25th March 2012, Southampton

The Connected Past symposium

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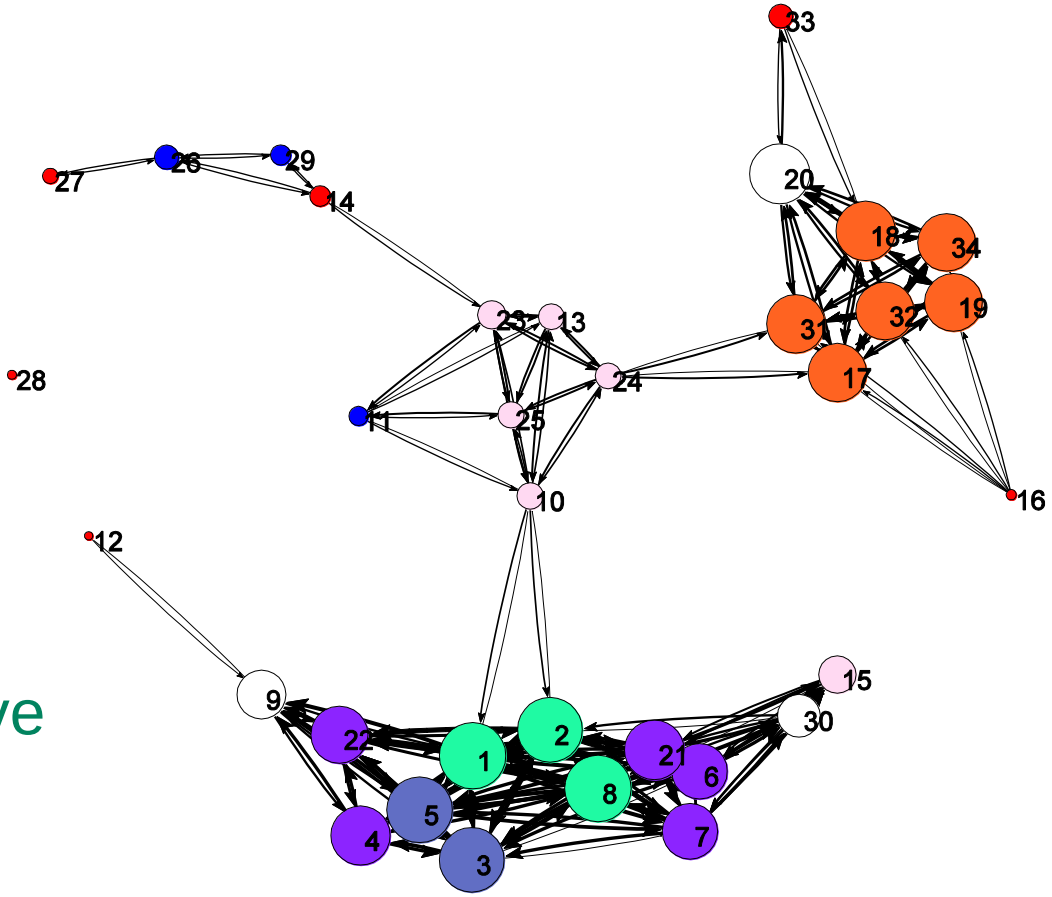
Which Network Model Should I Use?

TOWARDS a Quantitative Comparison of Spatial Network Models in Archaeology



Tim Evans, Physics Dept & Complexity and Networks Group

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The Problem

- Archaeology can be “Site Centric”
  - Regional and global interactions hard to consider
- Networks emphasise interactions

Given a set of sites and their locations can we understand their interactions by creating a network of edges?

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Island Archipelagos as an Ideal Network

- Vertices = Major Population or Resource Sites
- Edges = Exchange between sites
  - physical trade of goods *or* transmission of culture
  - direct contact *or* island hopping links

- ***Sea isolates communities → Natural Vertices***
- ***Interactions controlled by physical limitations of ancient sea travel → Simple Links***
- ***Coastal Sites often isolated like islands due to geography and difficulty of ancient land travel***

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Different Spaces

- We shall consider our sites to lie in two-dimensional geographical space

- Alternative is to consider sites located in some artefact space
  - Frequency counts of objects found at a site define a vector in a large-dimensional space

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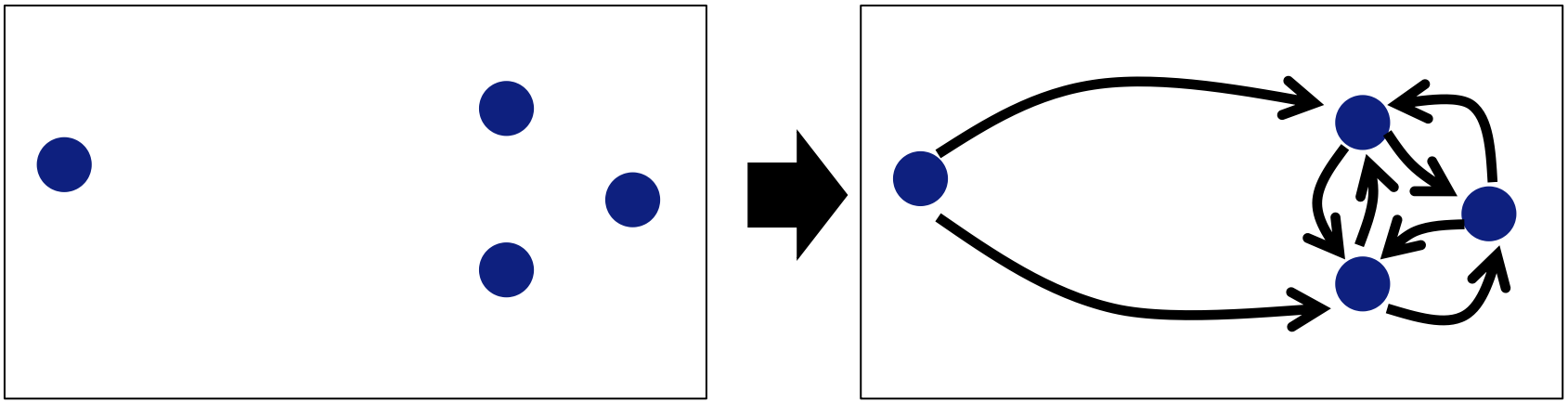
Edge Models

An edge model generates a set of edges between a given set of edges



Note a different type of model defines ‘Spatial Influence’ e.g. Theissen Polygons, XTent

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Deducing Interactions

- Geography controls interactions in models discussed here
  - As the crow flies
  - Accounting for geography by hand estimation
  - Accounting for geography compuationally GIS

  [Terrell 1977; Irwin 1983; Hage & Harary 1991; Broodbank 2000; Collar 2007; Bevan 2010]

- Artefact counts [Terrell 2010; Sindbæk 2007]
- Texts [Isaksen 2006; “Anskar’s Vita” Sindbæk 2008]

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Examples

- PPA – Principal Point Analysis
- MDN – Maximum Distance Network
- Gravity Models
  - Doubly Self-Consistent Models
  - Rhill and Wilson
- Stochastic Models
  - ariadne

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PPA - Proximal Point Analysis

- Equal sized sites
- Sites connect to ***k*** nearest neighbours
- Analyse graph
  - Often without directions on edges
  - Sometimes only local measures used *e.g. Degree*
  - Sometimes global measures used
    e.g. ranking, centrality, betweenness

Examples: Hage & Harary 1991; Terrell 1977; Irwin 1983; Broodbank 2000; Collar 2007

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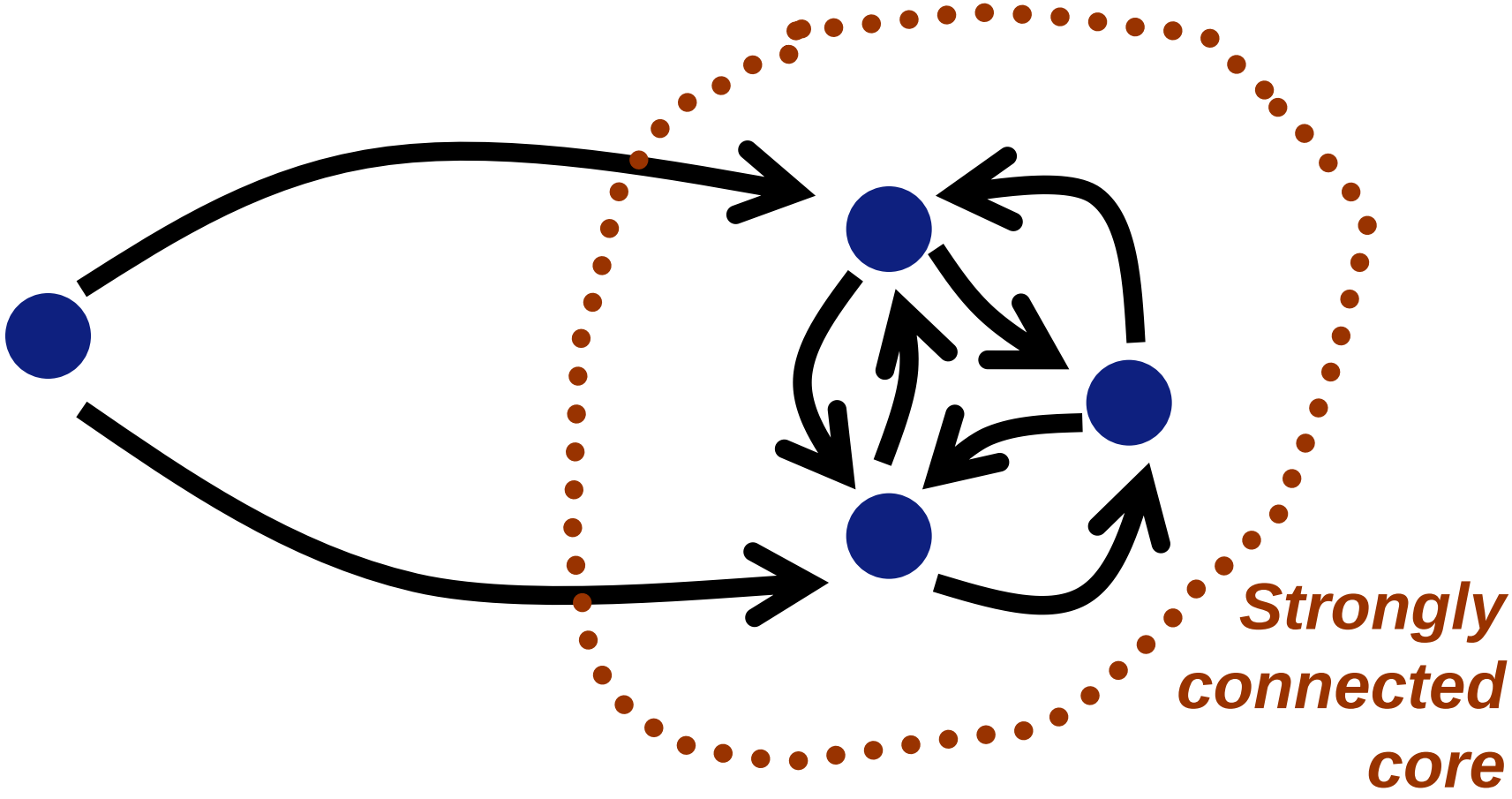
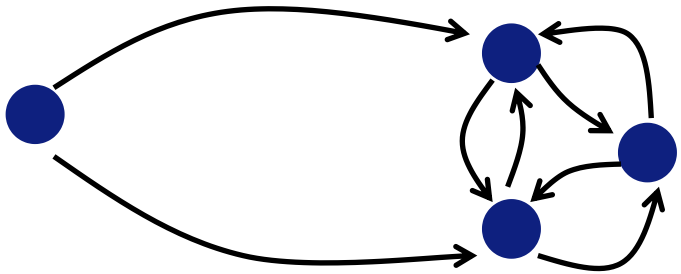
DPPA Example (Directed PPA)

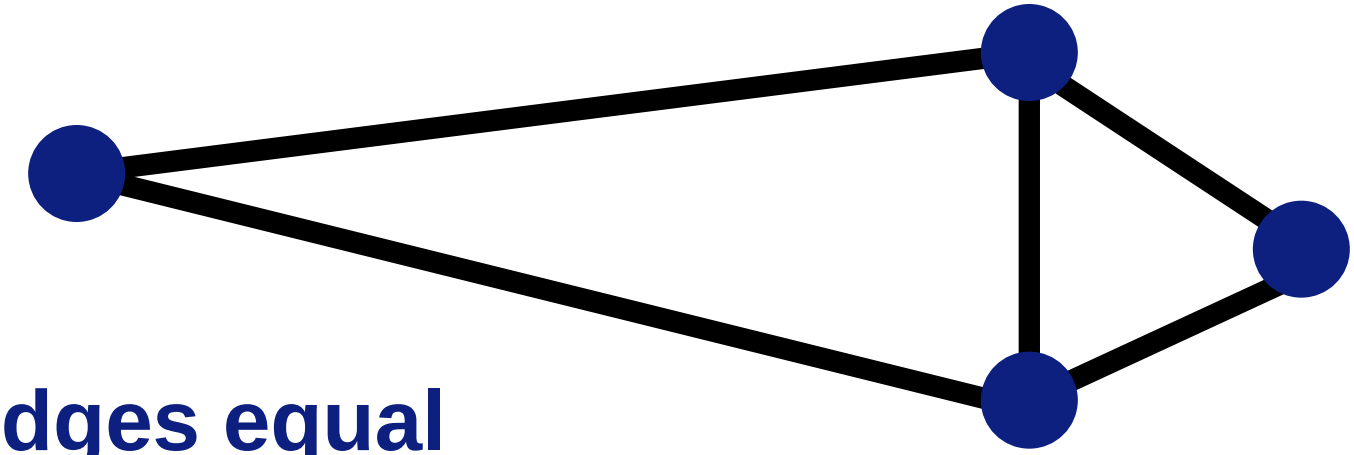
Connect each site to its ***k=2*** nearest neighbours



PPA Example



Ignore direction



- **All edges equal**
- **Network now simply connected**

MDN – Maximum Distance Network

Just connect each site to all sites lying within distance ***D***.

- Used as model for ad-hoc wireless models [e.g. Srinivasa & Haenggi 2010]
- Mathematical analysis possible as ***Random Geometric Graphs*** [e.g.Penrose 2003]
- Not much used in archaeology

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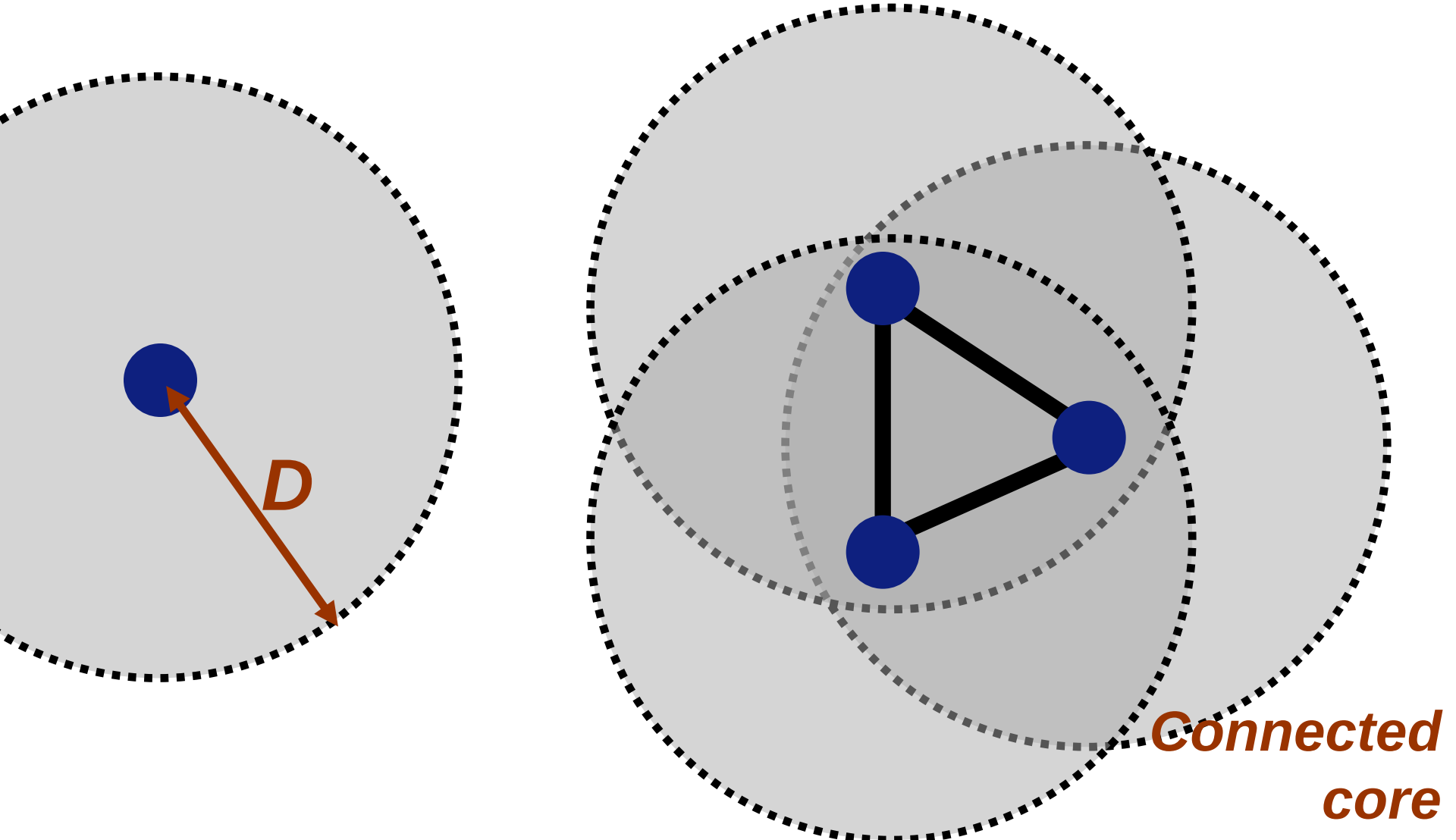
MDN Example

Sites ***D*** or less apart are connected



Doubly Self-Consistent Gravity Model

- Edge from ***i*** to ***j*** is flow F_{ij}

$$F_{ij} = a_i A_i \; b_j B_j \; V(d_{ij}/D)$$

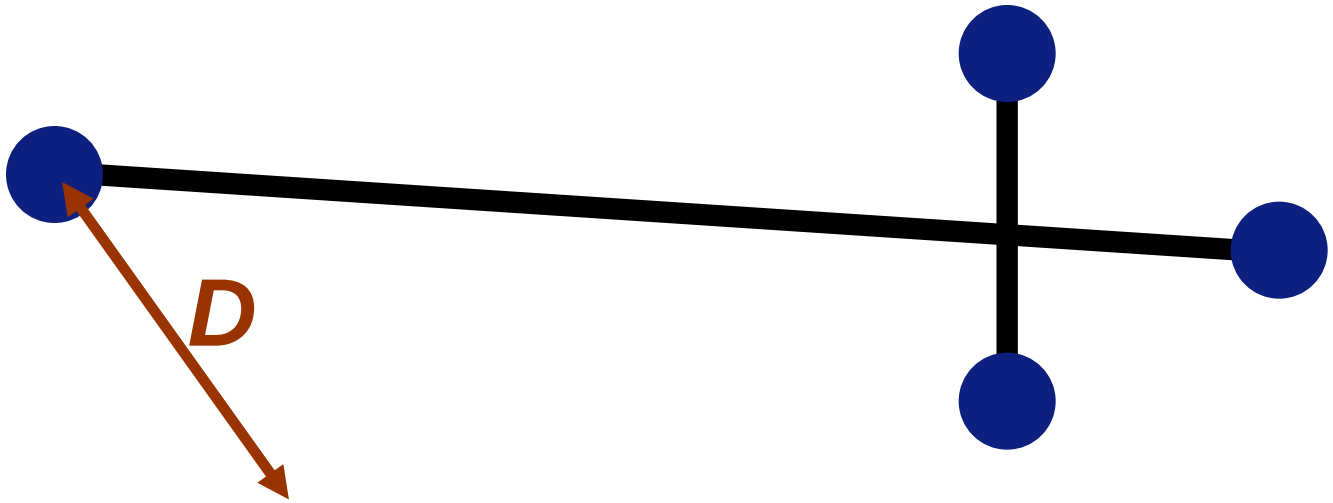
- Inputs
  - arrival and departure rates A_i and B_j
  - Distance cost function V and distance scale D .
- Solve by demanding self-consistent arrival and departure rates to fix a_i and b_j

$$\Rightarrow \quad \Sigma_j F_{ij} = A_i \;\; \textbf{and} \;\; \Sigma_i F_{ij} = B_i$$

- Equivalent to optimising a cost function

Doubly Self-Consistent Gravity Model Example

Distance scale ***D*** as before



Rihll and Wilson Gravity Model

- Edge from ***i*** to ***j*** is flow F_{ij}

$$F_{ij} = b_i B_i \ (A_j)^{\alpha} \ \mathrm{V}(d_{ij}/D)$$

where α is an additional model parameter

- Self consistent departure rate fixes b_i

$$\Rightarrow \quad \Sigma_j F_{ij} = D_i$$

- Departure rate D_i is either:-

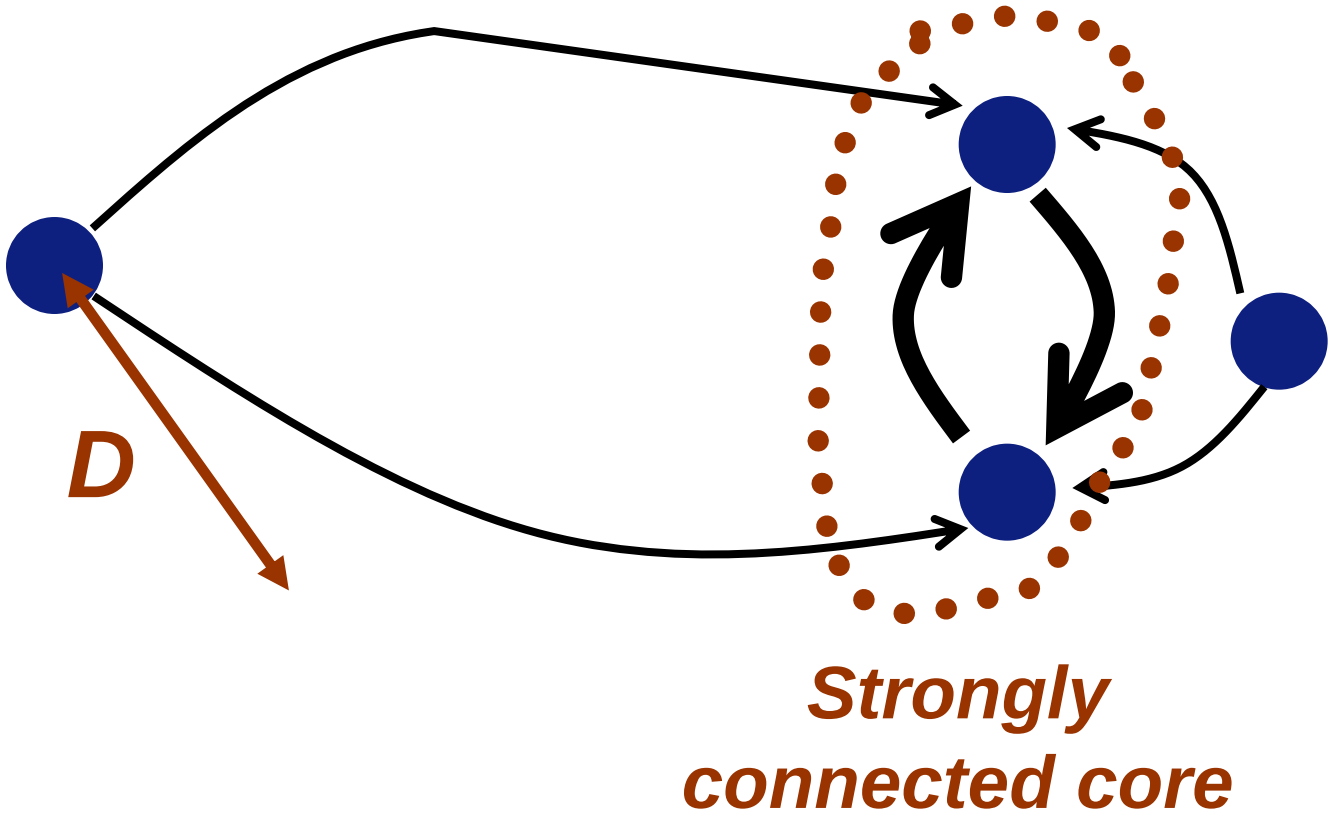
(a) a fixed input (size of site), or

(b) set equal to arrival rate A_j

- Find A_j and interpret as importance of site

Rihll and Wilson Gravity Model Example

Same ***D*** as before,
closest two sites have most connections



Stochastic Model – ariadne

[Evans, Knappett and Rivers 2008-2012]

- Has intrinsic volatility set by `temperature' parameter
- Allows sites to vary in size in response to network connections
- Network will give an low value of a `cost' function
  - includes costs for sites and edges sizes and ascribes benefits to interactions

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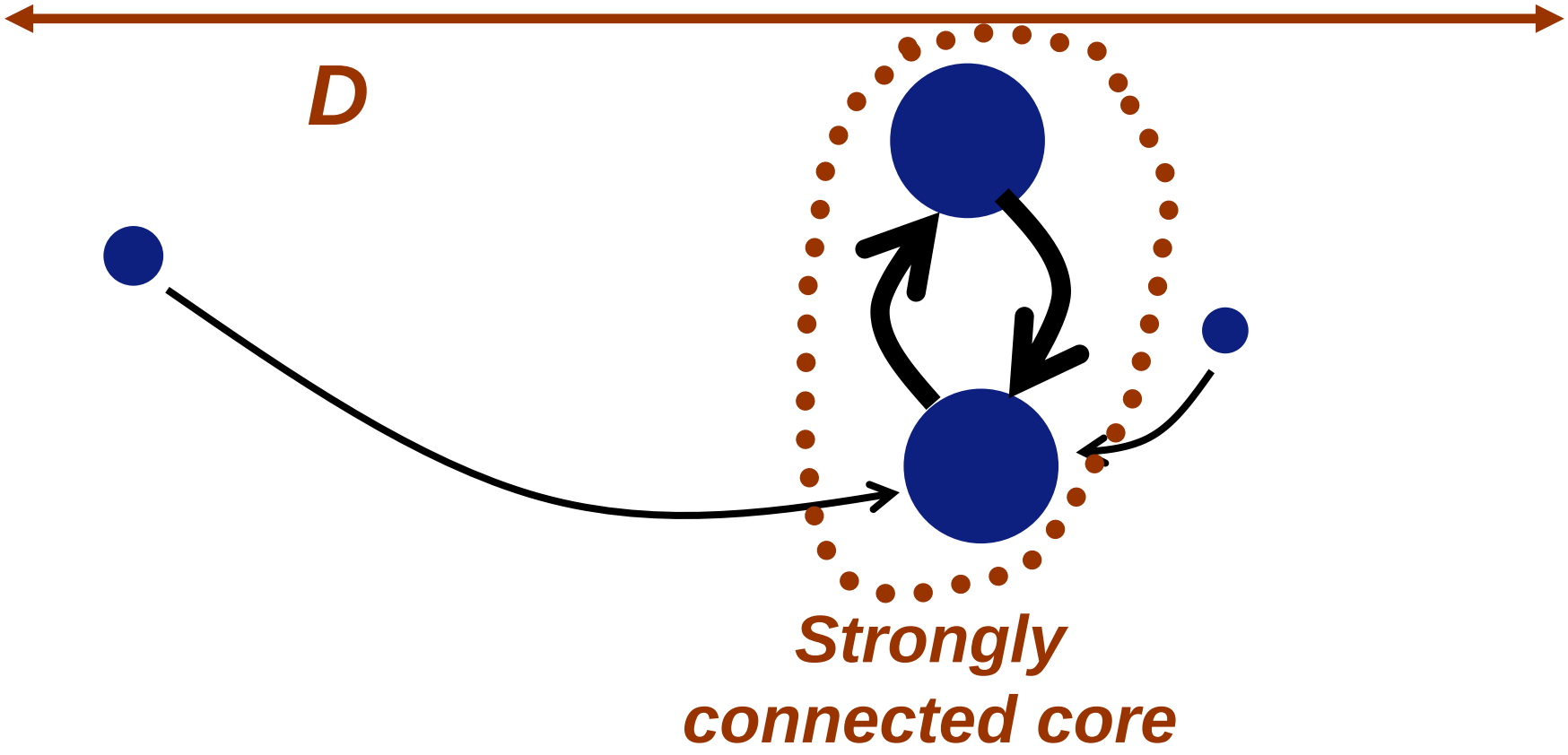
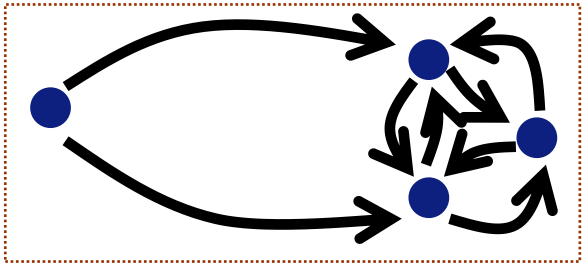
ariadne Example

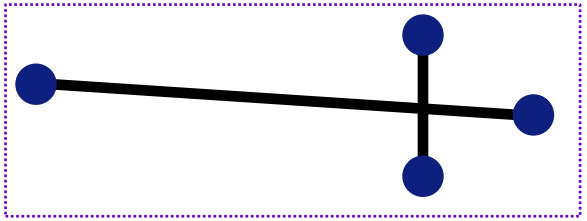
Bigger ***D***, 3 other parameters, one solution has the closest two sites large with most connections

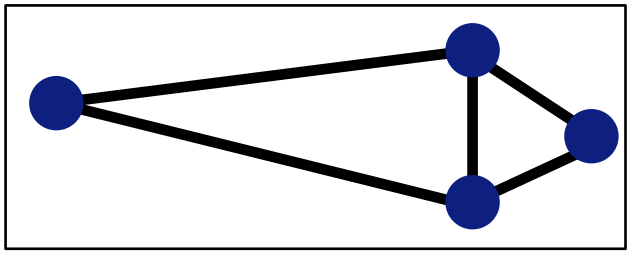


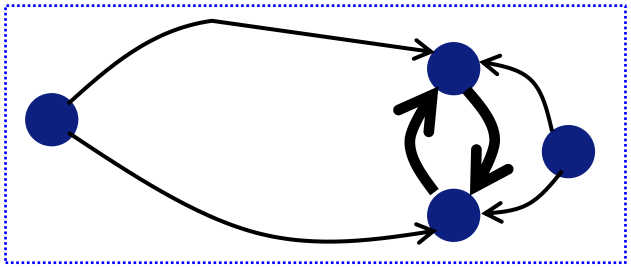
Comparing Networks

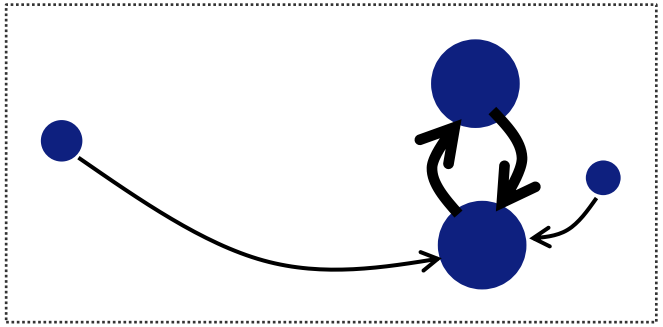
- Same arrangement of sites gives different networks
- How can we compare them?











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Comparing networks (I)

Measure a quantity associated with vertices

- Integer valued quantities poor *e.g. degree*
- Avoid quantities defined for simple networks
  e.g. average shortest path

$\Rightarrow$ Work with quantities defined on weighted networks

e.g. PageRank, clustering, betweenness'

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Comparing networks (II)

Measure similarity of each pair of vectors

- Pearson correlation coefficient if gaussian
- Rank values then compare ranks (largest value 1st, smallest last, then use Kendal's tau or Spearman) if have outliers
- Other less traditional schemes

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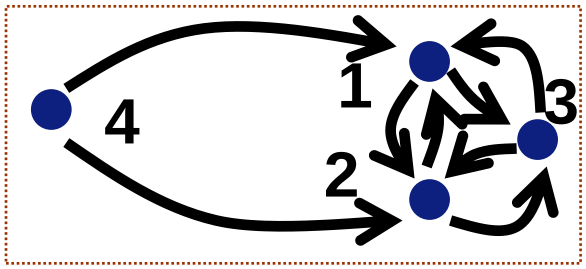
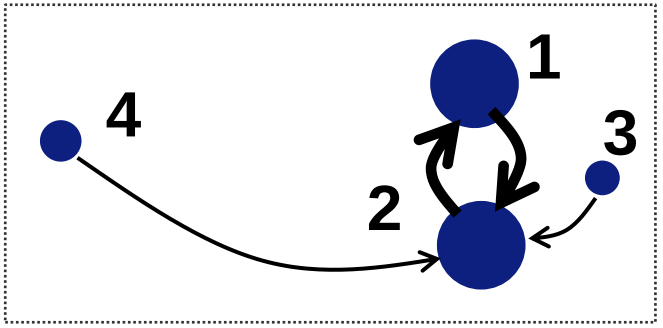
Comparing Networks (III)



Vertex	Page Rank DPPA	Page Rank ariadne	Rank DPPA	Rank ariadne
1	0.35	0.35	1.5	1.5
2	0.35	0.35	1.5	1.5
3	0.25	0.10	3	4
4	0.05	0.20	4	3

0.78



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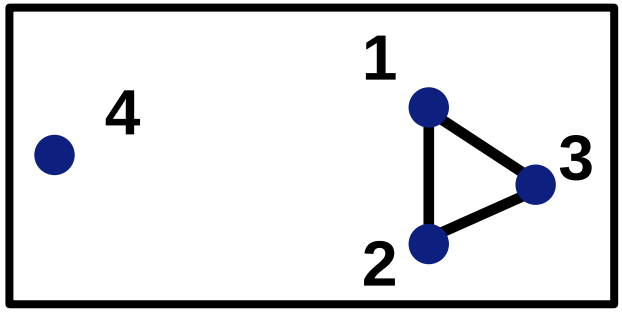
Example Networks



e.g. PageRank of vertices

(use averages for ties)

Vertex	PPA	DPPA	MDN	DCGM	RWGM	MC
1	1.5	1.5	2	2.5	1.5	1.5
2	1.5	1.5	2	2.5	1.5	1.5
3	3	3	2	2.5	3.5	4
4	4	4	4	2.5	3.5	3

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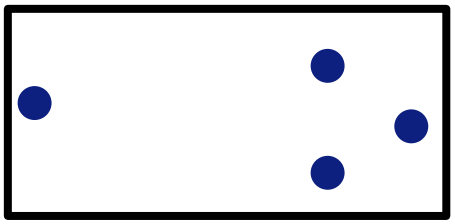
PageRank Correlation Matrix (Kendal method)



	PPA	DPPA	MDN	DCGM	RWGM	MC
PPA	1	1	0.82	NA	0.94	0.78
DPPA	1	1	0.82	NA	0.94	0.78
MDN	0.82	0.82	1	NA	0.58	0.27
DCGM	NA	NA	NA	1	NA	NA
RWGM	0.94	0.94	0.58	NA	1	0.94
MC	0.78	0.78	0.27	NA	0.94	1

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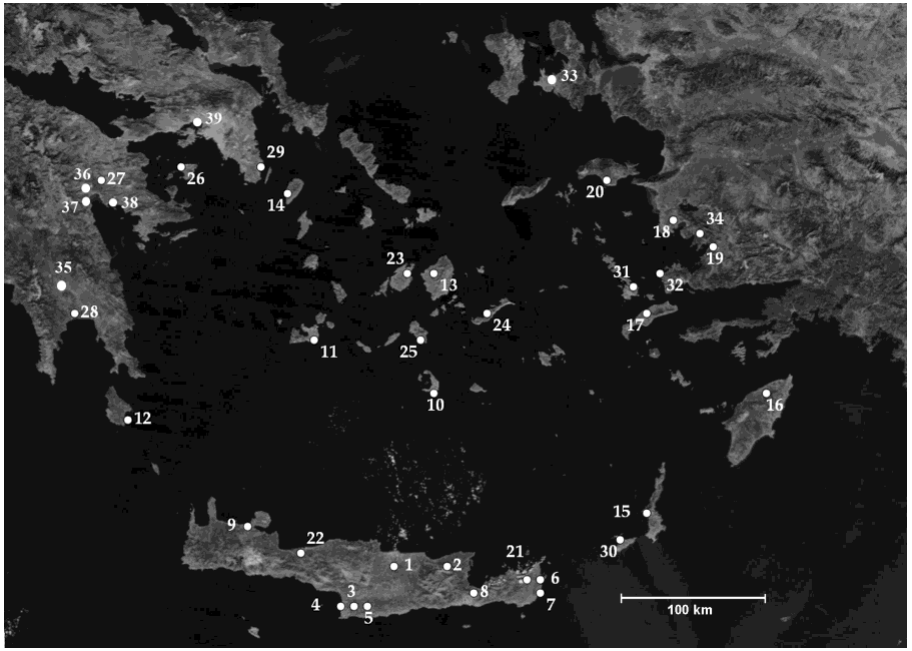
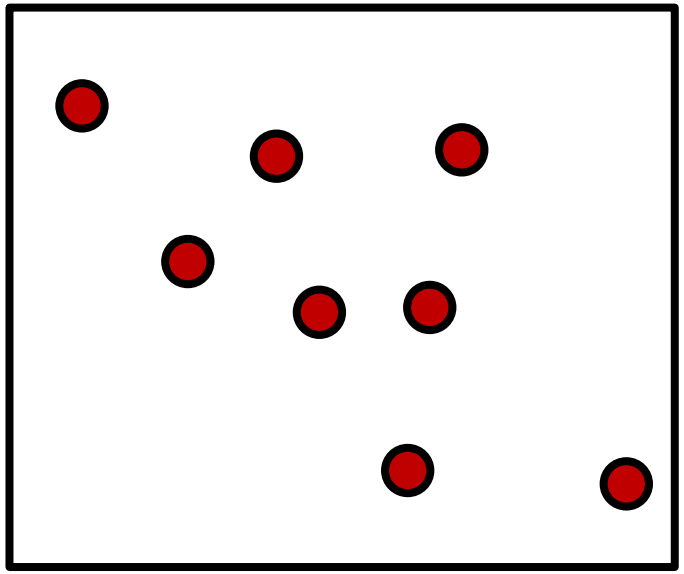
Test Data

- Use real data sets
  - 39 Minoan Aegean sites [Knappett et al]
  - 110 Geometric Greek Sites [Rihll & Wilson]
- Use artificial datasets
  - Random sprinkling of points constant probability density





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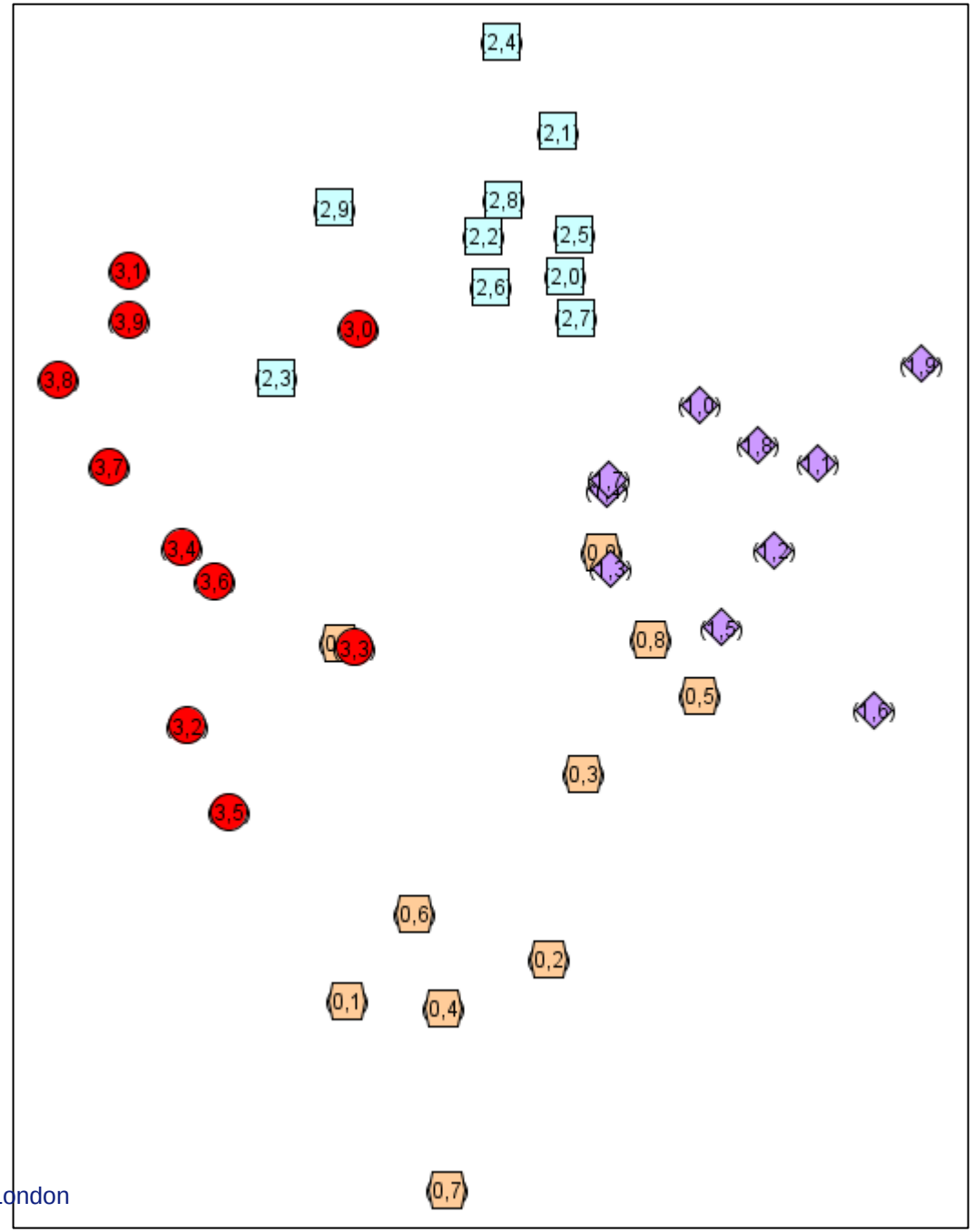
Test Data

More realistic:-

- 40 sites
- In 4 groups centred at points of compass
- 10 points per group scattered around centre



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Problem

Every model has some parameters

How do we choose values for different networks when we want to make a comparison?

e.g. is a k=4 PPA to be compared to a D=100km MDN network?

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Answer

Look for models with same `physical' characteristic:-

- Average Distance
  - usual definition does not apply to weighted networks
- Time scale to visit all nodes
  - use random walkers
- Others…?

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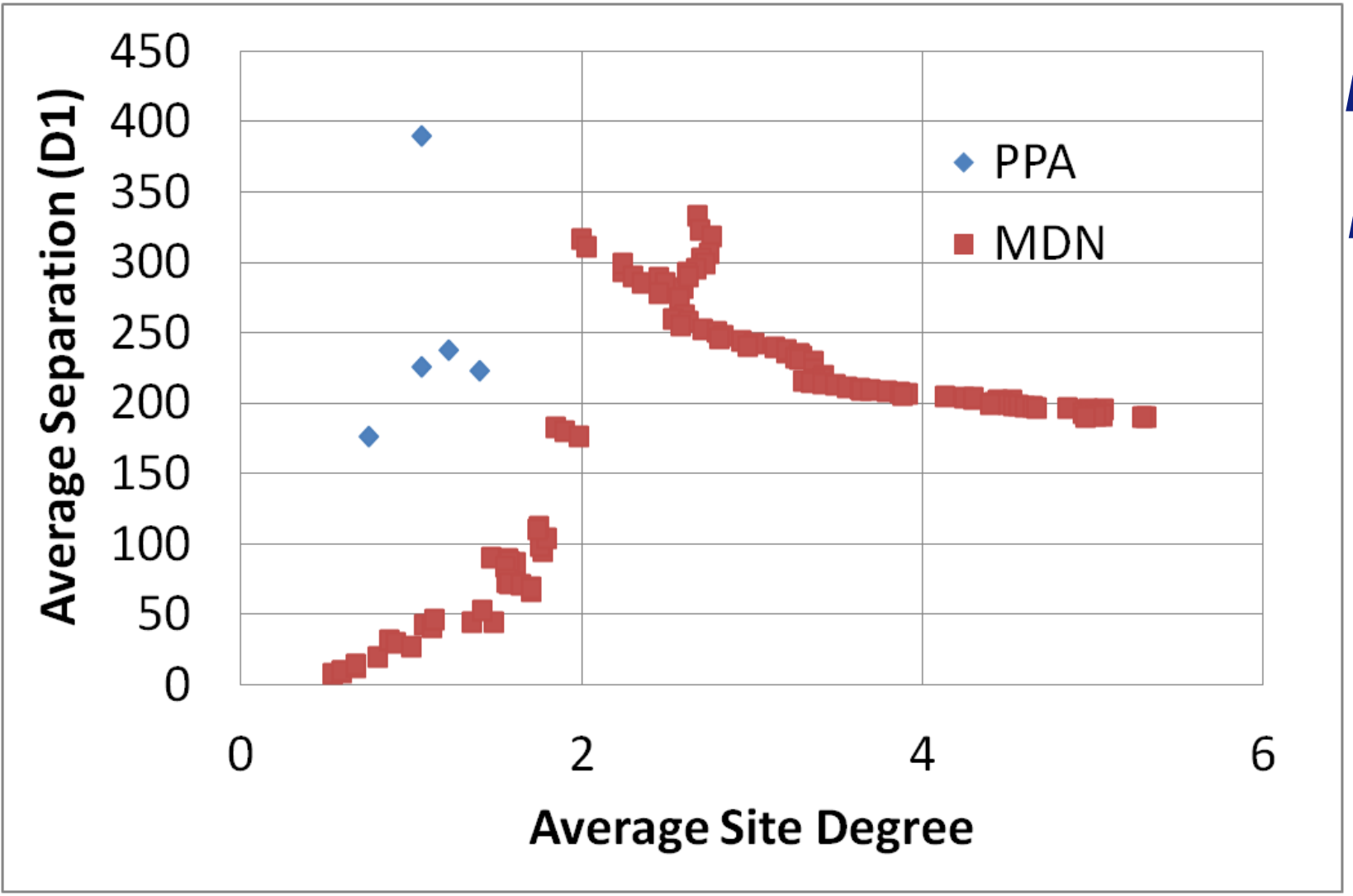
PPA and MDN: Distance vs Degree



Distance = Physical

Degree = Model

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Physical Characteristic

Measures of distance (time scales etc) still transformed (renormalised) from model to model

? Use network topological characteristics ?

e.g. choose parameter such that there is one distance/time is roughly proportional to geographical separation

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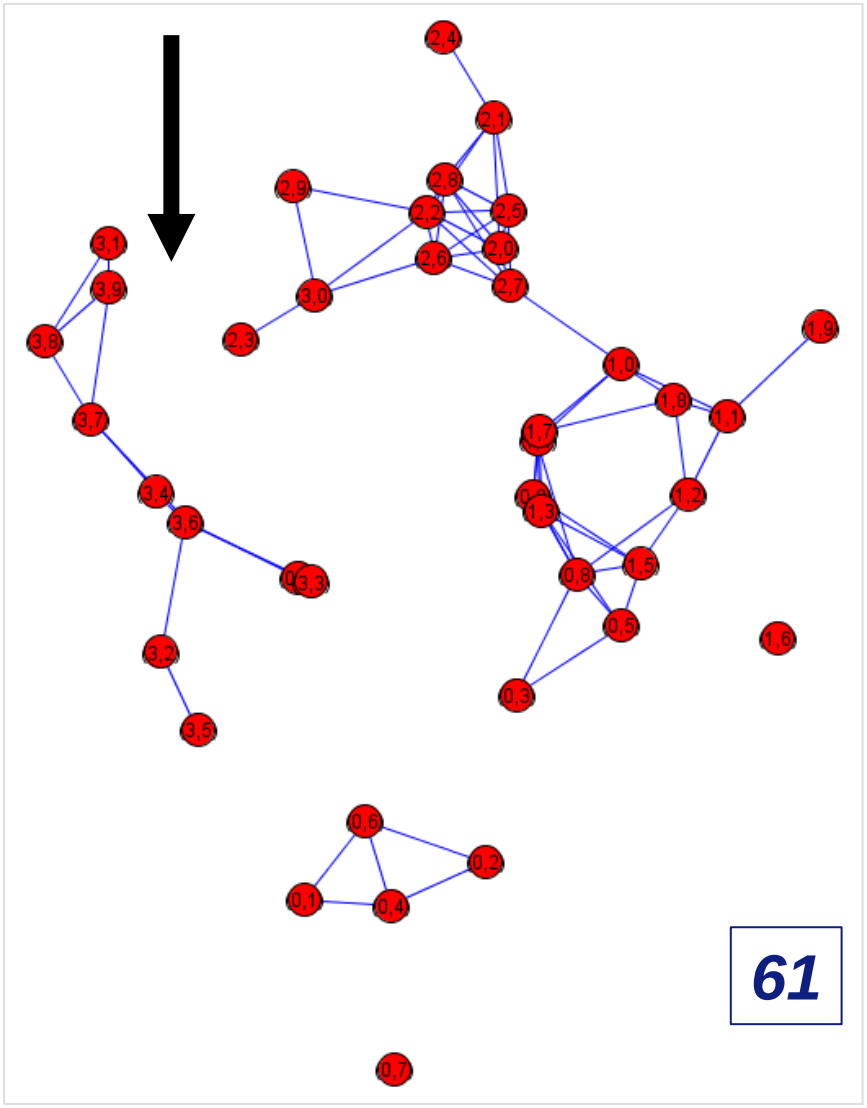
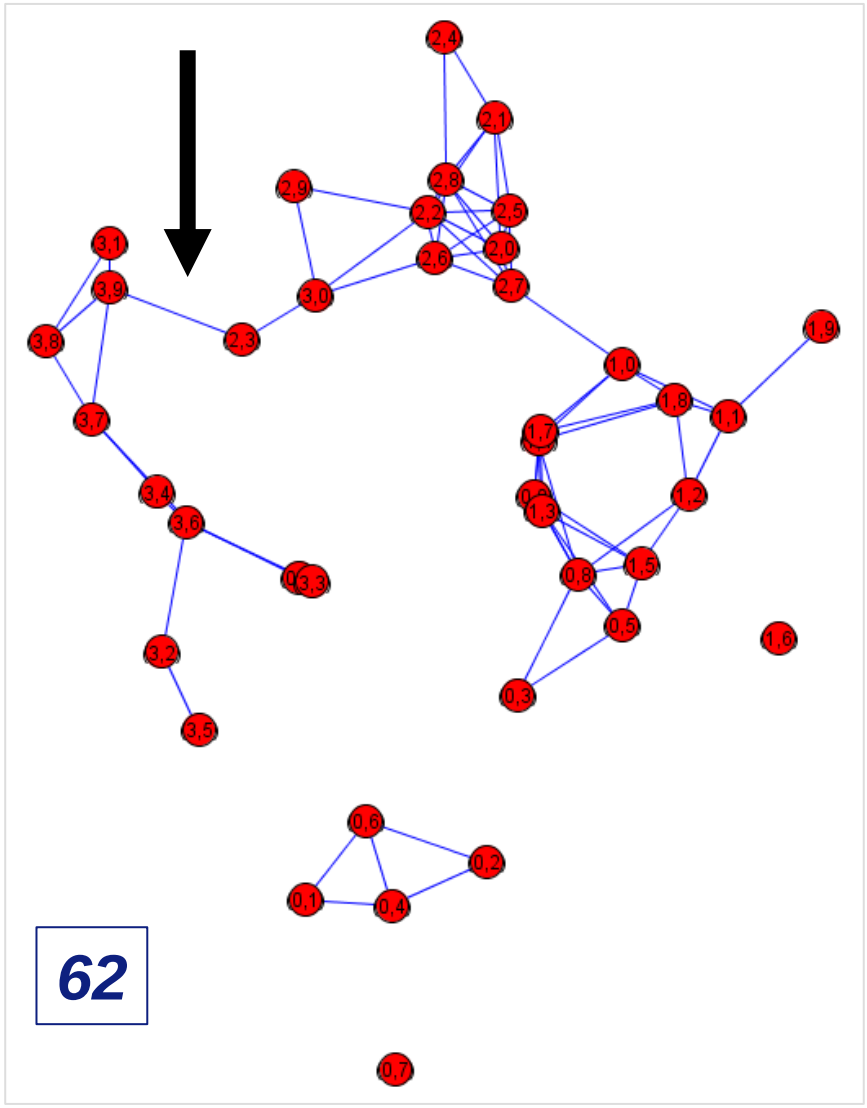
MDN for Distance 61.0 vs 62.0





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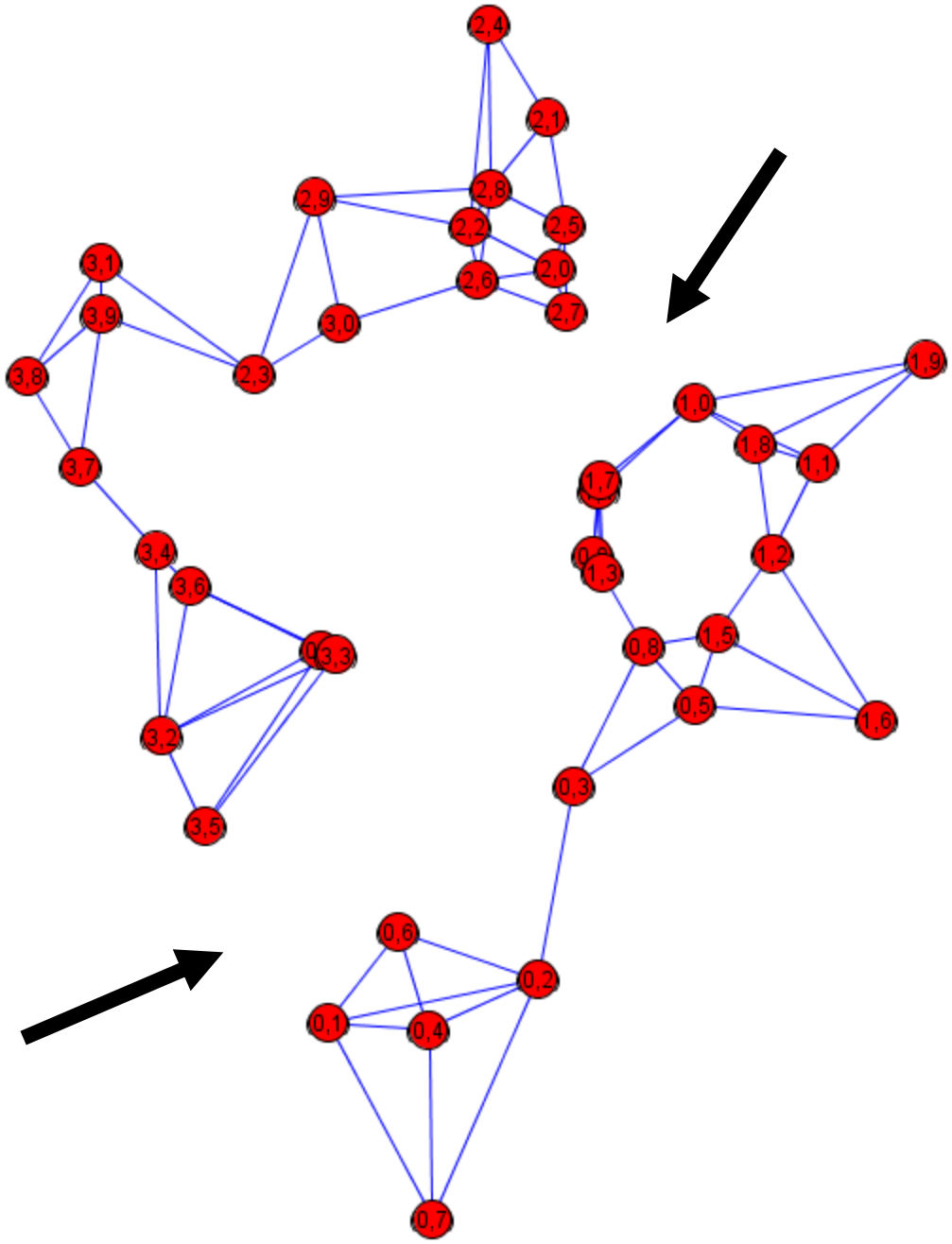
PPA $k_{out} = 2$



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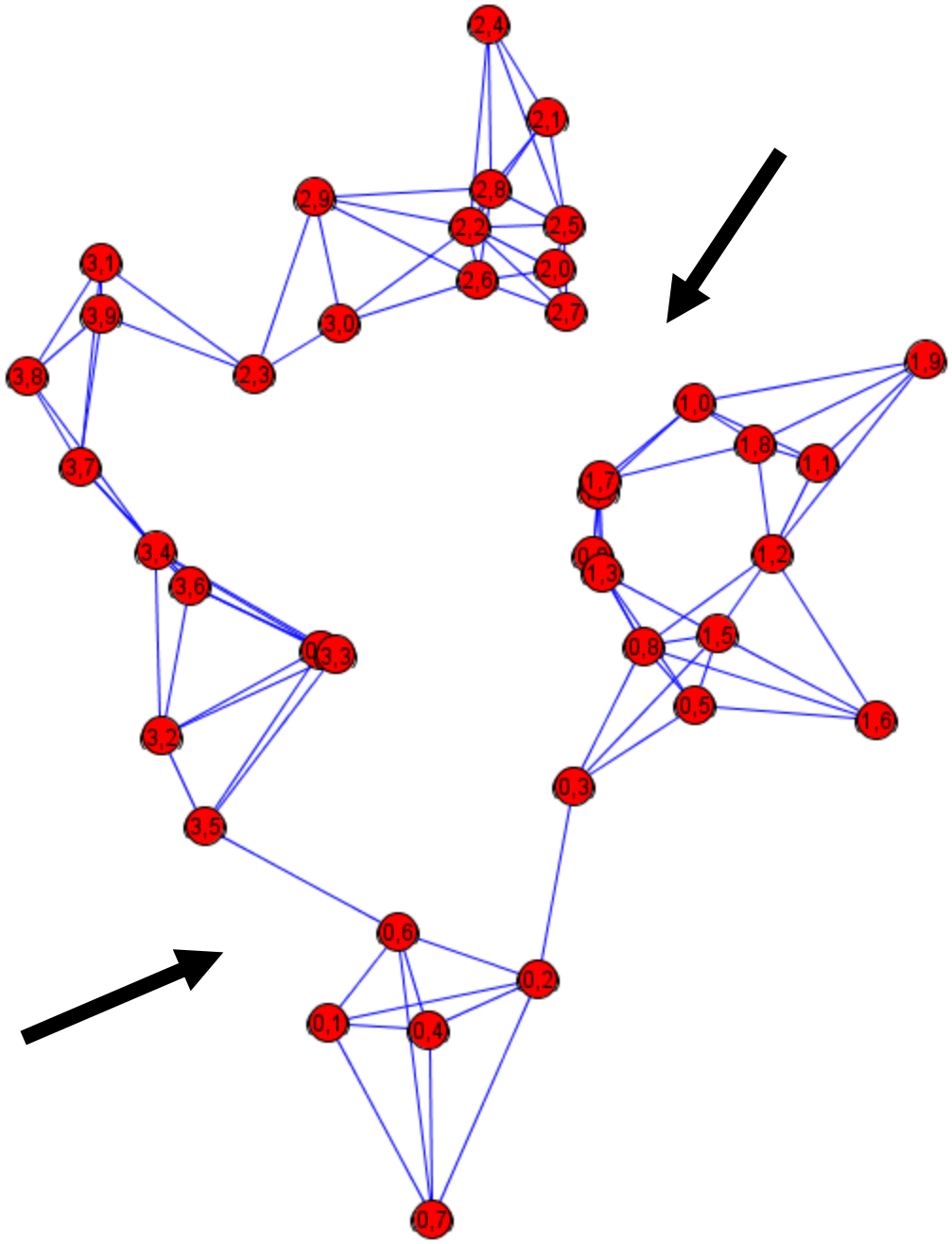
PPA $k_{out} = 3$



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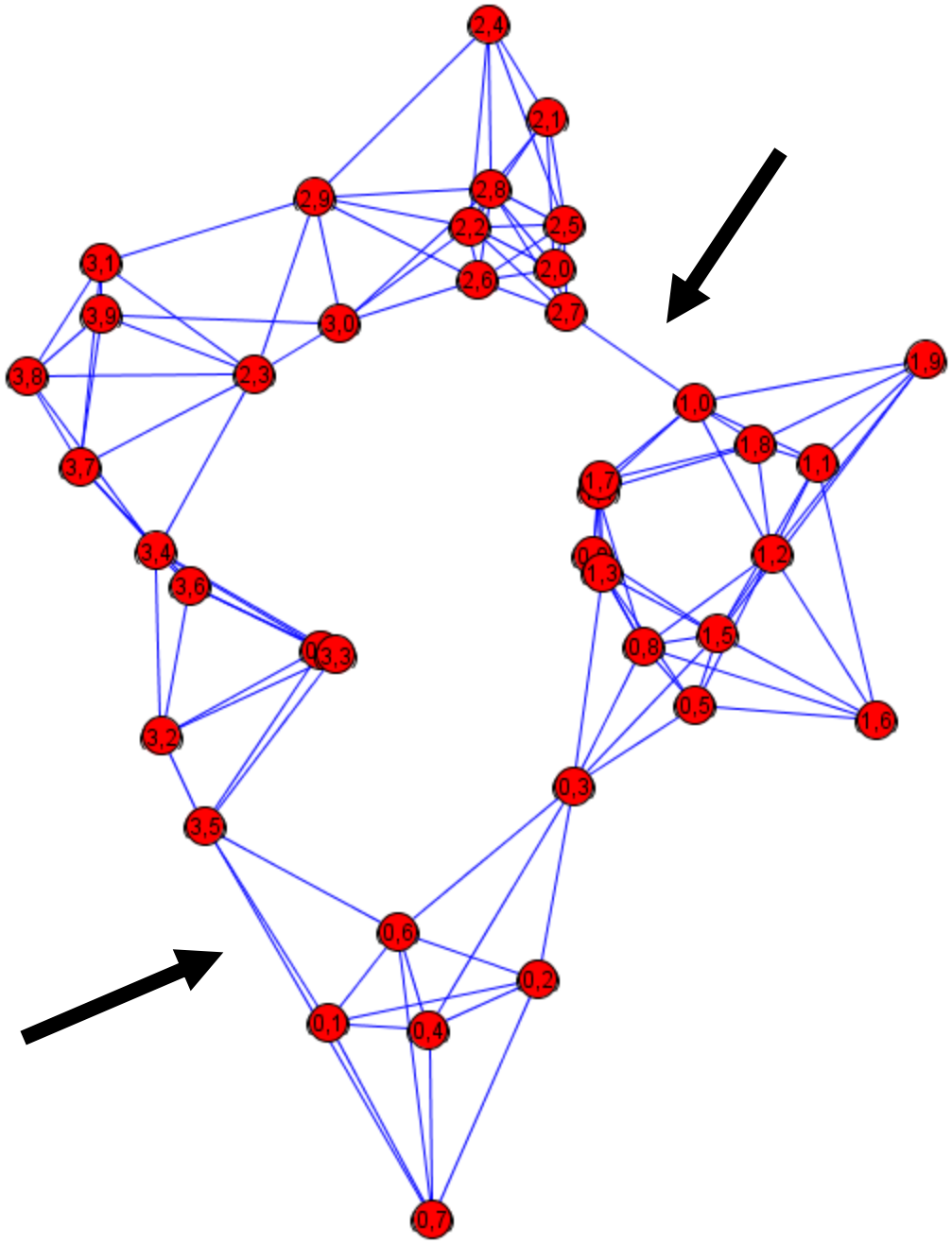
PPA $k_{out} = 4$



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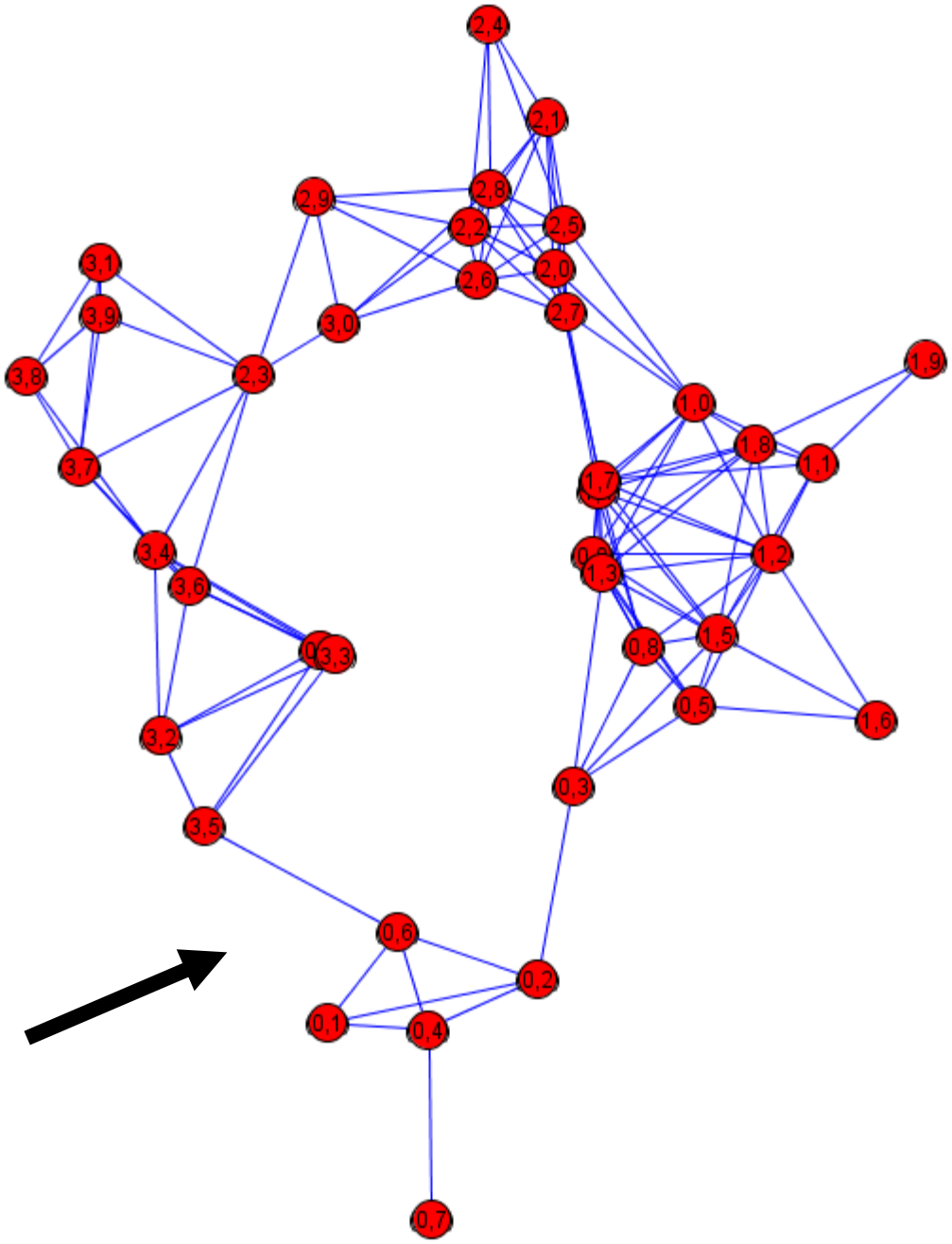
MDN
– first global cycle
D=82km



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TOWARDS a Quantitative Comparison of Spatial Network Models in Archaeology

Still work on going, no good recommendation as yet but basic ideas are

- Measure (several) vertex properties in each model
- Similarity of model defined through similarity of vertex properties e.g.Pearson, Kendal tau, Spearman coefficients
- Need criteria to fix parameters

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Acknowledgements

- All work done with
  - **Carl Knappett (Toronto)**
  - **Ray Rivers (Imperial)**

  some work also with **Edmund Hunt (Imperial)** and **Eric Beales (Toronto)**

- Publications

  `http://theory.ic.ac.uk/~time`

  or google “Tim Evans archaeology”

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