What makes a site important? Centrality, gateways and gravity

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I. Introduction

Analysing patterns of spatial organisation is one of the most basic procedures for any archaeologist. This is relevant across all spatial scales, from the household to the region, particularly as expressed in some of the most formative texts of processual archaeology (e.g. Flannery 1976; Clarke 1977). As the focus of this volume is regional interaction, we will here consider questions of spatial organisation at the macro scale.

Whether this concerns the distribution of sites, or the distribution of materials/artefacts, archaeologists have tended to work from a certain set of understandings about inhabited space, in particular in terms of zones of interaction around a material source or central site (cf. Smith 2005). For example, for many years archaeologists have used Thiessen (or Voronoi) polygons to tessellate regions containing known sites (e.g. see Conolly and Lake 2006). This can provide a sense of site importance if, in the process, it emerges that some sites have influence over larger domains than others. It is also possible that larger sites can subsume smaller ones. In particular, the related XTENT model allows larger sites to exercise an influence that will spread across the boundaries of smaller neighbours (Renfrew and Level 1979).

In these approaches, in which spatial organisation is conceived in a 'radial' manner (Jennings 2006), the 'power' of a centre—whether derived from a high concentration of people, or material—naturally lends itself to similes in terms of 'forces'. Thus tessellation methods suggest important centres as 'pushing' their influence towards their neighbours' boundaries. In this Chapter we shall present the complementary viewpoint that sees important sites as sources of *attraction* for 'trade', technologies and ideas, 'drawing in' from the same and further neighbours. We stress that these approaches are not necessarily antithetical since they may represent different categories of interaction, e.g. what we might loosely term the sociopolitical and the socioeconomic. Even within the latter the 'push' from one site's zone of influence upon its neighbour's may have much in common with the 'pull' on the site by its same neighbour. For this article we prefer to think entirely in terms of directional interactions between sites rather than complicating matters by inferring zones of influence.

Just as with tessellations, our main goal is to find workable criteria for what makes a site an important centre of attraction and secondly, to test them for a system for which we have a good archaeological record. It is clear that we cannot just use site size alone, since the importance of a site is also related to its interactions and position with respect to other sites. These can often be characterised as reflecting the site's 'centrality'. Centrality can take many different forms and exist for many different reasons. Do we mean a centre of population, a centre of agricultural redistribution or a centre of a trading network? For example, one kind of centre may be a 'central place' or a 'hub', and yet another a 'gateway' (Hirth 1978). Arguably archaeologists have not given nearly enough attention to the different kinds of centres that may exist, and more importantly, our means for identifying them. This article takes a step towards remedying this by considering centrality in greater detail, to the exclusion of other measures of importance.

If, to this end, we are going to both shift away from a 'zonal' understanding, and recognize the specific kinds of directional links that might exist between sites, then what methods are available? Network methods are eminently suitable (see Smith 2005) for providing measures of centrality. Yet perhaps because of the prevalence of zonal thinking discussed above, they have been relatively under-utilised in archaeology. In the 1970s, as archaeology came under the influence of the New Geography there was some sporadic use of networks, both to analyse trade patterns (e.g. Irwin-Williams 1977), and to assess centrality, with the use of graph theory on data from coastal Papua New Guinea (Irwin 1974; 1978). It is difficult finding other uses of networks for regional analysis from this period, and we have to leap forward a decade to the work of Peregrine (1991), distinguishing between degree, betweenness and closeness centrality in analyzing the role of Cahokia in the Mississippi river system and Gorenflo and Bell (1991) of how one might use network analysis to assess ancient road systems. Also at this time we find Broodbank (1993; see also 2000) inspired by the work of Irwin and colleagues (e.g. Terrell; Hunt) in Papua New Guinea and Oceania to assess interactions in the Bronze Age Cyclades using proximal point analysis (see also Hage and Harary 1991).

Despite the sporadic nature of most of these appearances of network analysis, the theme of centrality does seem to be a recurring one. In the one area where networks have been more consistently applied, Oceania (see Terrell this volume), centrality is one of the main features measured on the networks under analysis. So it should perhaps not come as too much of a surprise that in the recent return to network studies in archaeology, centrality again features prominently. Munson and Macri (2009) in their work on Maya networks based on epigraphic data also draw on Freeman in their focus on degree centralization; Isaksen (2008) uses betweenness and closeness centrality measures; Johansen et al. (2004) discuss degree and 'information' centrality; Mizoguchi (2009) uses a wider range of centrality measures (for which see Jackson 2008; Newman 2010).

As we see from these examples, networks can arise in several different contexts, with details conditioned by geography, exchange technology (e.g. modes and ease of exchange), social organisation, to say the least. Although there are some generalities, this conditioning is sufficiently explicit to require that their application has to be tempered to specific questions, concerning societies of a particular time and place. With this in mind we have chosen the Middle Bronze Age (MBA) S. Aegean, for which we have already developed network models (Evans et al., 2009, 2011; Knappett et al. 2008, 2011; Rivers et al. 2011). As we have seen above, archipelagos are particularly suited to network analysis, since islands provide natural choices of network nodes and, with a dominant means of transport (in this case, sailing vessels) the links between the nodes are simplified. Further, the MBA Aegean, characterised by a strong Minoan/N. Cretan presence, is approximately isolated in space and time, beginning with the rise of the 'palaces' and concluding with their burning, sometime after the eruption of Thera.

If we want to quantify the centrality of a site we need to introduce metrics which may seem too specific and, on occasion, inappropriate, but which can be used as a basis for discussion. In the next section we shall recapitulate some of the basic definitions of centrality before using them in subsequent sections to show how different network approaches identify the 'centres', 'hubs' and 'bridges'of the MBA 'thalassocracy'.

II. What do we mean by centrality?

In the first instance, we consider centrality in the sense of a 'central place', defined by Renfrew (1977: 85) as:

> *"The central place is a locus for exchange activity, and more of any material passes through it (per head of population) than through a smaller settlement."*

This concept of "central place" implies more than simply larger size, even though 'size' comes in several forms e.g. in the carrying capacity of the site (its resource availability) and its resource exploitation (a possible proxy for population). We need to know how the site in question connects to other sites. The archaeological networks we have in mind have sites connecting with each other in a variety of ways and with a variety of strengths, from the very strong to the very weak; the networks are *weighted.* Indeed, although we shall not pursue this here, the stability of many social networks is dependent on there being many weak links (Granovetter, 1973, 1983). These bring in rare but necessary contacts, practices and materials that enable innovation to thrive. Further, whatever the nature of the exchange, we do not expect exact reciprocity in the relationship between sites; the networks are *directed.* Each pair of sites is connected by two opposing links reflecting different levels of exchange. For non-directed networks, where the flows are identical we can, if we wish, replace the pair of links by a single undirected link.

For this reason we cannot talk simply of the 'degree' of a site—i.e. the number of links a site has to other sites, a conventional measure of centrality—as we can for simpler unweighted, undirected networks, in which links of equal strength are either switched on or off (e.g. networks of citations, simple kinship). We suppose instead that we can identify a single measure that characterises this inter-site exchange, small when the links are weak, large when they are strong. For the sake of argument we can think of it most simply as representing a flow of boats/goods and perhaps people, although we appreciate that there is much more to 'exchange' than this. The degree of a site can now be generalized to its *out-strength* or *outflow* (the total outward flow/exchange from the site to the other sites) and the *in-strength* or *inflow* (the inward flow from the remainder of the network).

Suppose the table of in- and out-flows is given by some means (we shall suggest several dynamical approaches for estimating it later). A sufficient reason for a site to be central in the sense of Renfrew is that the other sites have strong interactions with it. Whereas the *outflow* of the site is often limited, the *inflow* to the site is not. This suggests, as a first guess, that we rank sites by their inflows. The greater the influx, the higher we would rank the site. However, this first guess can be improved, in that site importance should be enhanced if the site is connected to sites that are themselves important. Thus, as a second guess, we can again rank sites by the inflows from the rest of the network, but in which each inflow is enhanced or diminished, according to the site rank of the origin of the flow as derived one step earlier. We can now repeat this procedure, using the new site rankings. On iterating the process, we converge to an unambiguous centrality ranking, the so-called *eigenvector centrality ranking*[1] (Newman 2010).

As we said earlier, in what follows we shall apply these ideas to Bronze Age Aegean maritime exchange. In a simple picture of island networks, harbours with high eigenvector centrality ranking will be the busiest, with the highest numbers of goods arriving and leaving, or the most people passing through them. We shall just term this *rank*. Further, rank per head of population, which we term *impact,* is just Renfrew's measure of central places quoted earlier. Should there be sites with significantly higher rank than their neighbours, then they are understood as the 'hubs' of the network. These are not necessarily the busiest sites of the network as a whole, as defined by the ranking tables proposed above, but those that are relatively the busiest within a region or neighbourhood[2]. In practice, we shall not make much use of them.

Rank can be problematical for those strongly directed networks in which some sites have strong outward flows and weak or no inward flows. Not only do such sites have low rank, but sites that connect to them can acquire low rank by contagion. In these cases it is sometimes helpful to introduce a qualified centrality ranking which interpolates between site size and site rank. To do this, we give the sites an initial 'centrality' value proportional to their size and make up by taking the complementary fraction of the incoming flows in determining final rank and iterate the procedure, as before. If we think in terms of journeys between harbours, we treat the network-wide activity as an aggregate of random exchanges/journeys, once the constraints upon them imposed by the model (e.g. distance, 'cost') have been taken into account. Whereas *rank* assumes perpetual travelling this qualified ranking effectively corresponds to giving a boat/traveller only a finite number of stopovers, Such a ranking is called *Katz centrality ranking*, and is the basis of *Google Page Rank*, used for ranking web pages. For the reasons given later we shall not find this ranking useful for our networks, although the concept is useful in helping to define other notions of centrality, in particular *betweenness centrality* (or, more simply, *betweenness*), which differ from that of Renfrew's 'central space' discussed earlier. A site with

[1] The ranks correspond to the components of the eigenvector with largest eigenvalue unity of the matrix whose elements determine the relative probabilities of exchange between sites.

[2] There is a more technical definition of hub centrality that refers to sites whose outflows are to sites of high centrality (e.g. see Newman 2010). We have a much more colloquial understanding of hubs in mind.

high betweenness may or may not have high rank or be a hub but, typically, could be an end of an important 'bridge' between parts of the network; a 'bridge' in the sense that, if it is broken, the connectivity of the network is damaged. It is understood as a measure of the influence a site has over the flows of people, goods, information through the network, insofar as it lies on important exchange routes between central sites.

In general, one might imagine that the most important routes between sites are those that are most easily (or 'cheaply') traversed, which often will be among the 'shortest'. Unfortunately, in its simplest form, defined as the fraction of shortest paths between sites which pass through the site of interest, *betweenness* does not generalize simply to directed networks of variable exchange strength. Further, the assumption behind this definition, that exchange between sites follows shortest routes, is unlikely to be true. The Late Bronze Age Ulu Burun shipwreck off the south coast of Turkey near the city of Kaş in the province of Antalya shows a passage in which cargo has been picked up and dropped off in anything but the shortest route. Although this is a different period and a different distance scale from that considered here we might expect something similar. With this in mind we adopt (and adapt to weighted networks) alternative measures of betweenness that relax the shortest-path condition to include all paths between sites, but which can be weighted as to give emphasis to the shorter paths. They are manifestations of what Newman (Newman 2005) has termed 'random-walk betweenness'. If conventional betweenness corresponds to the targeted transmission of exchange (goods, people, etc.) by the shortest route, the latter assumes exchange by vessels with no clear long-distance goals.

Even then there are two very different approaches that we can adopt. Our approach is to estimate the likelihood for travel along all paths that connect separated sites allowing for a finite number of stopovers, as used in Katz centrality[3]. From these we can construct the flow through a given site that is a generalisation of the number of shortest sites through it which takes some less directed travelling into account[4]. We might anticipate a good correlation between our betweenness centrality and what constitutes a central place – high rank goes along with high betweenness in many cases. Those sites with high betweenness can be thought of as 'gateways' if their relative rank is low, but we shall not be too prescriptive in our use of the term.

As we said in the introduction, our interest in these manifestations of 'centrality' is because they provide a sense of site importance that we expect to see reflected in the archaeological record. Exactly how is not clear but, among other things, we would expect sites with high rank to reflect this in the size of harbours, in the variety of artefacts and the technical innovation of production techniques. At a local level, hubs would show the same. Gateways are not necessarily large but, again, would expect to have their importance reflected in the variety of artifact types and the distances they may have travelled. In what follows we shall see how both central places and gateways arise in the MBA Aegean.

For the MBA S. Aegean of Fig.1, we have identified the 39 sites listed in Table 1 as some of the most significant. In later Figures we shall show networks imposed upon these sites in which the thickness of the directed links reflects site outflows but, by eye alone, it is impossible to estimate site centrality reliably. This, surely, is the point of the exercise; to what extent are sites 'central' that don't look so on simple grounds of geographical position and resources? However, for those

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[3] There is an additional free parameter (as in Katz ranking), the number of stopovers in a typical journey, from one onwards. Once away from extremes the betweenness ranking is not usually sensitive to its choice. We take the number of stopovers to be 'a few'.

[4] This differs from Newman's approach in his 2005 paper, which ignores the equilibrium flows of the network, the steady buzz of activity as goods and people flow across the S. Aegean. He defines betweenness in terms of how the network behaves when we put it under pressure, imposing a 'push' from initial and final sites. In the terminology of eigenvalues, both this and our definition of betweenness use the information in the eigenvectors for non-leading eigenvalues, but Newman's ignores the leading eigenvector completely

models, which include the familiar Proximal Point Analysis (PPA) in which the links are non-directional (or for which, more generally, inflow equals outflow at each site), site *rank* reduces to the simpler site *inflow*, our first guess, and can be read schematically from the Figures, as can hub rank[5]. Nonetheless, 'betweenness' remains more elusive.

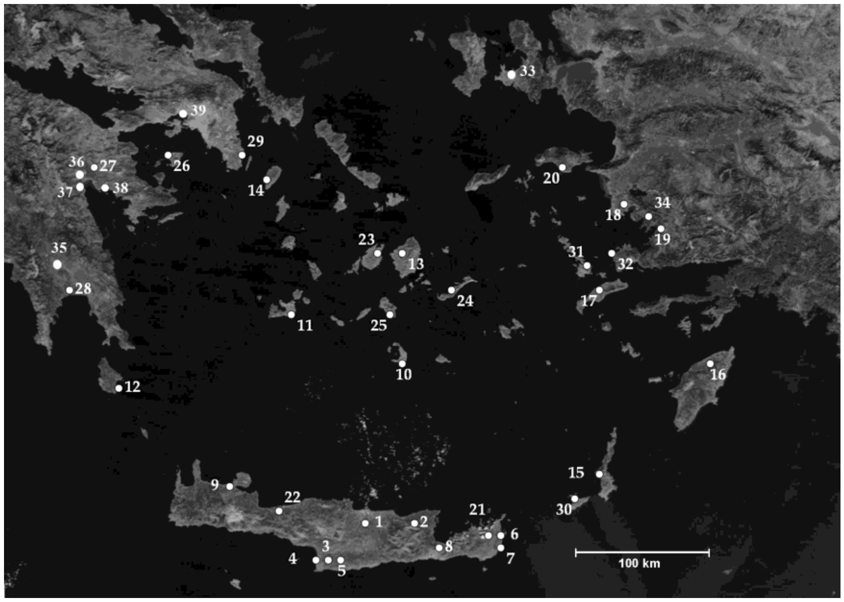

Fig. 1 Important sites, for the MBA Aegean, including Knossos [1] and Thera [10]. The sea journey from the N. Cretan coast to Thera is just more than 100km

1. Knossos (L)	14. Kea (M)	27. Mycenae (L)
2. Malia (L)	15. Karpathos (S)	28. Ayios Stephanos (L)
3. Phaistos (L)	16. Rhodes (L)	29. Lavrion (M)
4. Kommos (M)	17. Kos (M)	30. Kasos (S)
5. Ayia Triadha (L)	18. Miletus (L)	31. Kalymnos (S)
6. Palaikastro (L)	19. Iasos (M)	32. Myndus (M)
7. Zakros (M)	20. Samos (M)	33. Cesme (M)
8. Gournia (L)	21. Petras (L)	34. Akbuk (M)
9. Chania (L)	22. Rethymnon (L)	35. Menelaion (S)
10. Thera (M)	23. Paroikia (M)	36. Argos (M)
11. Phylakopi (M)	24. Amorgos (S)	37. Lerna (M)
12. Kastri (M)	25. Ios (S)	38. Asine (S)
13. Naxos (L)	26. Aegina (M)	39. Eleusis (M)

Table 1: The sites enumerated in Fig.1 and the size of their local resource base, with (S), (M), (L) denoting 'small', medium' or 'large' respectively in terms of their resource base *(input).* This is to be distinguished from their 'populations', which are *outputs.*

[5] Unfortunately, for such *non-directional* (or equal in-and out-strength) networks Katz ranking typically tracks site size for our networks. Even for our directional network models Katz ranking tends to follow site size and we shall not use it to discriminate between sites in the subsequent discussion.

Generating centrality

What the studies that we have cited earlier do is take a network designated by archaeological data and then 'measure' the centrality of sites on that network. With static networks, there is little scope for understanding what features might have generated different kinds of centrality. It may be possible to say, for example, by analyzing known connections, that a site is significant in the sense above; but how did that site come to have such connections in the first place? Rather than assume that network function follows from network structure, we should consider how structure is tied up with function. Network structure is emergent and dynamic. But how we can access this 'agentive' quality of a network?

In common with many other authors we assume not only that networks have functions, but that their structure will, in some sense, be approximately 'optimal' in fulfilling those functions. In particular, where regional 'exchange' networks are concerned, the likelihoods, costs and benefits of movement across physical space are important factors (e.g. see Barthélemy 2010) in characterizing optimal behaviour. We shall discuss several mechanisms for generating networks that encode some form of optimisation. Given our understanding of the archaeological record we can decide which models give the most plausible outcomes. However, all models work with a very broad brush and, even if the overall pattern looks plausible, which particular sites achieve the highest centrality within a model can depend on details (e.g. typical journey length) about which we have only imprecise knowledge. For that reason, our preferred model ('ariadne', see later) has stochastic outcomes that we can interpret as plausible 'histories' of the system that can arise from the same initial conditions and which have to be interpreted statistically. We shall turn to this later.

We organise our models essentially according to the number of assumptions that we make, the simplest (null models) first. There is no doubt that exchange networks are directional, but sometimes it is a convenient fiction to drop directionality, since it simplifies model-making. When trying to identify centrality we separate mechanisms into those which generate non-directional networks and those which generate directional networks, since they possess different behaviour with regard to rank, although less so with regard to betweenness.

III. Non-directional networks

As null models, we first consider simple unweighted networks, dependent either on physical geography or on limited exchange.

Geographical networks

The simplest assumption is that there is a typical distance *D*, determined by marine technology, beyond which single journeys become too difficult, almost whatever the nature of the exchange, but that distances shorter than this are regularly taken. Sites are linked if their separation is less than *D.* If *D* is shorter than the typical intersite distance then it will be very difficult for an exchange network to form. On the other hand, if *D* is much larger than the intersite distance we have almost a *complete* network, in which every site is connected to the majority of sites. This suggests an interesting picture, if we assume that *D* increases in time as a result of improved sailing technology. In that case we expect a strong large-scale exchange network to come to life once the technology is such that sea journeys can match inter-island separation.

This is a little simplistic. When considering the ease of travel between sites, we are really concerned with travel times, for which distance is not always a good proxy. In particular, we introduce a reasonable frictional coefficient for land travel in comparison to sea travel. Its main effect is to make S. Crete less accessible to N. Crete by land[6] . Even then, travel time can be

[6] Our conclusions are largely indifferent to this value as long as it is somewhat larger than unity. To be explicit we take a frictional coefficient of 3.

directional, particularly for maritime journeys with winds and tides. Despite that, we anticipate that over the period of a year such behaviour tends to average out, and we keep our ‘distances’ non-directional.

We find that, for travel distances *D* of 100km or less, there are four regional clusters, the Cyclades, Crete, the Peloponnese and the Dodecanese. As we increase *D* to about 110km the Cyclades connect to the Peloponnese, initially through Phylakopi. At about 120km N. Crete connects to the Cyclades through Thera and the Peloponnese through Kos and Kalymnos (see Fig. 1). Continuing to increase *D* then enables the Dodecanese to connect to E. Crete, via Rhodes. In the Early Bronze Age (EBA) these distances are too large for paddle/oar based vessels to make more than occasional journeys and there is no thriving network on this Aegean-wide scale. Nonetheless, there is an established Cycladic network, for which inter-island distances of the order of 30km are perfectly commensurate with distances of canoe travel (Broodbank 2000). However, in the Middle Bronze Age (MBA) the appearance of sail means that distances of 100km and more are possible and, unsurprisingly, a vigorous maritime network develops. [For this reason the sites chosen are MBA sites.]

Rank is purely geographical, the rank for distance scale *D* being essentially an ordering by the number of nearest neighbours to each site within a circle of radius *D.* High site density is correlated to high rank. This favours N. Crete and the Dodecanese, with Phylakopi prominent in the Cyclades. However, since there is strong regional grouping, even for small *D*, rank does not vary widely. Which sites are gateways is also a direct measure of geography, in particular for sites that form bridges between regions as *D* is increased and regions begin to connect. For the S. Aegean we see bridges between Phylakopi – Kea and Thera – Malia (with precedence, by distance alone over Thera – Knossos). Phylakopi and Malia have high betweenness.

Proximal Point Analysis (PPA)

While there is no doubt that geography informs network formation, ease of travel is only one factor in their composition. To adopt the very different viewpoint of Proximal Point Analysis (PPA), it may be that there is an optimal number of relationships between one site and its neighbours that can and need to be sustained properly, *independent* of site size and site separation. Most simply, we connect each site to its *k* nearest neighbours with outward links, for some small *k*. When the process has been completed, link direction is removed, to create an undirected final network. As a result, *rank* now becomes identical to site ‘degree’, provided we also give an equal weight to each link. Visual inspection is sufficient. This is how PPA was used by Broodbank (2000) in EBA networks for the Cyclades.

Although geography still defines neighbouring sites, distance is no longer the sole determining feature**.** By its nature, PPA tends to reproduce the strongly connected cores of the geographical network. In our Figure 2 (for *k=3*) these comprise the Cyclades, the Dodecanese and Crete. As before, the regions where there are sites with high rank are those regions with high site density. However, with distance less important, there are striking differences in the way regions connect to each other. Already, for *k=3* we can circumnavigate the S. Aegean in a continuous loop, part of a tendency in PPA for connected sites to form chains. Gateways are relatively simple to identify, as happens when geographically isolated sites like Cesme bridge the Dodecanese and the Cyclades, unlike the case where distance dominates connections. For the sites in hand, it is necessary to take *k=4* before Crete is connected to the Cyclades (through Thera), However, if we were to increase the number of sites for the same *k* value, this connection would disappear. This means that how connections form depends on the sites we have deigned to include[7]. For this reason, for the MBA Aegean there is sufficient ambiguity in the numbers and positions of sites for PPA to be a helpful guide for defining centrality.

[7] Even for the simple geographic networks rank depends on the sites that are included. However, connectivity between regions is controlled by spatial separation and is less susceptible.

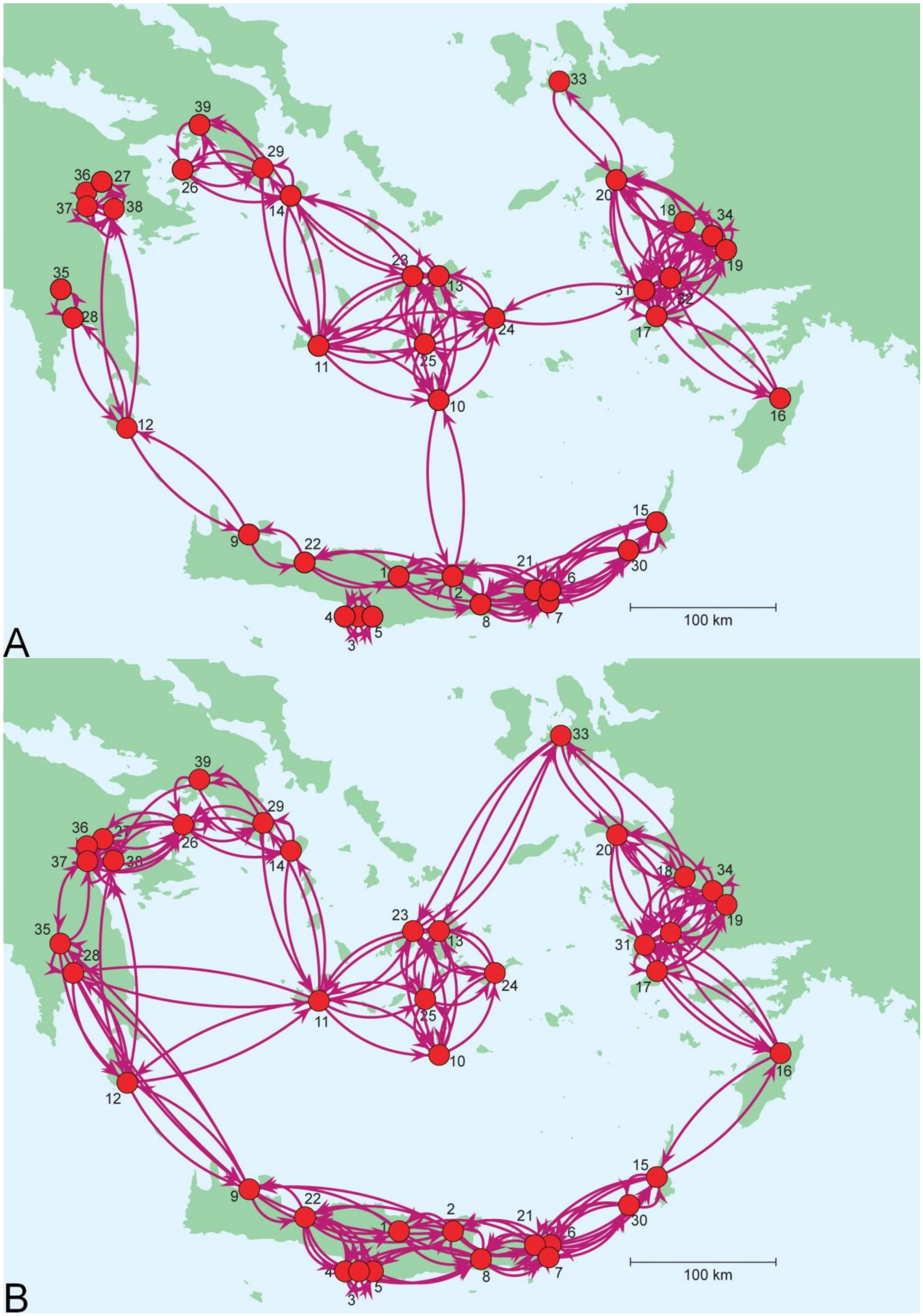

Fig.2. Contrasting a simple geographical distance network (*D = 125km*) in the top figure (A) to PPA (*k*=3) with its emphasis on connection in the bottom figure B.

Simple gravity models

Returning to our earlier comments on 'forces', the 'attraction' of centres is often construed as a 'gravitational' pull. Thus, for example, Renfrew (1977, 87) brings up the notion of gravity when he shifts from discussing artifact availability by a diffusive 'push' to the role of central sites in introducing directionality into material distribution patterns. In fact, as we shall see, in several approaches to network dynamics gravitational concepts arise very naturally.

In the first instance the pattern of distance scales in simple unweighted geographical networks has a natural extension into 'gravity' networks with equally undirected but *weighted* links, in which exchange also depends on site size as well as intersite difference, the larger the sites the greater the exchange, the larger the distance the smaller the exchange[8]. Following on from their work as planning tools from the '60s onwards (e.g. see Jensen-Butler 1972) gravity models have been recognized as a useful tool in regional analysis in archaeology since the 1970s (Plog 1976; Johnson 1977; Renfrew 1977; Conolly and Lake 2006). They mitigate against the problem noted above that we might expect different results if we were to include further sites as good MBA candidates, either as a result of new finds or a desire to be more thorough, in that they aggregate local sites into collective 'gravitational centres of resources and population' and aggregates exchanges accordingly. This can be construed as one aspect of optimisation in that, in reducing our need for detailed knowledge of local sites it minimises the effects of some aspects of our ignorance. It is for this reason that we have only allocated one site per island, for small/medium islands[9].

More relevant than site 'size' in the sense of resources (or carrying capacity) is site population which, when not requiring 'imported' resources to sustain the site, is a reflection of resource exploitation. For the moment we take one proportional to the other, although in our model to be discussed later we have carrying capacity as *input,* population as *output*. For brevity we shall term population/resource exploitation as 'population' alone, although we appreciate that the situation is more subtle. From this viewpoint, the simple geographical analysis earlier essentially corresponds to taking all sites in Table 1 as having the same population. This is unrealistic. In Table 1 we have classified a site's size as 'small', 'medium' or 'large' on the basis of archaeological evidence. Even this crude division, in which the ratio of resources is 1:2:3, is sufficient to show the effects of differences in resource availability. If we take the populations proportional to carrying capacities, then *rank* is no longer simply geographic and has to take site size into account. As a result, small sites struggle to achieve high rank, with Cretan sites on the North coast now dominating the table, both with regard to rank and betweenness (with respect to the latter, Naxos, Miletus and Thera have the highest betweenness after N. Crete). However, Thera has high *impact*.

If we use 'geography' as shorthand for the intersite distances and site carrying capacities, and 'technology' as shorthand for the ability to travel (*D*), all the models above have essentially used only geography and technology, even if the outcomes are not simply geographic. With essentially no freedom of choice there is little surprise that they produce the obvious. To go beyond this requires a more sophisticated sense of agency than how far, how many, how big?

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[8] The way in which exchange falls off with distance is controlled by *D.* For distances shorter than *D* we assume that journeys are relatively easy to make, and for distances larger than *D* they are difficult to make. The simple geographical distance model above takes this as a step function (unity for separations less than *D,* zero for separations greater than *D*)*.* A literal 'gravity' model would take it to fall off inversely with distance. We take behaviour that lies between the two with a smooth transition from one to the other regime. Results are largely independent of the details of the transition.

[9] Mainland coastal sites or Cretan sites essentially behave as islands because of the difficulty of land travel.

IV. Directional Networks

Hitherto, we have assumed undirected (reciprocal) links, which is not how exchange networks operate. Simple gravity models impose penalties on long single journeys, while PPA imposes penalties on sustaining too many links. With this in mind we consider more generalised models which accommodate 'gravity' while being directed.

Let us return to the notion that networks are, in some sense, 'optimal'. Usually, we think of this as an 'active' aspect of agency, so that the individuals/communities/polities make conscious, albeit imperfect choices, which optimise their exchanges in some ways. We shall turn to such optimisation later. For the moment we consider a passive alternative to optimization that has nothing to do with community behaviour, but everything to do with our knowledge of it, in which we minimise the consequences of our ignorance (as we have already begun to do with our use of gravity model aggregations). Suppose, given our aggregated sites, we only know a few features of the network, such as the necessity for exchange (related to site size) and a fixed overall 'cost' of exchange. We can construct many networks compatible with these constraints. If we give these networks equal statistical likelihood, on the grounds that to discriminate between them would assume more knowledge than we possess, we can ask what is the 'most likely' type of network to evolve, again in a statistical sense[10].

We borrow the idea from contemporary modelling of urban traffic flows (Erlander 1990; Ortuzar 1994), with potential parallels to historic and prehistoric exchange networks. The outcome of this optimisation is a *gravity model* (Batty 2010) where we have traded network cost for travel distance *D.* We shall give no details here because *rank* is (almost) simply proportional to size, giving all sites essentially identical centrality in the sense of Renfrew (as is Katz ranking). This is not a useful way to proceed, but models like this have the important ingredient that, as with PPA, even remote sites have to couple to the network, whatever *D*.

Urban retail models: The Rihll and Wilson model

Transport modelling networks are most simply taken as undirected since, in the absence of any prior information, it is sensible to take site inflows *equal* to site outflows. To introduce directionality we first consider a variant, which takes us *a* step beyond geography and technology, originally designed for urban planning (Wilson 1970, 1971, 1978). This has been adopted by Rihll and Wilson (Rihll and Wilson 1987; 1991) to an Iron Age archaeological system, that of Mainland Greek city states, but we shall apply it here to our MBA sites. In this variant the site outflows are initially taken as proportional to site capacity (as for simple transport models) but the inflows are now *outputs* determined by the search for most likely networks, the networks characterised by a new attribute termed 'attractiveness'. In introducing this new feature we have broken the simple connection between rank and size. It is, of course, highly unlikely that we can do more than provide more than the simplest caricature of an evolving network with just a single new free parameter, as we now see.

The generic behaviour of the network as attractiveness changes is straightforward (Dearden 2009; 2010). For low attractiveness we have many small competing sites. As attractiveness increases a handful of these sites show increasing inflows until a very few sites dominate the ranking tables so strongly that there is no ambiguity about defining them as hubs, drawing in all the 'trade' (outflow) of their weaker neighbours[11]. The rapidity of this change of behaviour as the network changes from a collection of little sites to a few major hubs is stronger for large *D*,

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[10] Technically, what we are doing is *maximising* entropy, a statement about the information encoded of the system, subject to constraints.

[11] Remember that this model was devised, among other reasons, for the placement of shopping centres, needing to draw in their clientele, for which 'goods' above equates to 'money'. In this context we are seeing the replacement of local stores by a few retail centres.

weakened as *D* is diminished, when there are more, but less dominant, hubs. For the relevant range of *D* from 80km to 130km for the S.Aegean the *rank* profiles vary smoothly with *D*. Each regional grouping contributes to the creation of central places. Larger *D* favours the Cyclades, smaller *D* favours Crete and the Peloponnese. See Fig. 3 for examples.

When there are many competing sites, as in Fig.3a (in which site size is a measure of betweeneess) N. Crete, Phylakopi, Kalymnos, Thera, Asine have high betweenness and N.Crete, Thera and Phylakopi have high rank. Since several small or medium size sites have high rank, they have even higher impact (transactions per head -- Renfrew's centrality). Joining Thera and Asine are Ios and Kalymnos. As we move to fewer competing sites (larger 'attractiveness') as in Fig. 3b, highly ranked sites become regional hubs. Depending on *D* the most likely hubs are one from each pair Phylakopi/Ios, Gournia/Petras, Myndus/Kalymnos, and Asine/Lerna. For such directional networks betweenness is strongly correlated to rank although hubs can be linked by bridges such as Phylakopi – Cesme that we have seen before. This is for the same reason, that PPA and constrained entropy models enforce some exchange over single journeys of large distances that less prescriptive models would expect to be achieved with stopovers. However, important as they may seem to be, in practice bridge ends can have low betweenness by our definitions (sites in Fig.3 are displayed by betweenness). We see that, by virtue of its other links, it is Phylakopi that has the high betweeness (and rank) and not Cesme and other outlying bridge ends.

How do these outcomes match our expectations of maritime networks? Superficially, this picture of hub sites, carving up the network into competing zones of influence, battling it out with each other for the outflow of the remaining sites, is reminiscent of the XTENT model in the patterns it produces. However, it differs dramatically from XTENT in that is describes an implosion of the components of 'trade' to the hubs from other sites, rather than an outward diffusion. In the language of maritime exchange, vessels from neighbouring sites leave with goods, return empty (see footnote 11). This is more like tribute than trade and not how we understand maritime networks to behave. For that reason we shall not consider the model further, despite the detail with which we have examined it.

However, before passing on, it has brought an issue to the fore; the relationship between site size (carrying capacity) and site weight (population) which, except for the explicit gravity model introduced initially, has played no direct role to date. Insofar as they have been relevant, site populations have been part of the model *input,* to be chosen more or less at will. This is not surprising, given the origins of entropy-maximising models in transport and retail models, for which flow is all and the notion of residential populations is meaningless or unhelpful. The historical record shows that we should not correlate population to resource availability too closely (i.e. assume constant population density) since we see sites with limited local resources able to sustain greater populations because of their exchange with the rest of the network. In fact, a necessary condition for a model to be realistic is that it should give rise to varying population densities. For this we need further ingredients in our models.

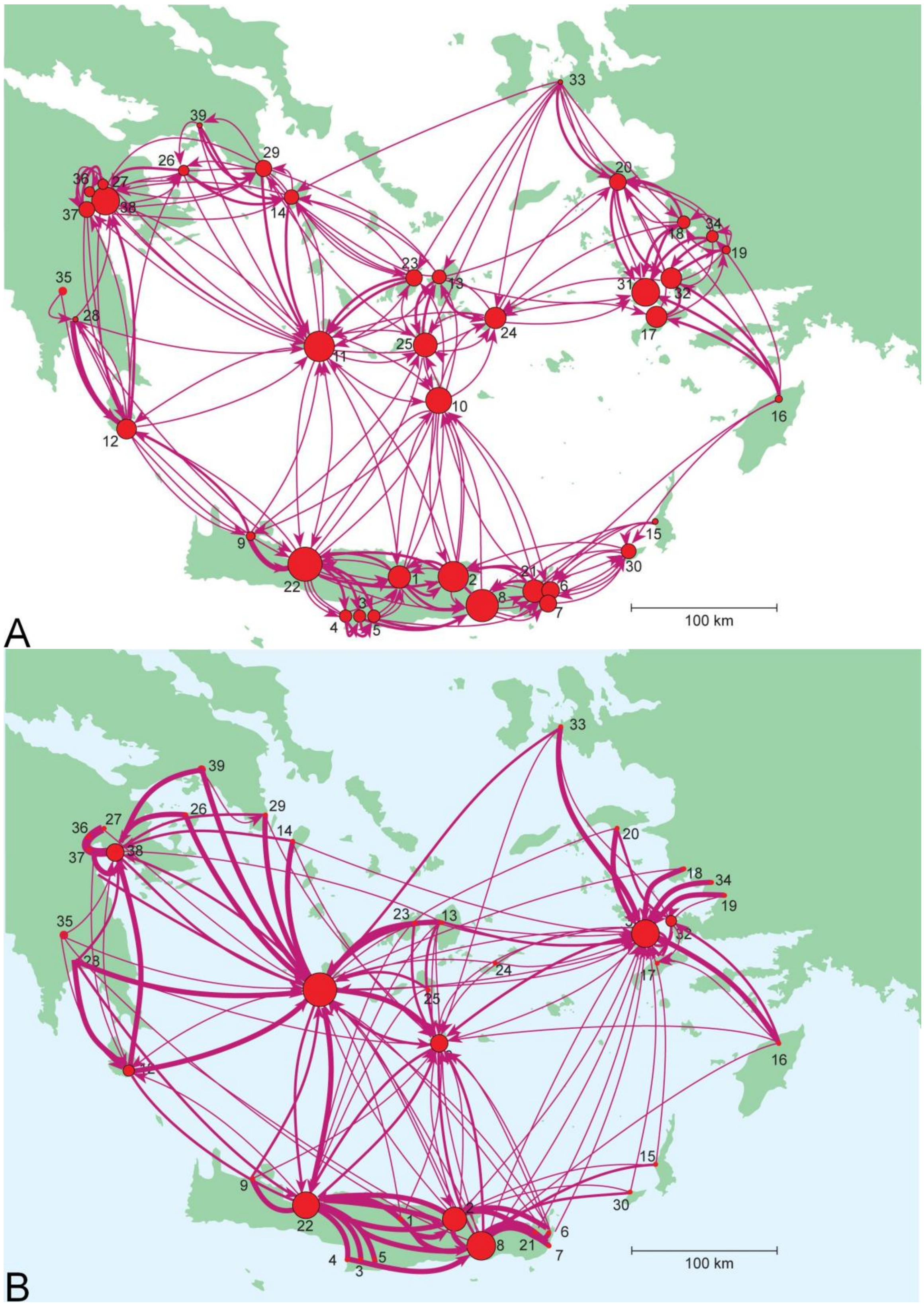


Fig.3. Two networks in the Rihll and Wilson model. The top figure A has lower *attractiveness* than the lower figure B. Size of vertices is given by *betweenness*

Cost/benefit analysis: *ariadne*

We shall now examine optimisation in the context of 'efficient' networks. By 'efficient' networks we mean *actively* 'optimal' networks, seeking to maximise the benefits of exchange, while seeking to minimise their 'costs'. In practice our networks are 'almost optimal' in that it is difficult to find the 'best' choice and it is sufficient to choose between several comparably good options. This stochastic element distinguishes this model from the deterministic models discussed previously. The model we shall describe here, named *'ariadne'*, has been given in some detail in papers and articles published elsewhere (Evans et al., 2009, 2011; Knappett et al. 2008, 2011; Rivers et al. 2011). We refer the reader to these papers for a fuller explanation.

The way to quantify 'almost optimal' is to reinterpret 'optimisation' as the 'minimisation' of a quantity we term the 'social potential' or 'utility function'.[12] We can think of this potential as describing a 'landscape' whose coordinates are site populations and the strength of the links between them. 'Optimisation' then corresponds to looking for the lowest point in this landscape (and identifying the network corresponding to this minimum). This landscape has very high dimension and there are many points (networks) competing to be the lowest. To take only the lowest of the minima is unnecessarily restrictive and suggests a more stochastic approach, in which we take comparable minima into account in a statistical way so that, for example, we might say that (on rerunning 'history' several times) Philakopi is a dominant hub three times out of five, Ios two.

The social potential in *ariadne* must contain at least two terms, one for the benefits of exchange and one for the costs of sustaining the network. As for the former, we take the benefit/utility attached to an exchange link to be conventionally 'gravitational', proportional to the product of the 'centre-of-mass' 'populations', with an intersite 'potential' falling off strongly at distance *D,* using the ease-of-travel interpolating function discussed earlier (footnote 8). We stress that here the populations will be *outputs* of the optimization. The mutual homophily of the gravity input means that large sites benefit hugely from interacting with large sites. Empirically this seems a necessary condition for the generation of the wide range of populations seen in the record. Further, as we have already observed, by adopting gravity inputs, we can approximately aggregate local resources into larger (e.g. island-wide) sites without needing local knowledge. As for costs, two obvious candidates are the cost of sustaining the total population or the cost of sustaining total exchange ('trade') and we assume that the total cost will be a combination of these.

While these terms may be necessary, it is straightforward to show that just these costs and benefits alone are still not sufficient to give the desired range of population densities. Specifically, provided the costs are not too large with respect to benefits, populations are almost proportional to site size, with not the wide variation we are looking for, despite the latter's gravitational nature. Also, there are very few of the weak links necessary to stabilise the network against change. In fact, as we now increase the costs an increasing number of sites collapse, with effectively zero weights/populations. Although there are some differences in how this comes about, according as how costs are allocated, the surviving sites in the network still maintain weights/populations remarkably proportional to size[13].

However, just as the urban retail model that we have discussed above for siting shopping malls breaks down if consumers also have their own vegetable gardens, livestock, etc., models for realistic trading networks do need to take the benefits of local resources into account. Their contribution to the social potential is a benefit, a cushion against population costs (although it can incur a cost if the resources are over-exploited This suggests that the simplest social potential

[12] Although we did not introduce it at the time, for the Rhill and Wilson model this potential is the negative of the entropy.

[13] For example, increasing the costs of sustaining the network gives rise to chains of strong links rather as in PPA.

required on general grounds should include all four terms (benefits from exchange and local resources, costs for sustaining trade and population). The aim is to find the configuration of the network that makes the social potential as *small* as possible. This takes us further beyond geography and technology, in having to estimate the relative benefits of local resources to those of exchange, the *total* population and the relative costs of sustaining the network. In practice, we can (almost) compensate for changes in one through changes in the other two, giving an effective model that is effectively only *two* new steps. The output is now not just outflows but also site populations. The 'social landscape' through which we hunt has, again, many minima (valley bottoms) that correspond to networks that are comparably optimal.

Ariadne: centrality

As we anticipated, not only do we break away from the proportionality between site size and population for functioning sites, but we have weak links that betoken stability. However, even with only 'two' new parameters it might be thought that there is too much freedom for the model to be useful given the relatively fragmented record. This is not the case. Just as we saw, for the Rihll and Wilson model, the need to steer away from instability by restricting attractiveness, *ariadne* can equally show instability, such as when the costs of sustaining the network become too high[14]. This is an almost inevitable consequence of the model's built-in homophilic tendencies, in which the non-linear benefits of large sites exchanging with large sites and thereby making large sites larger, is in precarious balance with the non-linear costs of over-exploitation of local resources. Avoiding this behaviour restricts us to a limited range of parameters.

Our approach has been to begin with a choice of parameters that give a plausible network. As we have observed, if we ignore local resources we have networks with population proportional to site size and large sites need to exchange so strongly with other large sites to compensate for inadequate local resources that the system breaks down. On the other hand, once we invoke local resources, if the benefits of exchange are too small, then islands try to become largely self-sufficient, but again making individual island collapse frequent. This is commensurate with the observation by Broodbank et al. (2005):

"For the southern Aegean islands in the late Second and Third Palace periods, an age of intensifying trans-Mediterranean linkage and expanding political units, there may often have been precariously little middle ground to hold between the two poles of (i) high profile connectivity, wealth and population, or (ii) an obscurity and relative poverty in terms of population and access to wealth that did not carry with it even the compensation of safety from external groups".

Although the freedom in the model is not so large as to enable us to get any behaviour we wish, there still is considerably more choice in the way that central places can be generated than in our earlier models. Nonetheless, despite the uncertainty introduced by our statistical analysis, there are a set of orderly patterns for the creation of centrality according to the scenario in mind. We have already mentioned the case in which, all other things being equal, increasing the costs of sustaining the network leads to instability as the sites concentrate on fewer and stronger links (Knappett 2011). We consider a different situation, far removed from instabilities, that arises as the benefit of exchange increases, all other things being equal.

By trial and error we find that some types of network are impossible to achieve, or extremely rare. Those networks that arise easily have a dominant Crete, connected to the mainland by a western string through Chania and Kastri and to a weaker Cyclades primarily through Thera and Phylakopi. There is a strong Dodecanese that is only intermittently connected to Crete directly through Rhodes, and not particularly strongly connected to the Cyclades through Naxos.
This is more or less as we might have expected, from simple geography alone. The interest lies in the detail. As we said, our model is not strictly deterministic in that we choose statistically

[14] We have invoked this (Knappett 2011) as one of the plausible reasons for the collapse of Minoan influence some time after the eruption of Thera.

between comparably efficient networks. This is to the good, given the inevitable fuzziness in attempting to quantify the 'best'. Typically N. Crete connects to the Cyclades through Thera, with W. Crete and Kastri connecting to Phylakopi. We give an exemplary network in Fig.4 in which the site sizes correspond to 'population', rank (busyness) and betweenness, as we read from left to right. We see that there is a strong, but by no means exact, correlation between them. We should pay more attention to the general pattern than to the details, since different runs of the programme give different fine structure (see Fig. 5).

In this particular network Knossos plays no major role. However, on rerunning the simulation several times *from the same parameter values* we find that, for a substantial fraction of the time (about 25%) there is a dominant exchange between Thera and Knossos or Malia. In Fig. 5 we show one network in which the link between Knossos and Thera is overwhelming, with Thera an unambiguous gateway to the Cyclades. Such a network is not to be expected on simple ideas of geographic space alone. Its behaviour is equally optimal as that of Fig.4, but the latter is more likely to have occurred. The former is more susceptible to contingence, although we do not know what the contingencies might be. We stress that we should not put too much emphasis on individual networks. Nonetheless, we find it striking that such behaviour arises in our model without too much difficulty.

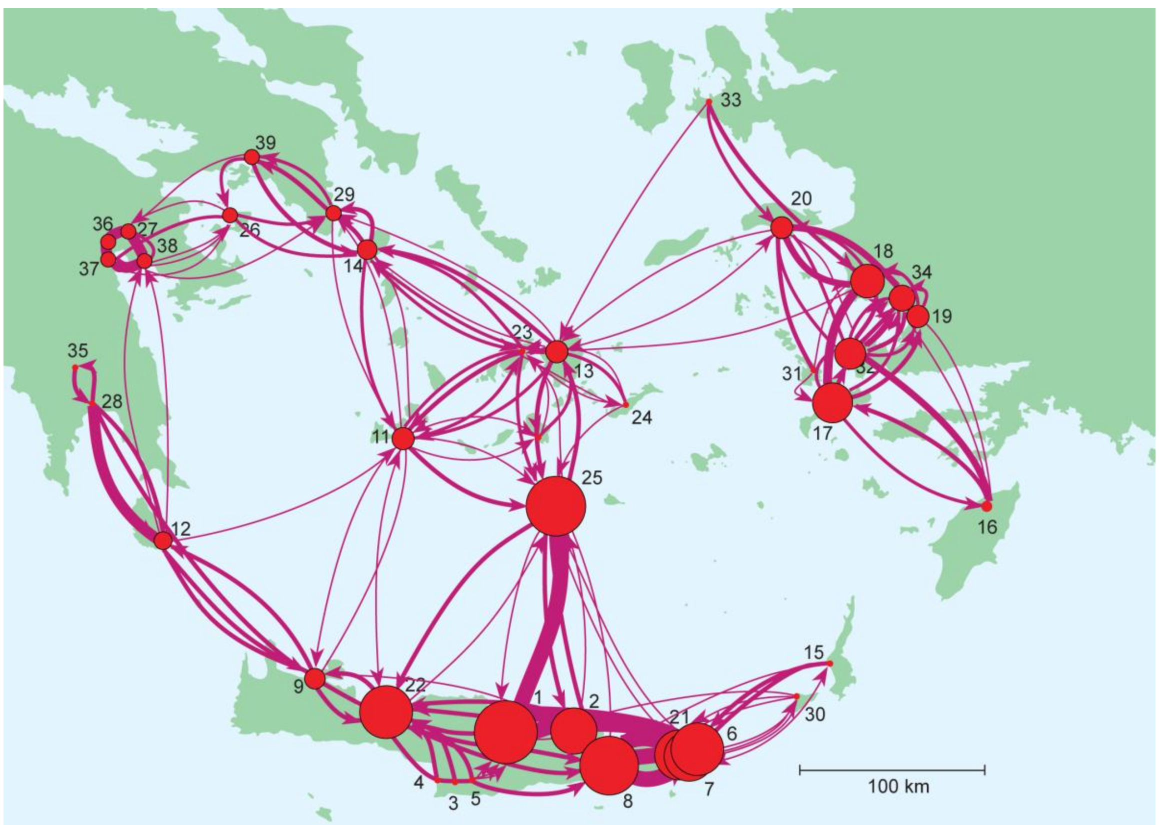
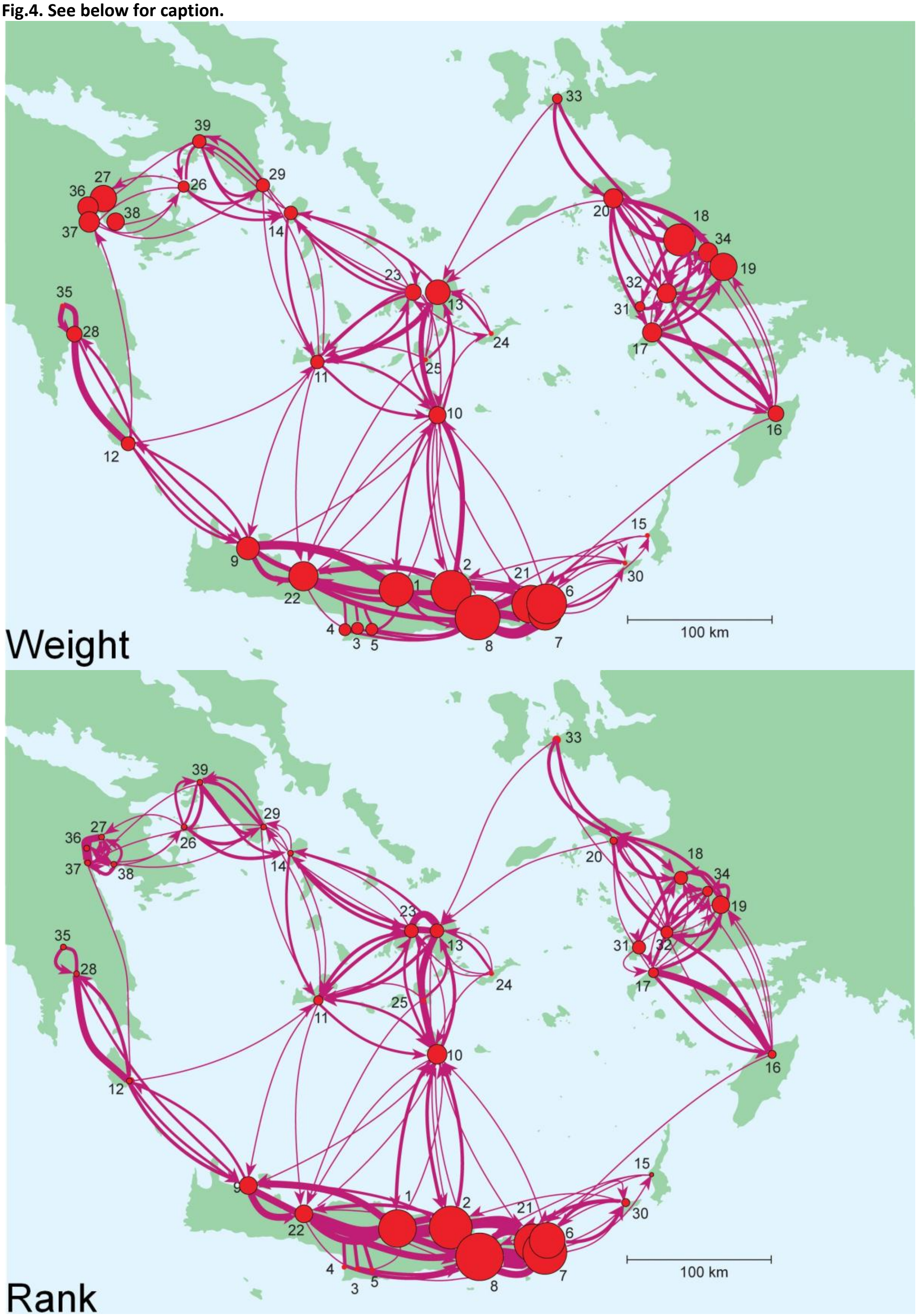


Fig.4. See below for caption.



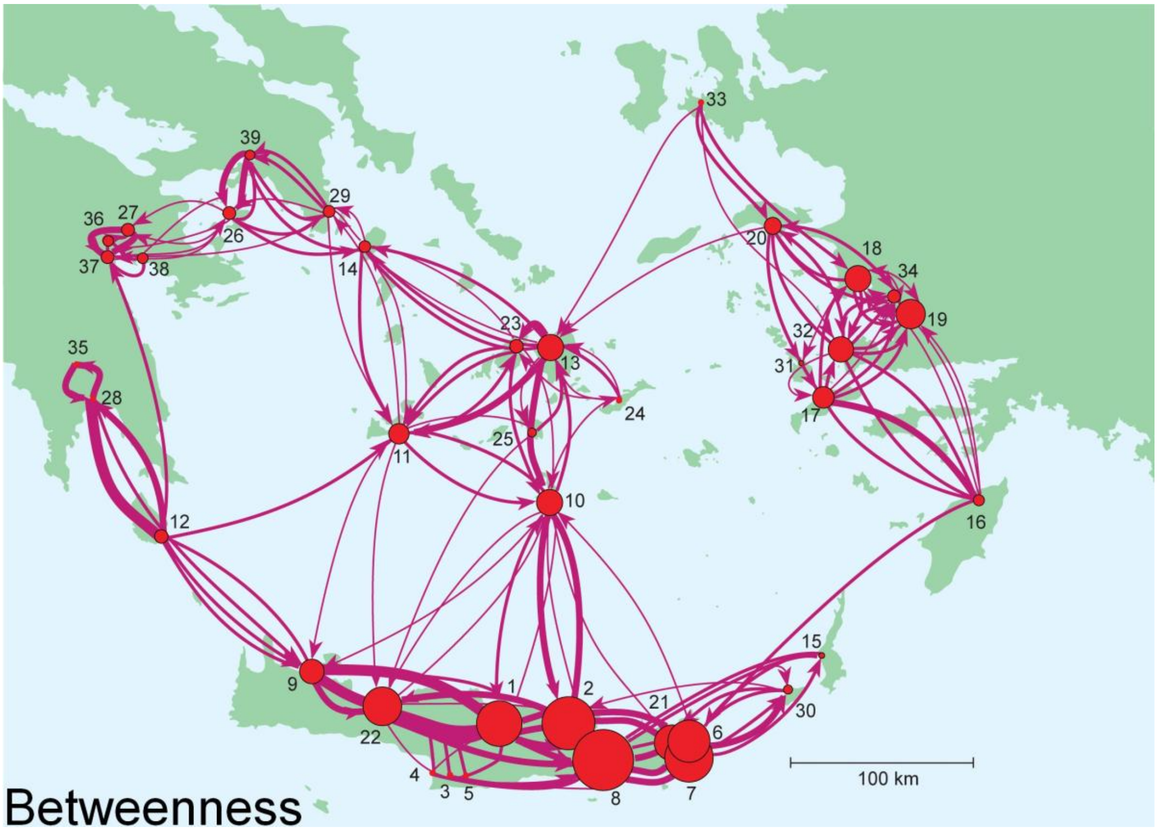


Fig.4. An exemplary network from *ariadne*, showing site rankings with respect to population (i.e. weight), busyness (i.e. rank) and betweenness respectively, as we read from top to bottom. We see the strong correlation between them. Population is an output in *ariadne*. Site resources, or carrying capacities are inputs, listed in Table 1.

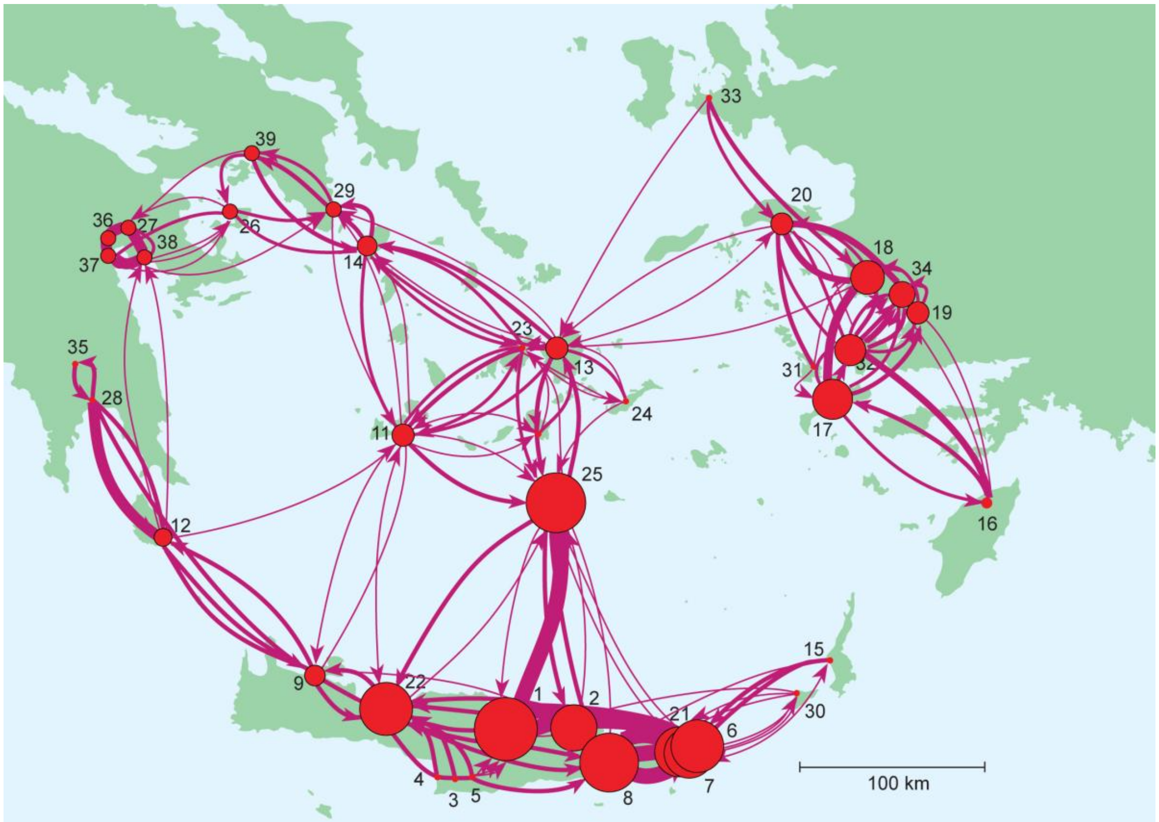


Fig.5. A network in which the link between Knossos and Thera is very strong. The model parameters are *identical* to those used in Fig.4. Sites are labelled by their betweenness, but there is significant correlation between all measures of centrality.

I. Summary

In this article we have shown how one might identify important archaeological sites by means of their 'centrality'. Colloquially we know what this means; sites with substantial resources in proximity, or manageable contact, to several or many other sites, preferably also important, with which exchanges is conducted. An important question is whether such sites can be simply estimated by the spatial geography of the network and some knowledge of site size, without having to be more sophisticated.

We have argued that we need to do more than look at the map, the map in question being that for MBA maritime networks of the S. Aegean, about which we have written at some length elsewhere, but not in the context of centrality. Central sites include both those that are the most active in the exchange process and those that mediate the networks flows. To this end we have introduced two kinds of centrality. The first is Renfrew's notion of a 'central place', understood as eigenvector centrality, which we have termed *rank.* For such networks a passable proxy for *rank* is the busyness of harbours as a measure of the flow of goods, people and ideas between them. Our second measure of centrality is a version of betweenness centrality, termed *betweenness,* from which we can infer 'gateway' sites.

Empirically (and sometimes analytically), if the links between sites are not directional, site *rank* is the effective site degree, assuming that links are essentially unweighted (strong links counted, weak links not). Models of this type include simple geographic and gravitational models and PPA. With effectively no free parameters beyond estimates of site carrying capacities and ease of travel by sailing vessel they paint with a broad brush, in this case the wrong picture.

There is more to agency that ability to travel and a desire to maintain links. We have assumed that networks arise that, in some sense, are optimal. We have considered both passive and active optimization. The former corresponds to looking for the most likely type of network compatible with our limited knowledge of the system (entropy maximization). Adopted from transport models and models for contemporary urban planning, the simplest application of these ideas is in the network modelling by Rihll and Wilson for Iron Age Greek city states, which introduces a variable construed as site 'attractiveness'. As applied to the MBA it provides for more interesting scenarios in which dominant hubs arise, typically one in each of the four geographic regions of the S.Aegean. It provides a good counter-example to our null models with their emphasis on geography, and leads to outcomes that are not predictable from map-reading. However, this is still a highly constrained model which, in the directedness of its links, is at variance with the record, a reflection of its origins for describing the transition from shopping streets to shopping malls.

For the reasons above we abandon it for our most substantial model, named *ariadne,* which is also optimal, but adopts a cost-benefit approach, looking for networks in which the benefits of exchange and local resources exceed the costs of sustaining exchange and the local populations. This is less prescriptive but, even then, has relatively few parameters. We have shown the role that our different understandings of centrality play here, particularly in the connection of N. Crete to the Cyclades and beyond. This, while being by no means predictive, in particular because of our stochastic analysis, with the emphasis it gives to Cretan sites and their connection to the Cyclades and the mainland via a Western link, it is not in obvious disagreement with our expectations. For example, it is relatively easy to find networks in which Knossos and Akrotiri have high betweenness and are highly ranked. Other sites that play an important role include Phylokapi and Kalymnos (for example) and we need to see to what extent this is reflected in the archaeological record.

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