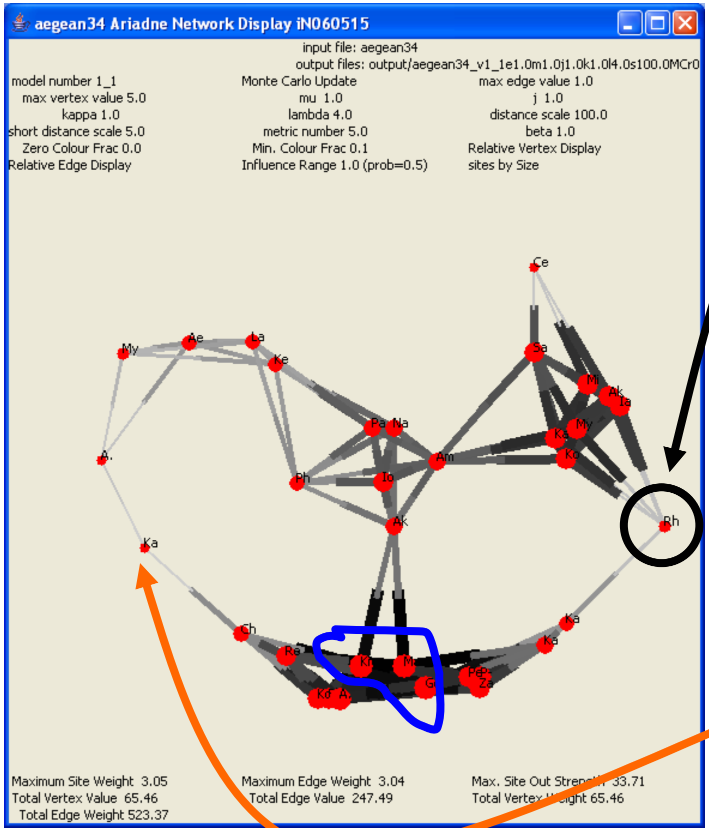
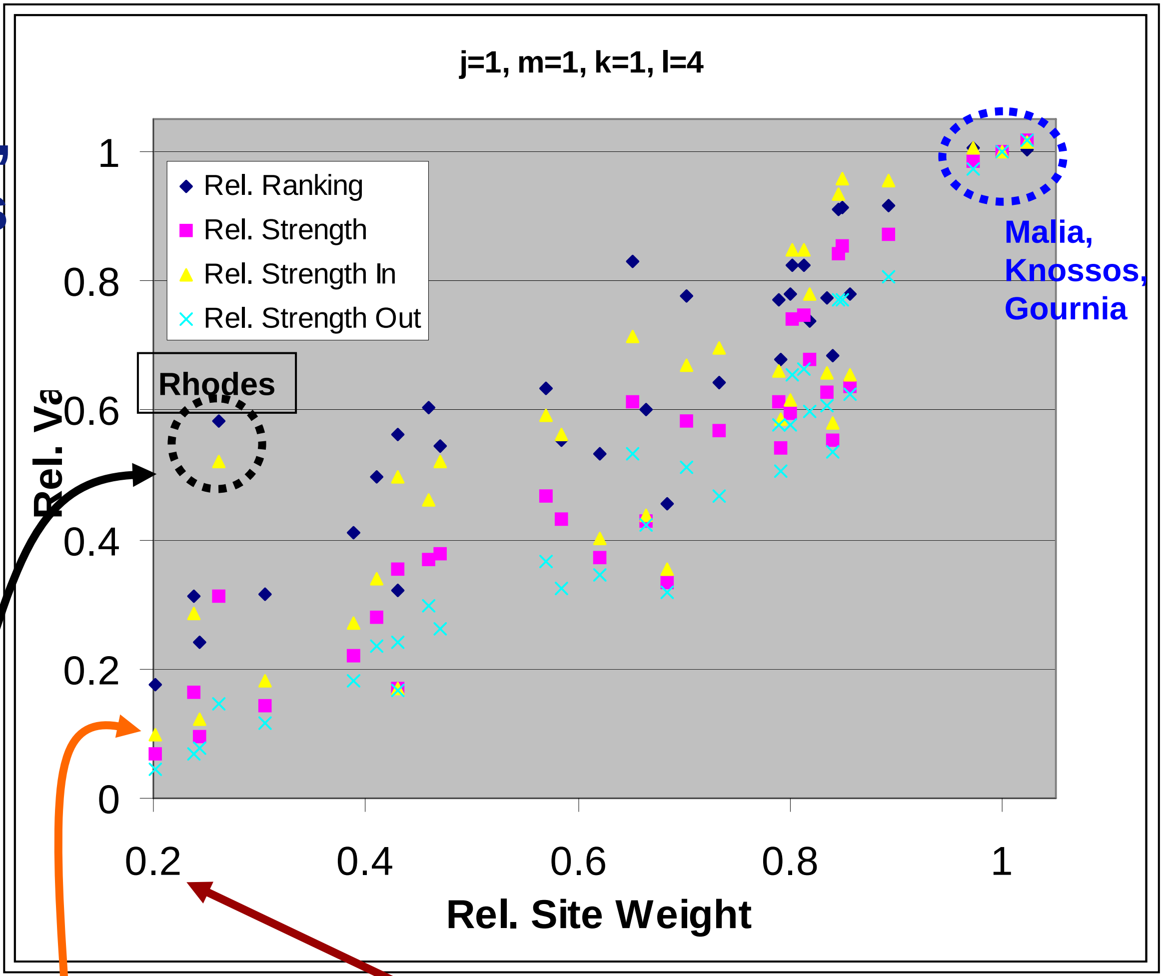
Interaction Favoured = Large Range in Site Sizes

Largest range in site sizes occurs for large λ, that is when trade gives great benefits





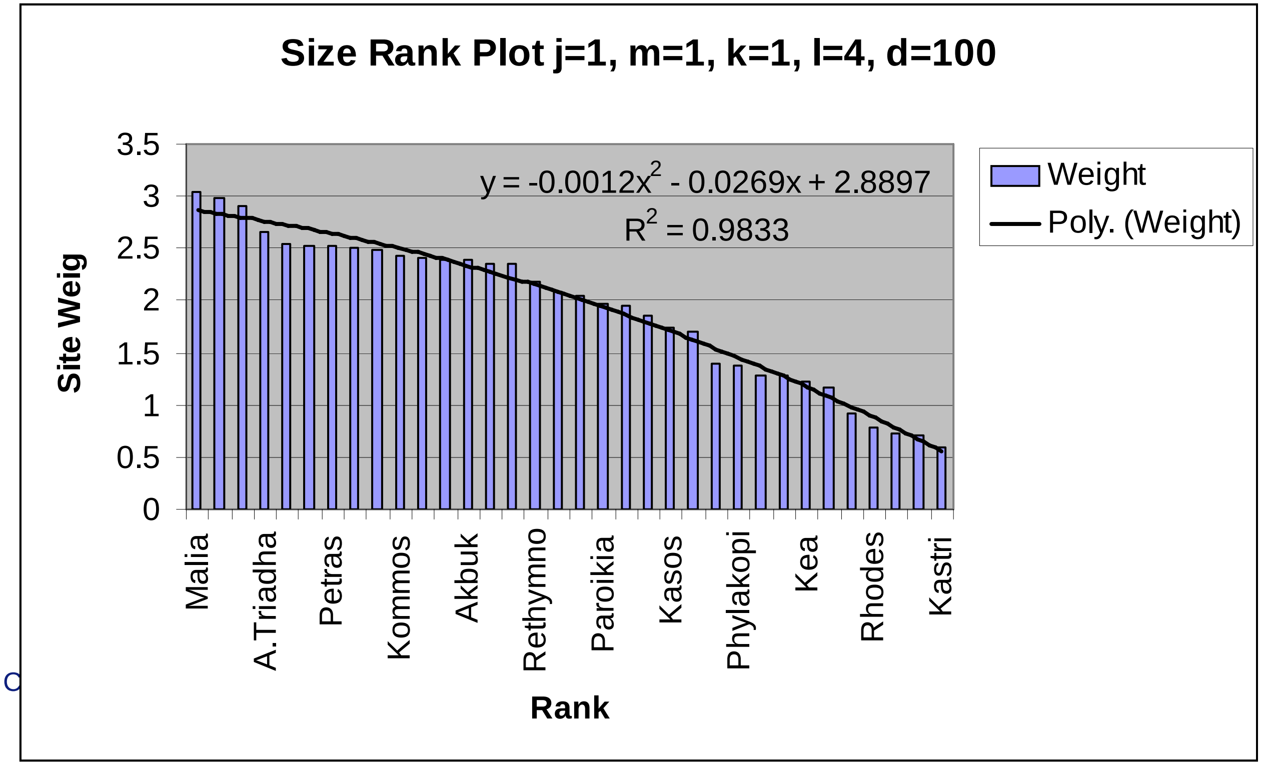
Kastri

Max./Min. site size =4.0

j=1.0, μ=1.0, κ=1.0, λ=4.0

Site Size Hierarchy

- Increased benefits or lower costs for trade (interactions) needed to bring wider distinctions in site sizes
  largest/smallest size ~ 4
- Still no large macrocephaly (big head)
  no Zipf-like size/rank distribution

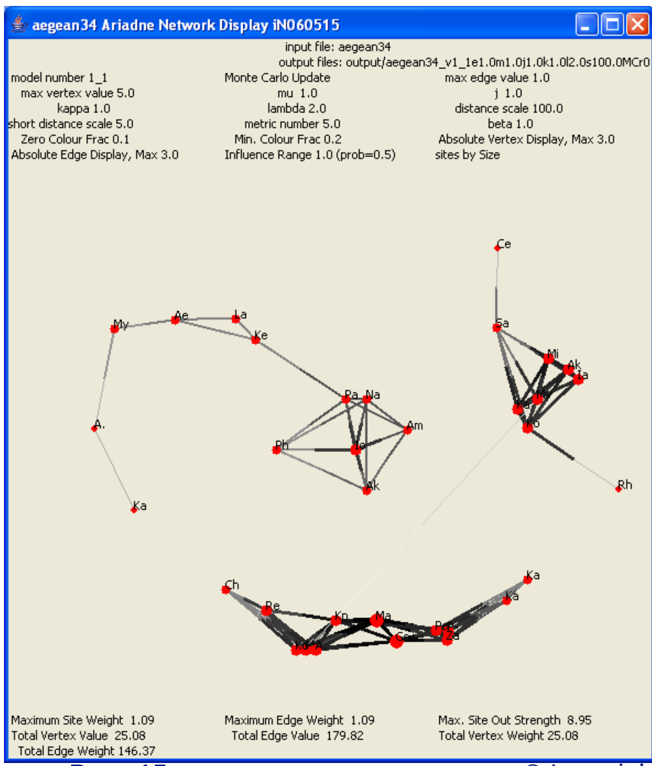


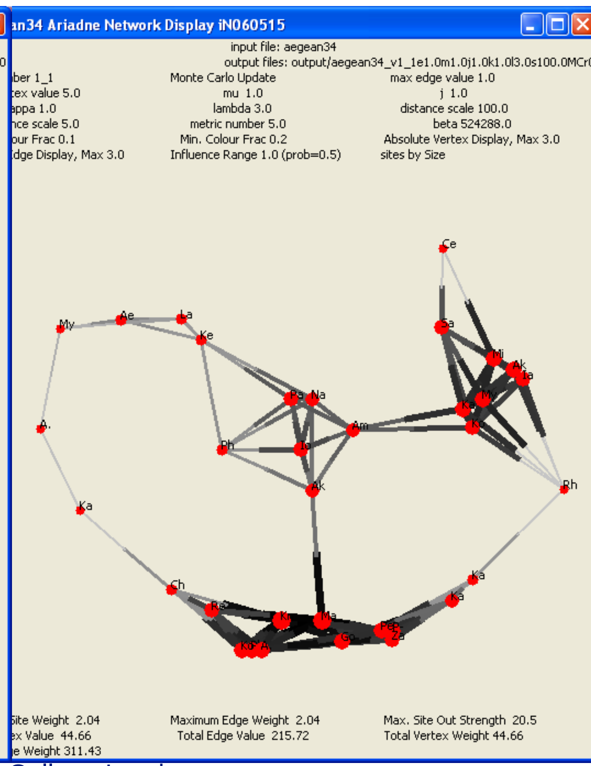
⟹Alter site term?

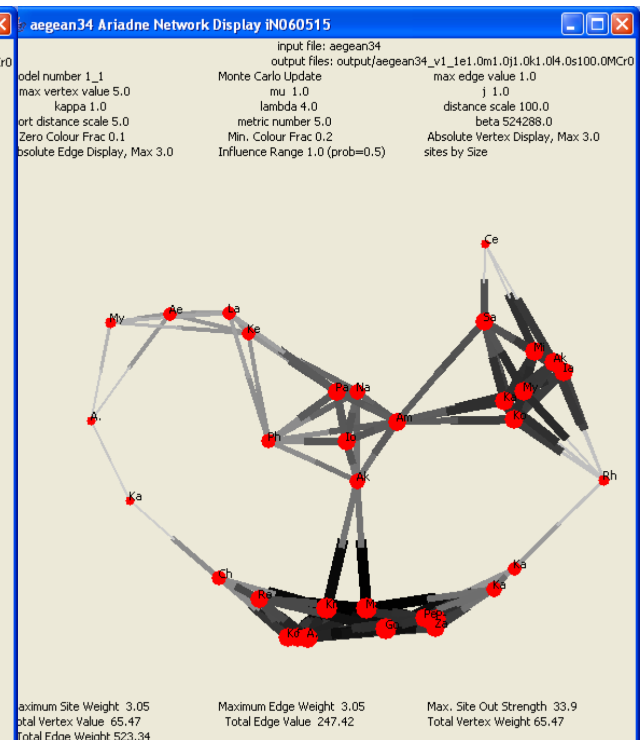
Making Trade Benefits Greater – Increase λ

- More links and larger sites
- Same shapes as before when changing trade costs
  $\Longrightarrow$ robustness of predictions

λ=2.0 **λ=3.0** **λ=4.0**



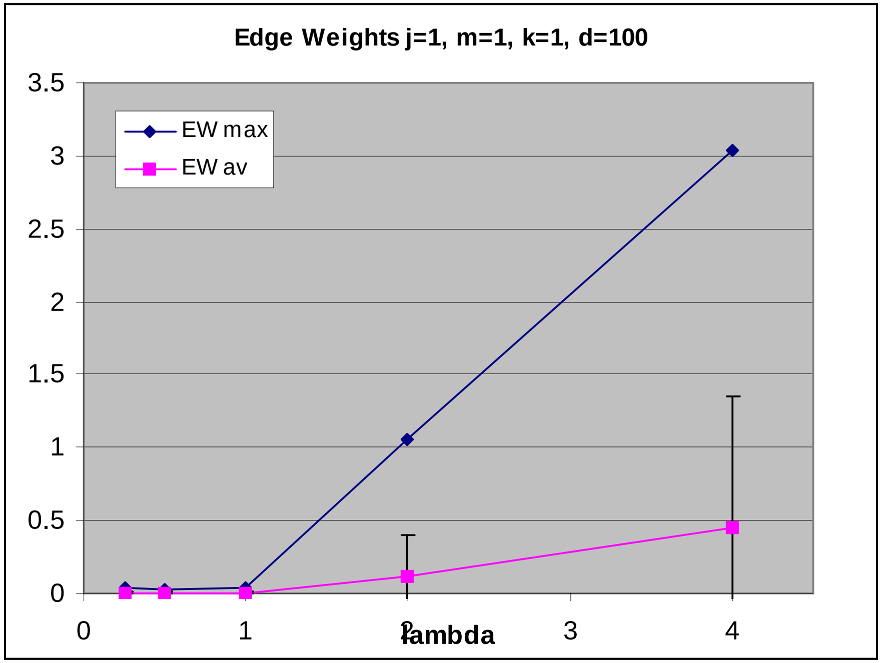


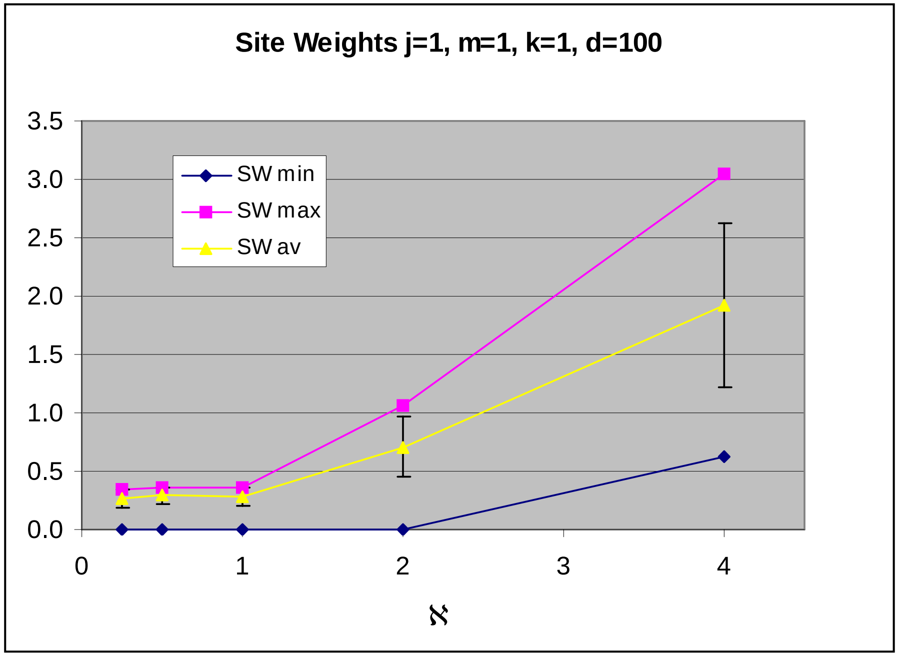


j=1.0, μ=1.0, κ=1.0, λ=4.0, d=100km (absolute value display)

Making Trade Easier – Increase λ

- Increase in site sizes and in number of edges starts suddenly when **λ/κ~1**

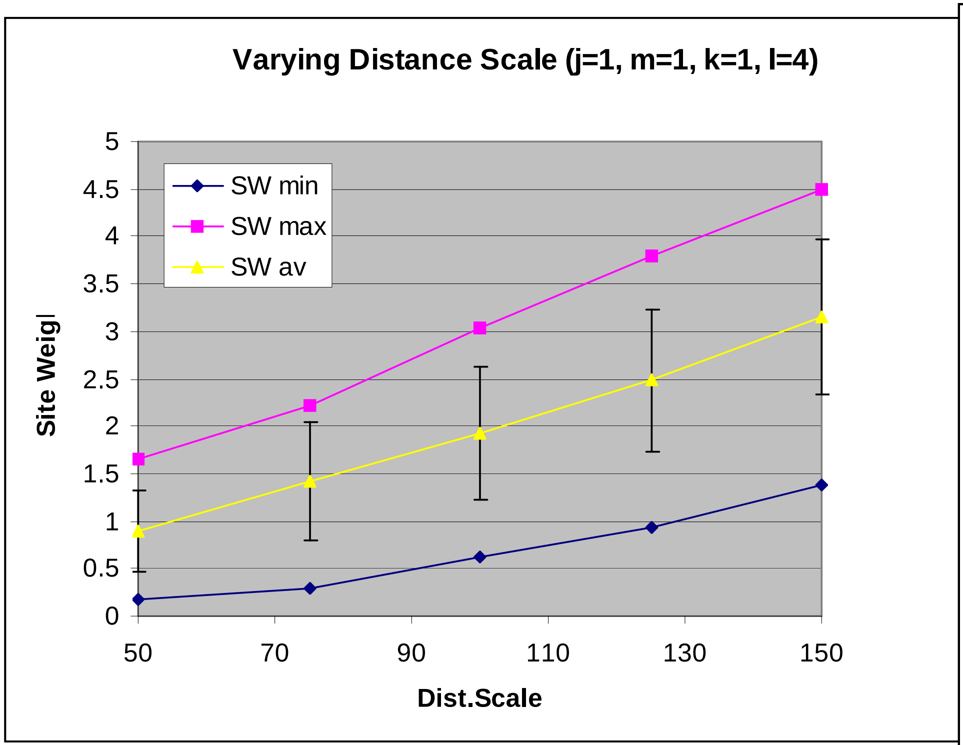


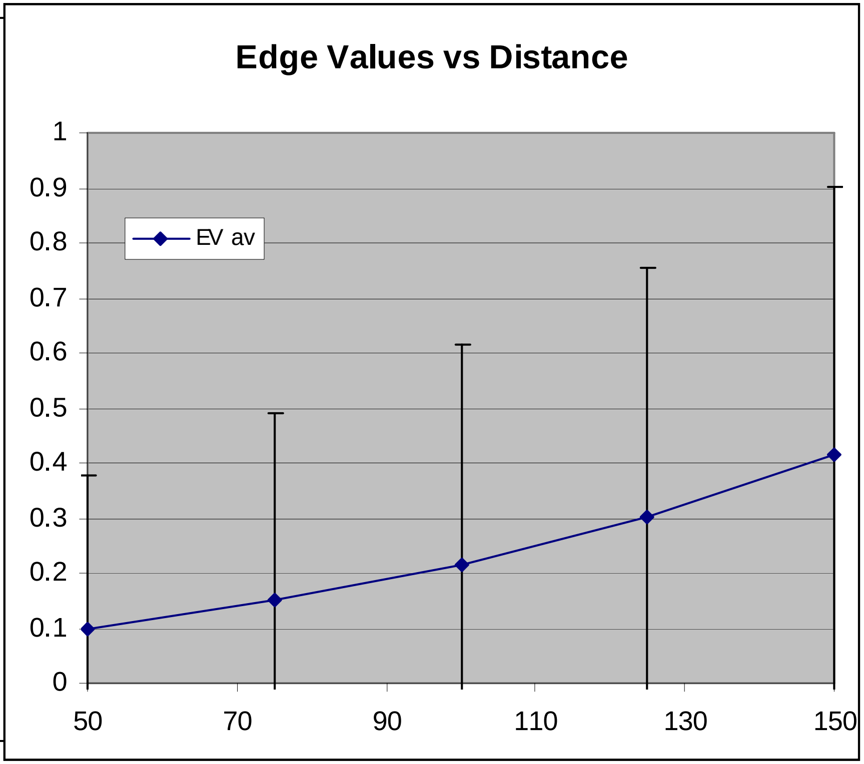


j=1.0, μ=1.0, κ=1.0, d=100km

Varying Distance Scale

- Both sites and edges increase in size





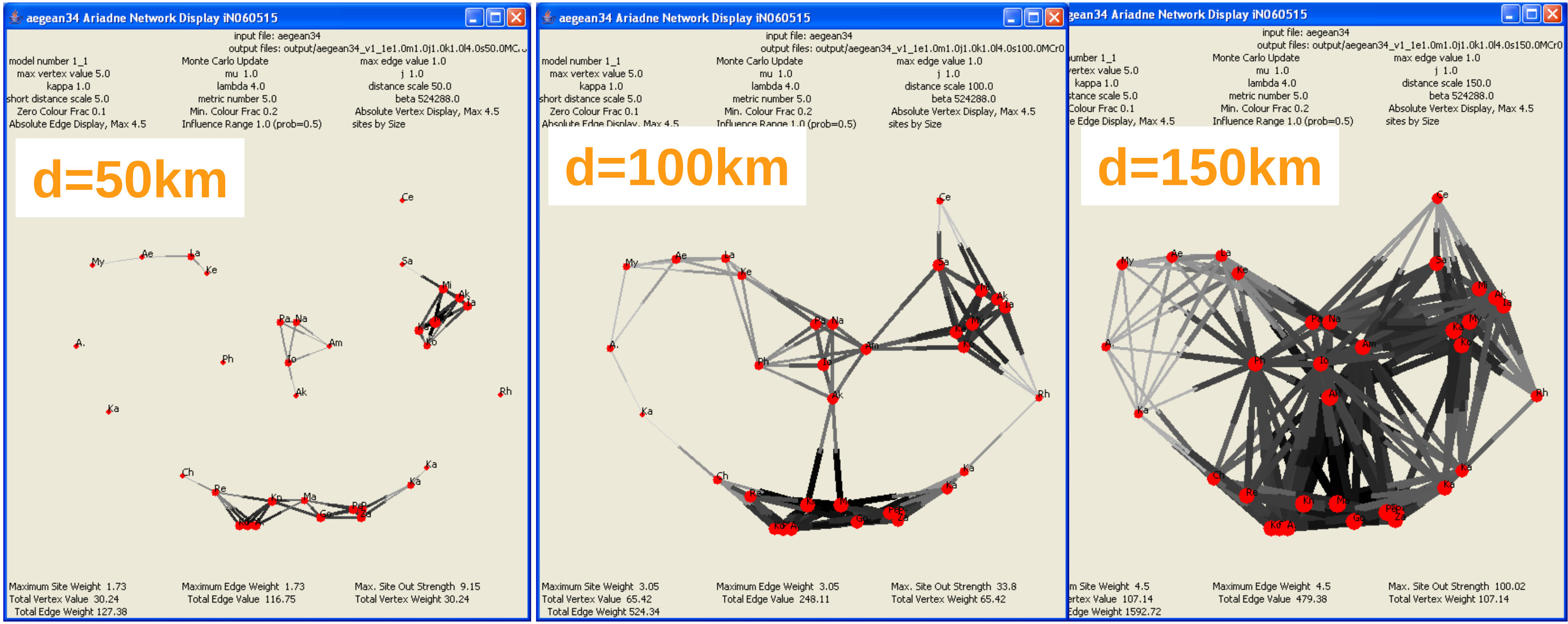
j=1.0, μ=1.0, κ=1.0, λ=4.0

Vary Distance Scale

- The distance scale of about 100km (rather than 50km or 150km) is critical to these networks ⇒ **SAIL is crucial**

Precise value will depend on exact form of potential,

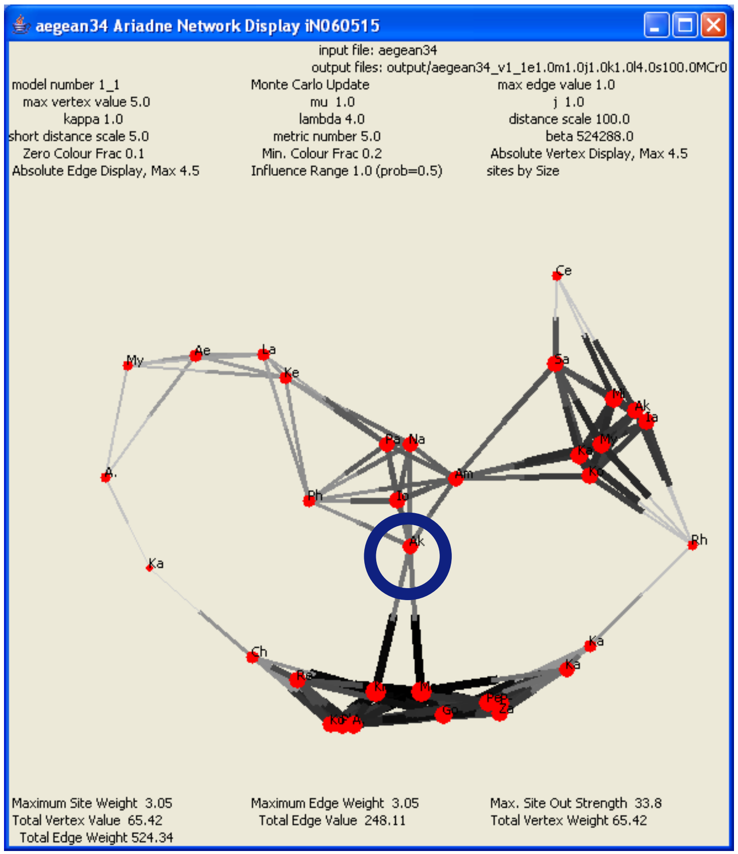
importance is in rough scale and robustness to changes in potential

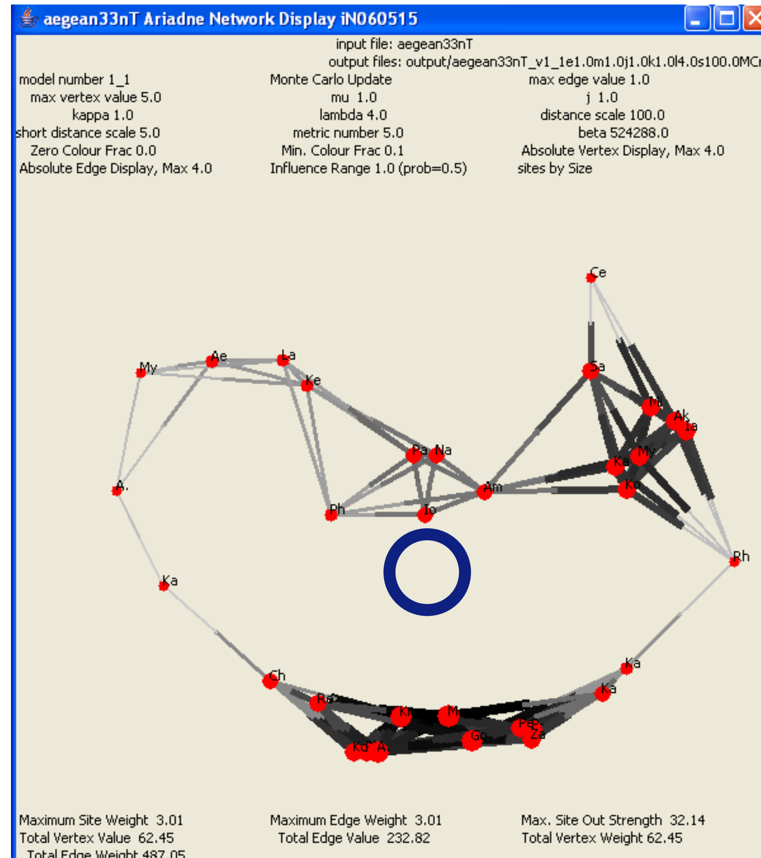


$j=1.0,\ \mu=1.0,\ \kappa=1.0,\ \lambda=4.0$

Thera eruption – remove Akrotiri

- A simple removal of this one site does not seem to produce much change in even in central Cretan sites
  - Knossos, Malia, Gournia still largest sites



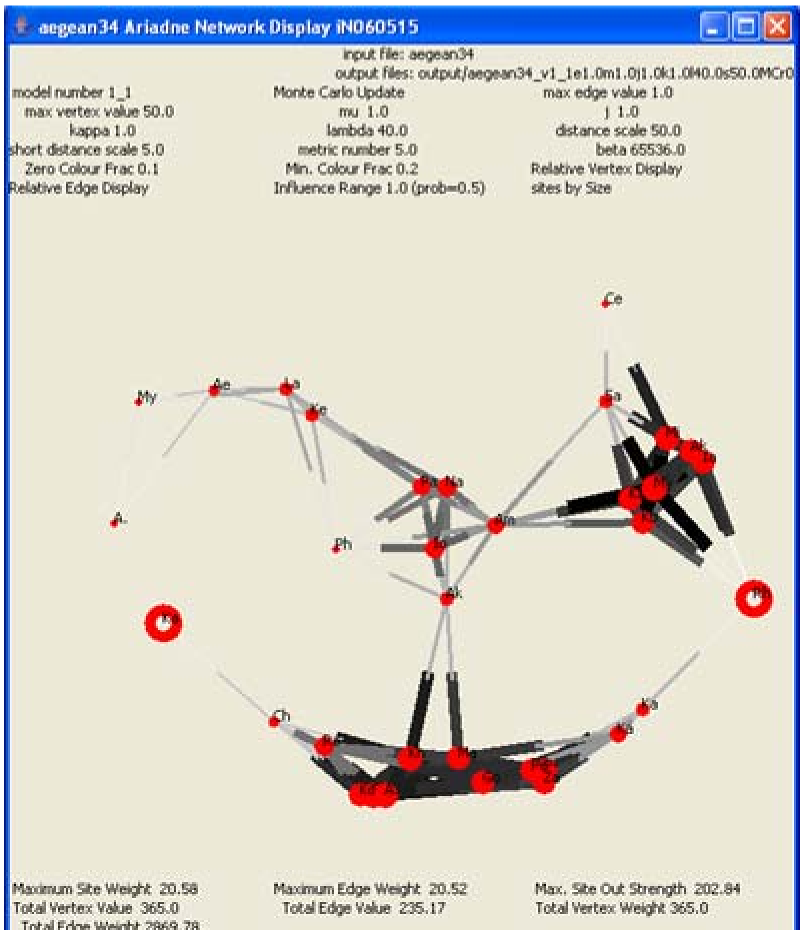


Summary

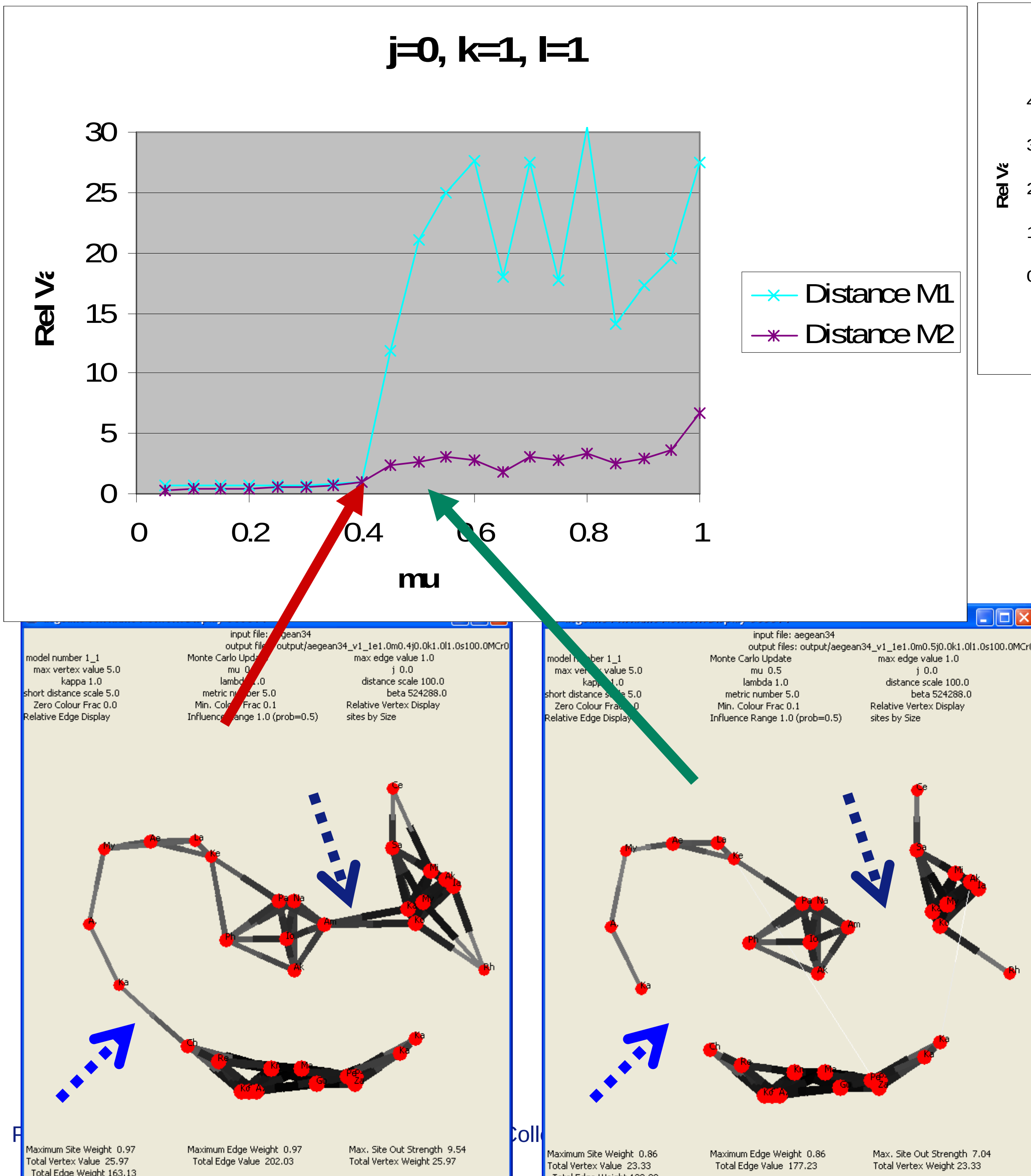
- Starting to extract basic results systematically
- Some behaviour looks interesting to an archaeologist – difference in the behaviour of the links from Crete to the rest of the system
- Some types of behaviour do not appear to be possible – no large site size hierarchy appears but is this good or is this bad?
- Some factors seem to be playing a key role – small differences in physical distance from Crete may be significant
- Many options remain to be explored – more analysis tools, more what if scenarios, EBA vs MBA, general time evolution, other data sets

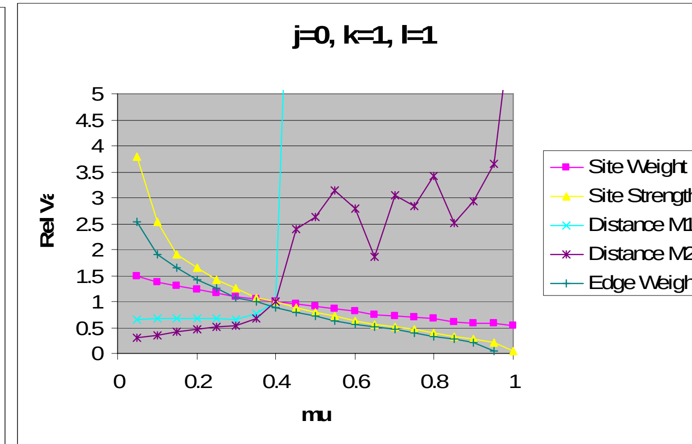
Can we get anything we like?

- No – may be able to get many things but by no means all



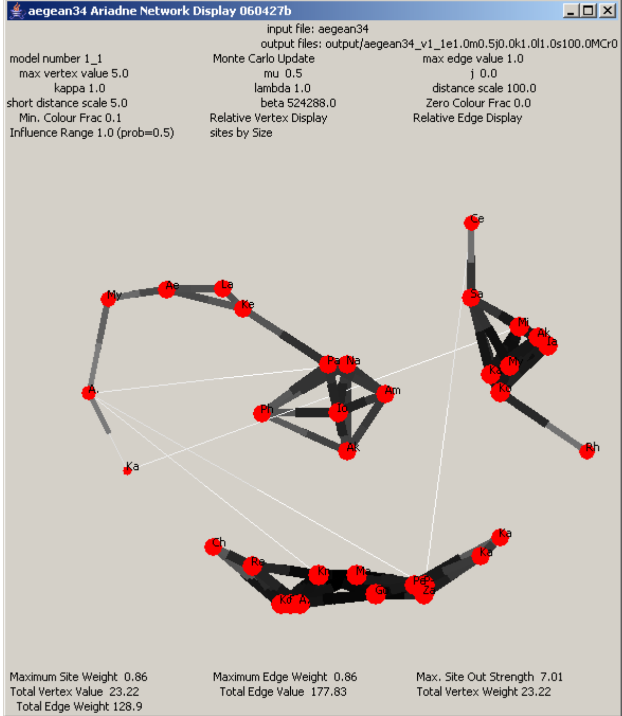
d=50km,
j=1, μ=1, κ=1, λ=40

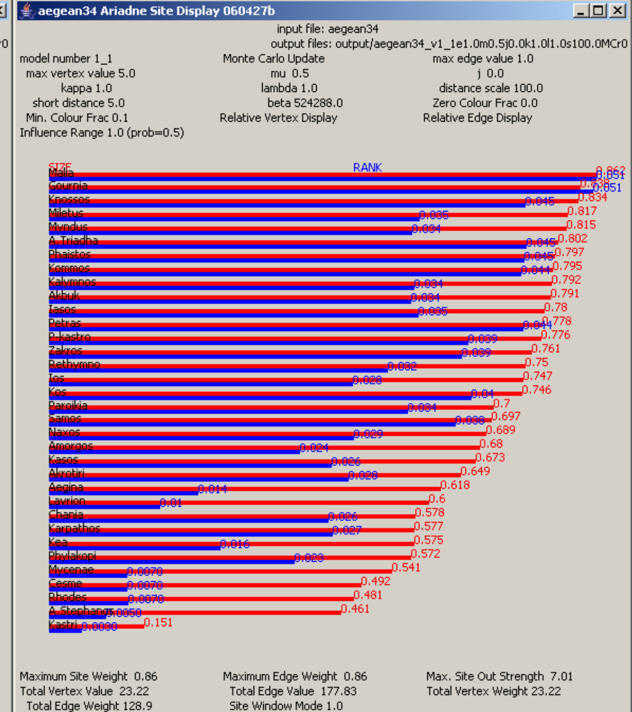




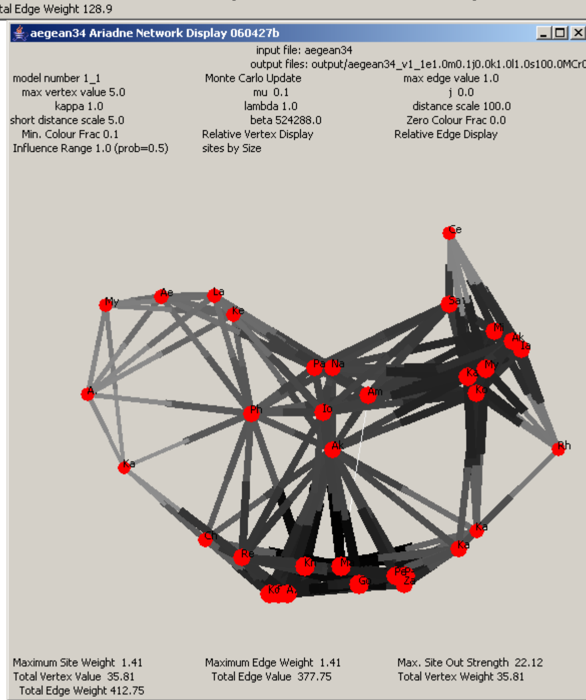
Fixed j, κ and λ

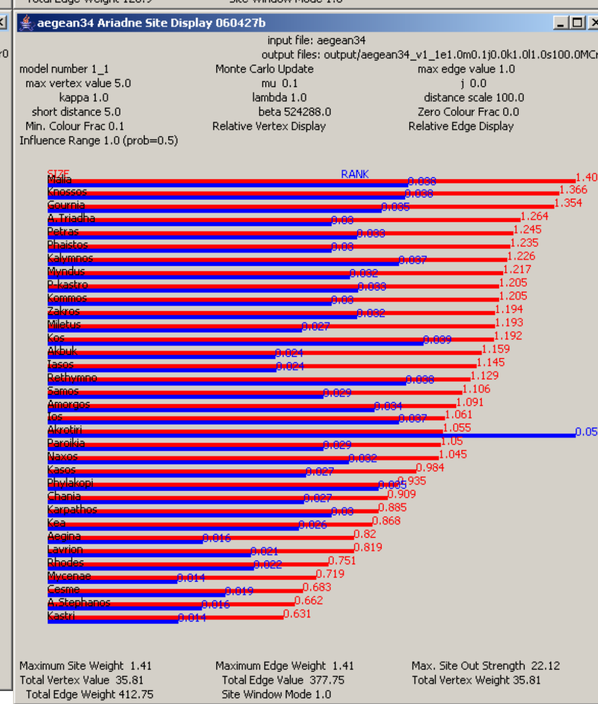
μ=0.5, j=0, κ=λ=1.0

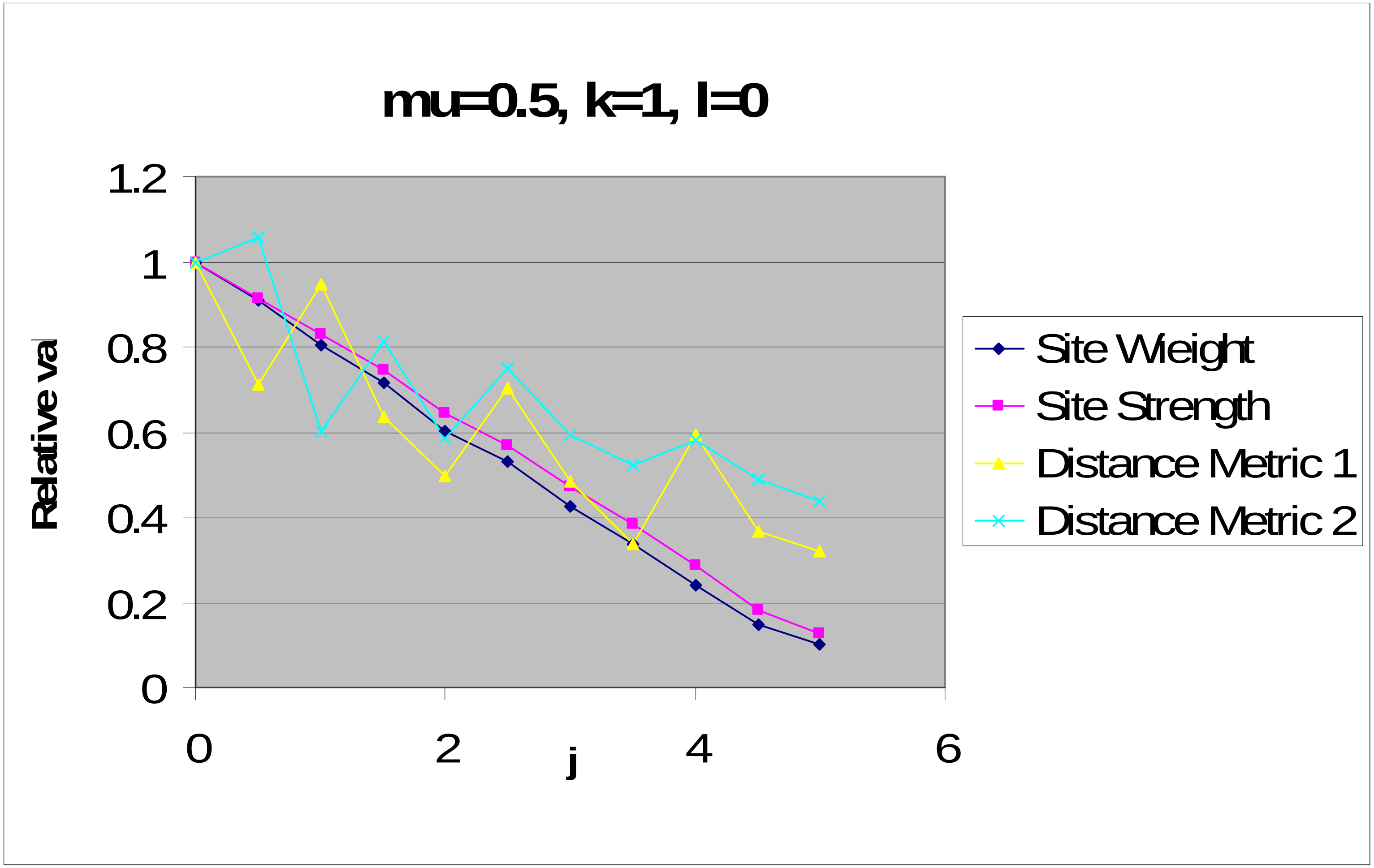




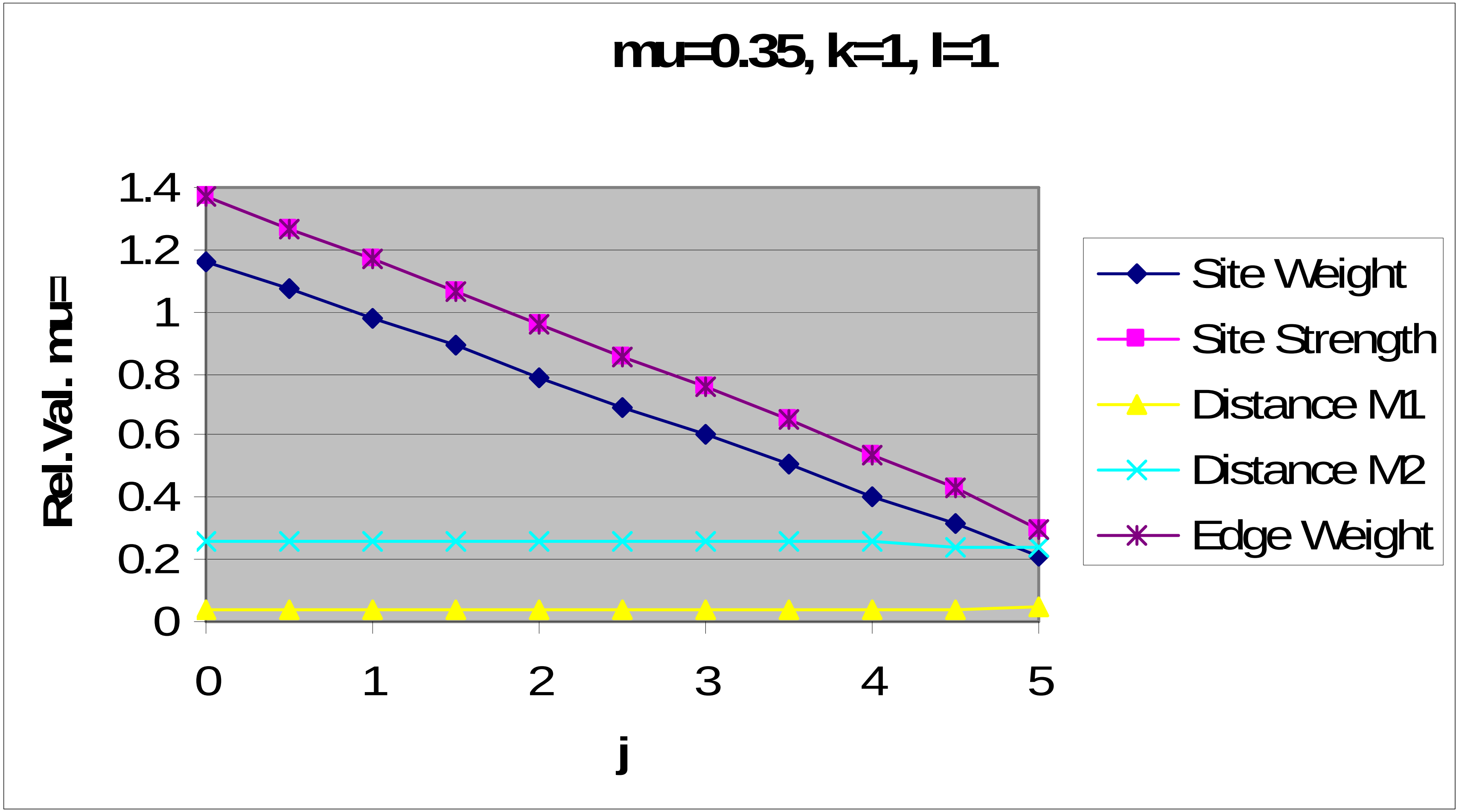
μ=0.1, j=0, κ=λ=1.0



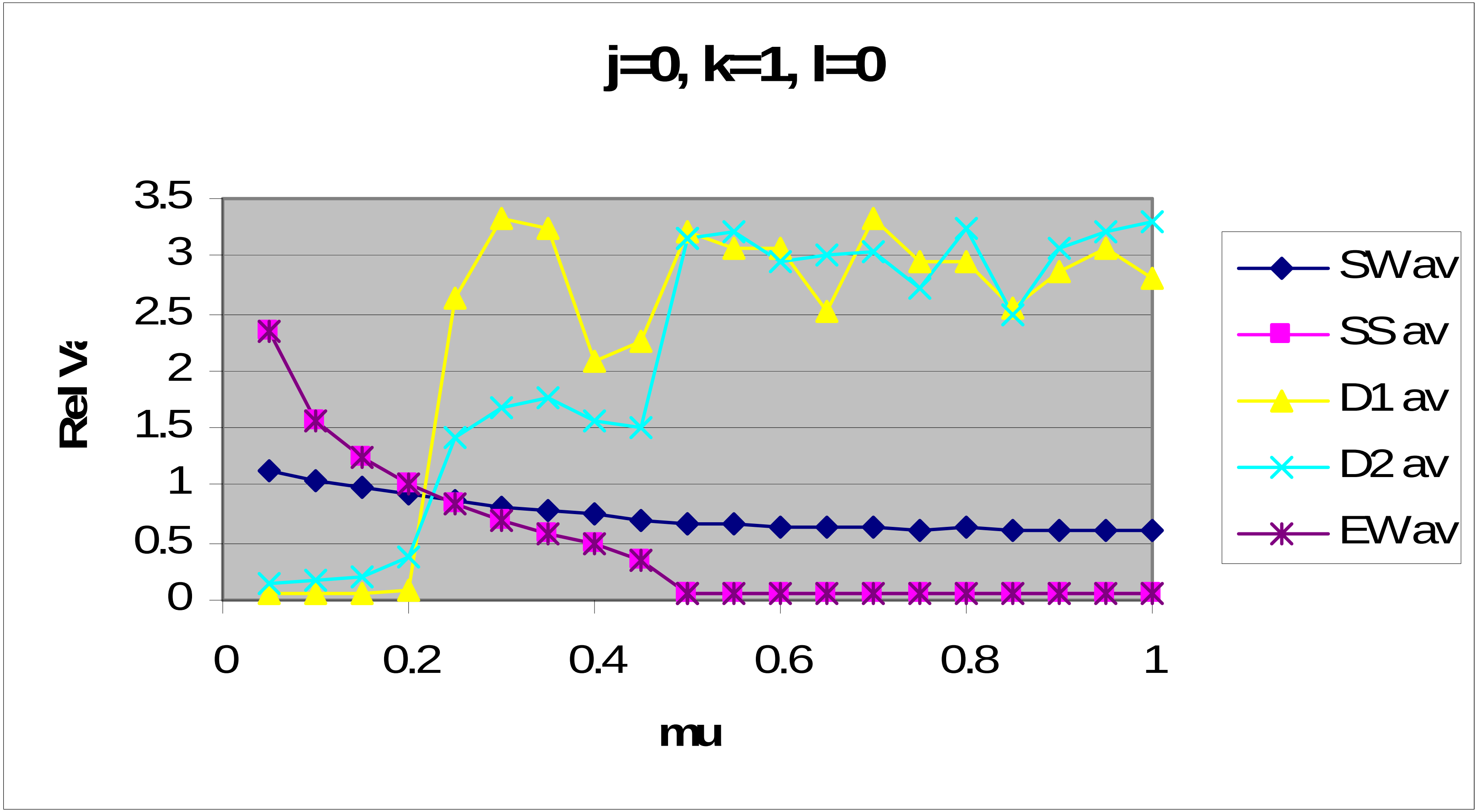




mu=0.5, k=1, l=0
1.2
1
0.8
0.6
0.4
0.2
0
Relative val
0
2
j
4
6
Site Wieight
Site Strength
Distance Metric 1
Distance Metric 2



mu=0.35, k=1, l=1
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Rel.Val. mu=
0
1
2
3
4
5
j
Site Weight
Site Strength
Distance M1
Distance M2
Edge Weight


j=0, k=1, l=0
3.5
3
2.5
2
1.5
1
0.5
0
Rel Va
0
0.2
0.4
0.6
0.8
1
mu
SWav
SS av
D1 av
D2 av
EWav

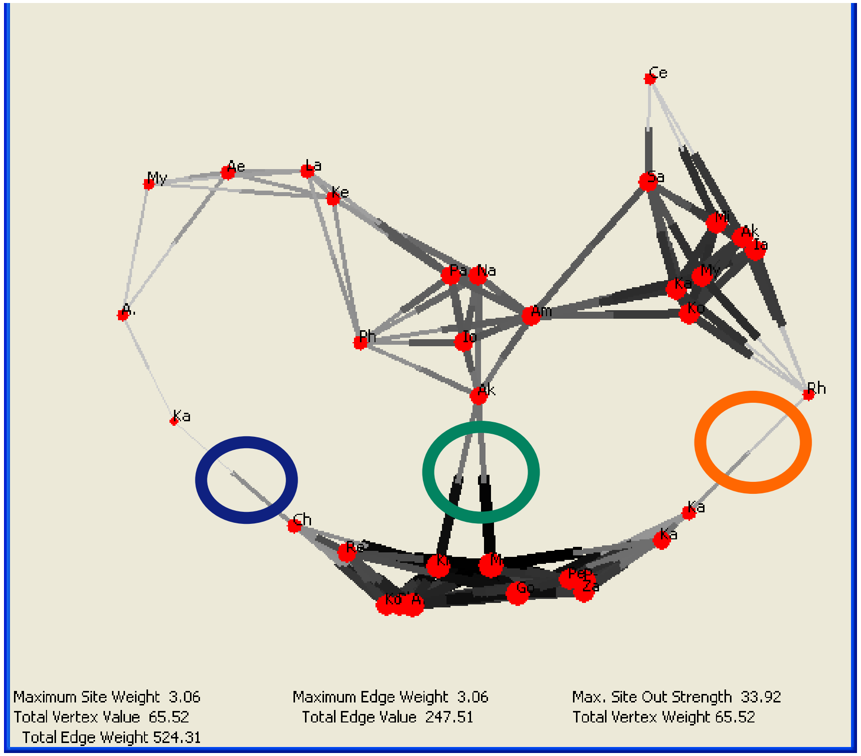
Varying μ

- Reducing μ produces more edges (trade)
  total edge weight 130 to 410 as μ goes from 0.5 to 0.1
- Site sizes go up in response but ratio of largest to smallest does not
  23.5 to 35.8 for total
  but ratio biggest/smallest goes from 1.8 to 2.3
- At extreme we make central geographical location important Akrotiri and Knossos swap from top ranking (2rwf) with mediocre one ranking (1rwf)

μ	Smallest Site	Biggest Site	Total Pop.	Total Trade
0.5	0.47	0.86	23.5	128.6
0.35	0.53	1.03	27.3	176.5
0.25	0.57	1.18	30.9	246.9
0.1	0.6	1.4	35.8	412.2

Increasing Cost of trade (link cost – increase μ)

- **Links slowly disappear.**
- **The weakest links (link variable e_{ij} small) *not* the first to go.**
- **Small differences in *distance* just beyond daily range make a huge difference**



Knossos & Malia – Akrotiri	120km
Karpathos – Rhodes	116km
Chania – Kastri	110km