

Dynamic networks and Hamiltonian landscapes: a case study from the Aegean Bronze Age

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This is a working paper on the use of graphs and other techniques from physics to describe and explain prehistoric interaction networks. Graphs¹ are used to represent basic information about settlements and their interrelationships: the settlement sites are the *vertices* of the graph, and their interrelationships are the graph's *edges*. There is no intrinsic reason why this methodology should not also be applicable to other contexts, such as historic and modern interaction networks. Nevertheless, prehistoric settings present two immediate advantages in terms of methodological development. First, the dataset is relatively incomplete and patchy, thereby encouraging reductive approaches. Secondly, we can chart changing network structures and dynamics over the long-term.

Archaeology has not been particularly successful in tackling the emergence and behaviour of regional interaction networks. The core-periphery models employed within world-systems theory² would appear to engage with such interactions, but there appears to be relatively little detailed study of the bottom-up emergence and local behaviours of regional networks. It may be that world systems theory is too homogenising to account for particular historical trajectories on the local level (Gosden 2004, 17). Or perhaps there is a wider problem in that we are simply far too site-centric: sites come first, established on local grounds, and interactions with other sites follow. It is difficult to entertain the thought that site interactions might themselves contribute to the size and status of the sites in question. But what if we do turn the tables, and treat interactions as primary and sites as secondary?

One excellent example of such a 'turning of the tables' is Broodbank's work on the EBA Cyclades. It is the only *systematic* attempt thus far, for any period of the prehistoric Aegean, to explain the growth of certain sites (in the Cyclades) in terms of their interactions. This approach was perhaps encouraged by the fact that some important Early Cycladic sites are very hard to explain in terms of local resources, occurring on small rocky islands with limited agricultural or mineral resources. Indeed, some are only inhabited for the first time in the Late Neolithic. Broodbank thus sought to attribute site prominence to the degree of 'centrality' in Cycladic interaction networks. Interestingly, Broodbank turned to a mathematical technique, a simple form of analysis taken from graph theory, Proximal Point Analysis (PPA), already used effectively for other archipelagos in Oceania (Terrell 1977; Irwin 1983; Hage and Harary 1991, 1996). Broodbank assigned hypothetical sites to islands on the basis of

¹ See Evans 2005 for a review of basic graph theory and bibliography of exemplary applications in a variety of fields.

² For example, Schortman and Urban 1992; Peregrine 1996; McGuire 1996; Stein 1998; Chase-Dunn and Hall 1997; Kardulias 1999.

population estimates derived from site survey data. He then drew edges from each site to its three nearest neighbours (in physical, not cultural terms). If the sites were all evenly distributed in space, like a regular crystal lattice, then each site would have the same number of connections as every other, and no single site would be more connected than another. However, due to their uneven distribution, some sites emerged as more connected than others, with five or six edges to other sites. These sites possess greater 'centrality' in the network. When one changes certain parameters, such as site density (as Broodbank does, simulating population increase over time), or the number of nearest neighbours, then one might expect the texture of the network to alter and other sites to emerge as central. When Broodbank compared the results of his PPA with the actual data, he found that it did indeed predict that a site on Keros, for example, would possess centrality in such a network. Of the five major EC sites, three were 'central' in the PPA. Of course, Broodbank also had to suggest some motivation for these interactions – communities don't just interact without motives or goals. He cited basic demographic processes and the need for social storage networks (Broodbank 2000, 81-96), with power and prestige emerging consequentially out of network interactions.

In terms of the methodology Broodbank employs, his use of graph theory is instructive. Using a rather mechanistic form of analysis of this kind seems at first surprising given the 'humanistic' tone in Broodbank's work. Yet this disjuncture is no fault of Broodbank's, but is, I think, part of a much wider problem: archaeologists seem not to have developed sophisticated and subtle means of investigating dynamic interactions occurring in physical space. Spatial analysis in archaeology has for decades employed concepts from human geography, such as gravity models, central place theory (Christaller), and Thiessen polygons, which tend to be site-centric. Yet it is most illuminating that Broodbank's use of PPA seems enormously innovative. His approach marks an interesting reversal of how archaeologists usually reason – i.e. a site grows for its own internal reasons, and then once it reaches a certain size it may be able to start participating in wider networks. In Broodbank's scheme, it is the links that explain the nodes rather than vice versa.

Although Broodbank has made the all-important first step, there is scope for developing this kind of analysis much further.³ The need for more sophisticated approaches to interactions soon becomes apparent when one looks to later periods. Broodbank was able to assume equal site size in his analysis, and also equal connections, in terms of weight and directionality. Furthermore, he restricted the scale of the system to the Cyclades alone. When we come to the interaction networks emergent in the late MBA, a very different picture confronts us:

³ although note that Davis (1982) was the first to use graph theoretic techniques in the Aegean, examining the oscillating centrality of Delos from 1600 to 700 BC. Davis' key idea is that Delos emerged as an important regional centre in the Archaic period because of its central location not in the Cyclades but in the larger Aegean network of city-states that stretched from the Greek mainland to the mainland of Asia Minor; it is only when these major landmasses come into the picture that Delos acquires a locational advantage with respect to centrality. This is discussed by Hage and Harary (1996, 197-201) who emphasise Davis' use of median centrality to assess relative accessibility.

1. nodes: we *know* that there are sites of substantially differing sizes and roles, quite unlike the situation in the EBA.⁴ Note the assumption that large sites developed due to local internal processes (e.g. access to agricultural surplus).
2. links: we can also see that there are very different kinds of links existing simultaneously, varying in directionality, length and weight (exchange and affiliation).
3. scale: not only the Cyclades, but also the Dodecanese, Crete, and the landmasses of Asia Minor and mainland Greece.

Graph theoretical analysis may help show us some unexpected patterns in the Aegean Middle Bronze Age. The current methodology has been largely developed through considering this particular period, spanning c. 2000-1500 BC. The main dynamic to concern us is the emergence of 'Minoanisation' at the end of the Middle Bronze Age. In this process a number of sites across the south Aegean, on both islands and mainland, develop increasingly complex exchange links and shared cultural traits. The driving force behind this is the large island of Crete, with certain central sites, and Knossos in particular, seemingly most involved. The similarities in material culture between sites on and off Crete are so pronounced that some have been led to speak of colonisation. This interpretation is connected with the idea of a Minoan sea-empire ('thalassocracy'). There is no direct evidence that the fleet needed to maintain such an empire actually existed; the source of the thalassocracy idea can actually be traced back to reports by Thucydides, which were of course made more than 1000 years later than the period they purport to describe.

Nevertheless, whether through direct (colonisation and military might) or indirect (acculturation) means, the Cretan palaces capitalised on their regional dominance and extended their influence beyond the island. Essentially this represents the earliest ever occurrence of state-led expansionism in the prehistoric Aegean (and by extension, Europe). Present interpretations are, however, inadequate, at many levels. Apart from the continued reliance in some quarters on Classical sources (i.e. Thucydides), there is a general tendency to explain first the growth of individual sites in local terms (good land, resources, etc), and then to extrapolate connections between sites from there. In other words, the 'vertices' (sites) always precede the 'edges' (links). There are, naturally, some exceptions to this, with Davis' work on the 'Western String' route through the Cyclades linking Crete to the mainland (Davis 1979), and Berg's assessment, using world-systems theory, of Southern Aegean

⁴ One cannot chart a simple evolution from EBA to MBA because there is a significant gap in occupation in the Cyclades. It is as if one is starting from scratch. Some of the most important EBA 'vertices', such as Chalandriani on Syros or Dhaskaleio-Kavos on Keros, are never again occupied. New interaction networks develop in the MBA, with a quite different focus, and enabled by a new transport technology – the sail. Crete becomes increasingly central to these networks, but these too eventually collapse (not a particularly robust or resilient system, it would seem). The focus subsequently shifts to the Mycenaean mainland, especially the Argolid (Mycenae). But these networks prove to be no more resilient, ending cataclysmically with the onset of the so-called 'Dark Ages'.

interactions in the Middle to early Late Bronze Age (Berg 1999). These and other studies, while focussing on interactions, have tended not to use explicit network models composed of nodes and links (in these cases, the nodes are undefined). Before exploring these phenomena further using graph theory, let us continue with our initial examination of conditions in the Aegean Middle Bronze Age.

Two key aspects of the Minoan networks briefly described above are:

1. an evolution **from exchange to affiliation** – initially the connections between islands involve exchange of goods, but eventually these are supplemented by actual imitation of artefact styles and technologies, suggestive of some process of cultural affiliation. It is interesting that this latter process appears to correspond in time with the probable emergence of a single political centre on Crete – i.e. Knossos. This central site can be regarded as a hub, and so we are inclined to investigate the possible link between hubs and strong ties in networks of this kind. *Do hubs serve to standardise functionality attributions, with innovations given clearer attributions and hence subject to different dynamics of transmission?*
2. a relatively rapid **emergence and collapse**. Interaction networks only endure for a mere two hundred years or so. This does not sound like a particularly robust or resilient system. More interesting still, they were preceded by different kinds of network in the Early Bronze Age (EBA), and indeed followed by others in the Late Bronze Age (LBA). In each case the focus of the networks is different, as is their geographical extent. In the EBA, the focus seems to be the Cyclades, and in the LBA, the Mycenaean mainland, especially the Argolid (Mycenae). Like the Minoan networks, each only lasts a few hundred years. The EBA networks are followed by a gap in occupation at many sites, if not total abandonment; some of the most important 'vertices', such as Chalandriani on Syros or Dhaskaleio-Kavos on Keros, are never again occupied.

As for the LBA, three significant shifts can be said to occur: first, with the transfer in the balance of power from Crete to the Mycenaean mainland; second with the changes within LB III that signal the demise of the Mycenaean palaces; third, the subsequent of the Late Bronze Age and the onset of the so-called 'Dark Ages'. Sherratt (2001) argues convincingly that the Mycenaean palaces were quite different to their Minoan predecessors. She considers them to be somewhat epiphenomenal, owing everything to the primary trade networks originating in the Near Eastern empires. They arose on nodal points in long-distance route networks. Still a kind of island-hopping system, until increasingly direct Cypriot exploitation of long-distance routes – leaving Mycenaean palaces as 'pigs in the middle', bypassed by a proliferating east-west traffic.

This process seems to also be felt on Kythera which, although not home to a Mycenaean palace, was nonetheless part of trade routes in the LB IIIA period (Broodbank, Kiriati and Rutter in press). But then its position as a hub is bypassed, presumably due to those same processes posited by

Sherratt (2001). There is population decline, and the apparent abandonment of the most prominent site on the island, Kastri. The resurgence of Chania (in west Crete) in the LB IIIB period has no positive effect on Kythera's fortunes, despite it being the nearest Cretan centre. Chania's connections in this period seem to be with the central Mediterranean, with apparently little use for Kythera as a stopping off point along the way.

In the abovementioned cases, archaeologists such as Berg, Sherratt and Broodbank/Kiriati/Rutter have recognised that the complex interactions between sites may have a dynamic of their own, with individual sites or regions sometimes at the mercy of the global dynamics of the wider networks. On the whole, however, archaeologists often fail to recognise that site interactions can contribute in a substantial way to the very character of the sites involved (because vertices and edges are always deemed to be in a hierarchical relationship, the former certainly primary). Exceptions to this tend to arise in particular circumstances, when, for example, it is possible to spot 'gateway' communities lying at the boundary between island clusters and mainland interiors; or when particular routes can be traced between areas of supply and demand in raw materials. But such patterns often remain at the level of descriptive observation because archaeologists are not well equipped to understand the interaction between local and global dynamics in networks of this kind.

So, why use network models? What may emerge is that the configuration of large-scale exchange and/or affiliation networks confers centrality on certain sites in unexpected, counter-intuitive ways. And if certain sites do emerge as central, are they to be found evenly distributed across the network? Or do we see some clustering of central sites? It is the large scale of these networks, and their complexity, that creates difficulties of understanding. Their 'long distance' properties are critical. And whereas there has been much emphasis on 'actors' and 'agents' of late, I think we should not lose sight of the fact that networks too can have their own behaviours that might not be simply predictable as aggregates of individual actions.

How far should we go with the use of techniques taken from graph theory and particle physics? We might just use them as heuristic devices, useful to think through; this can be an important first step, forcing an explicit focus both on interaction dynamics, and on the complex interface between local and global behaviours. In the application of systems theory to archaeology in the late 60s and early 70s, the idea of small change in one subsystem leading to substantial change at the overall system level was already in place; however, the character of the subsystems and their interactions was prescribed mechanistically. With the new generation of network analysis it is possible to conceive of order emerging from the bottom-up, in a far more fluid and contingent manner. And these new analytical techniques could be of considerable utility not only in helping us to understand more fully the Early Cycladic scenario presented by Broodbank, but also in extending our study to deal with the added complexities of the Middle and Late Bronze Ages in the Aegean.

Thus here we choose to conceive of an Aegean network, with its complicated constraints and interactions, as explicable in terms of an 'energy landscape' through which the system moves. This is based on a general principle that systems want to minimise their energy (hence conceiving of the system as having agency or behaviour of some kind). A system with low energy is close to some optimal solution in which all the different constraints and interactions are balanced. Rather like the stock market (e.g. the FTSE 100), evolution has both long-term and short-term characteristics which, most simply, can be thought of as a smooth general trend upon which is superimposed volatile short-term fluctuations. Although *the* optimal solution is rarely if ever reached, there may exist numerous different solutions that approach the optimal. So, one of these solutions may have Knossos as a key central place. Some small changes in certain parameters might then jog the system and cause it to fall into another configuration, equally close to optimal, but in which Knossos is no longer central. Or perhaps it might transpire that Knossos is again central, but for different reasons and with a different set of connections. That there are significant 'jogs' to Aegean interaction systems seems quite clear – the innovation of the sail at the beginning of the MBA could be one, and the destruction caused by the Thera eruption might be another. This latter most likely corresponds to a major change in the nature of the 'landscape'. In such circumstances, what were stable site exploitations in the valleys of this 'landscape' can become unstable configurations on its hills, which lead to a major readjustment in site use as the system migrates to the new 'valleys'. We might eventually go on to investigate why these kinds of 'kicks' caused fluctuations that led to new system configurations, while other fluctuations did not. We know of many examples in physical systems in which it is not necessary to have large changes in initial conditions to produce very different outcomes. One could simulate different kinds of fluctuation in order to see what systems are affected by what kinds of kick – for example, what else would it have taken for Mycenae to supplant Knossos?

From heuristics we can move into analytical and quantitative approaches. These are currently being developed with some initial application to archaeological data included below.

MODEL-MAKING

Generalities

The energy landscape that we wish to describe has two types of coordinates; site variables and link variables (generalisations of latitude and longitude). The energy of the landscape is denoted by its altitude. The assumption is that the system will evolve in the valley bottoms. Pursuing the simile further, the long-term evolution of a network can be thought of as a slow buckling of the terrain (*plate tectonics*), while the short-term volatility corresponds to shaking it (earthquakes).

Without wishing to assert primacy of sites (vertices) over links (edges), or vice-versa, we take the site co-ordinates of the landscape to be the fractional resource exploitation of individual sites (vertices), with one co-

ordinate label per site, and the link coordinates to be a measure of the relative link strength (in a way to be defined later), with one link between every pair of sites. The parameters that control the contours of the landscape are measures of site independence or self-sufficiency, and constraints on population size, etc. Thus, for example, as populations grow or total trade volume increases, the landscape changes, and the positions of the valleys into which the system wishes to fall changes. This is rather like Broodbank's increase in the number of links per island as population increases. Volatility here would correspond to short periods of drought, or unexpected local population changes (e.g. losing the longboat crew, as in the Pitcairns). This is accommodated in these models by the introduction of a 'temperature', whereby high volatility is 'hot', low volatility 'cold'.

Specifics

In all models, the 'Hamiltonian' energy function H which characterises each configuration of the system separates into four terms:

$$H = -\lambda E + \kappa S + (j P + \mu T). \quad (1)$$

In some roughly defined way, H measures the 'cost' (in manpower, resources, etc.) of organising the system of sites and their trading links. The aim is to find the configuration of the network that makes H as small as possible, for fixed values of κ , λ , j and μ .

The individual terms that constitute H are understood as follows:

S only depends on the properties of the sites in isolation. As such, it is a sum of terms, one term for each site, that describes the exploitation of the site as a function of the fraction v of its total resources. Over-exploitation is undesirable while under-exploitation is a wasted opportunity. By itself, S takes a minimum at some intermediate value⁵. Initially, we assume that all sites are equally easy (or difficult) to exploit. We could distinguish between rocky sites and sites with pasture, for example, at the loss of simplicity but at no cost to the numerical work.

E is the exchange/trade term which shows how the sites interact with one another (trade, influence) in a way that depends on both the properties of the sites and the network and weight of their interactions. Most simply, it is a sum of terms, one term for every pair of sites that is linked by trade or for other reasons.

The final terms (in brackets) enable us to impose constraints on population size, total trading links (and/or journeys made).

⁵ It is not the value of S that is important but its derivative (slope). We have to ask whether it is better to place a small number of extra people at one site or another which means we need to know how much better the system becomes when we increase the population at different sites by a small amount. We should put them where they do the most good. This ought to be when there are a lot of resources underexploited at a site, when it is under-populated and indeed the rate of return for putting some extra people at a site steadily decreases in our models and is highest at zero population and is zero (cost neutral) when S reaches its maximum.

All other things being equal, increasing λ increases the importance of inter-site interaction, whereas increasing κ increases the importance of single site behaviour. On the other hand, increasing j effectively corresponds to reducing population, and increasing μ reduces exchange.

Robustness

There is a general issue here, which applies to any modelling of social, cultural or economic phenomena by algebraic methods. Such methods can seem naïve, while simultaneously being over-prescriptive in that definite (but potentially arbitrary) functional forms have to be chosen so that calculations can be performed. As to the former, the reader must decide, but the latter is less a problem than one might think. One resolution to being over-prescriptive is the notion of a *universality class*. By this is meant that, rather than try to prescribe a 'fuzzy' function to accommodate our uncertainty, we can hope for a family of 'crisp' functions that, provided we ask the right questions, will all give us the 'same' answer. In the context of the Aegean, such questions will most likely be of a general, rather than a specific nature. The notion of *topological congruence*, taken from population biology, is most helpful. Functions which can be deformed into one another by stretching and squeezing are topologically congruent⁶. Although we consider a specific function, we expect similar general results from different functional forms in a family of functions, as long as they are congruent. This is one way to characterise robustness, which is essential if we are to believe that our conclusions are realised by realistic systems.

Despite the occasional demonstration to the contrary (Easter Island) we assume that any community will work towards an efficient use of resources. Thus S in Eq.1 encourages growth when there is under-exploitation but penalises a site when it stretches its local resources.

However, when it comes to modelling E several possibilities suggest themselves, and we shall consider two main options below.

1. The first choice is whether to adopt 'gravitational' models or not. In our context, for a 'gravitational' model the energy cost attached to an exchange link between sites with resources s and s' , separated by 'distance' d is proportional to

$$s.s'/f(d),$$

where $f(d)$ is some function of distance d , larger for large distances. The term 'gravitational' is applied to these models in analogy with Newton's law of gravity, for which the gravitational energy of two masses is proportional to the products of these masses, divided by a power of the distance between them. For such models it is advantageous, in cultural exchange, or trade, for both a site and its exchange partner to have large resources. We realise that the

⁶ Topology is concerned with our ability to squeeze and stretch, without tearing.

cultural exchange/transmission that we are considering here is by no means simply economic but, in contemporary economic parlance, we would say that this model embodies the advantages of a large consumer market and producer power.

In non-gravitational models it may be advantageous to connect to bigger sites, without any further advantage if one is big oneself. More simply, it could be that an exchange/trade term at a site only depends on the existence of links to other sites, and is insensitive to the resources/population available on the site itself. In contemporary economic parlance, we might term this a purely market led view which ignores consumer demand.

2. The second major choice is how to measure the 'distance' d . Most simply, it is the physical distance between the sites, perhaps modified to take into account currents and prevailing winds, land versus sea travel. However, in the absence of an advanced marine technology, a more realistic option might be to define d as the network length. This is the distance between sites when only links in the network are followed, the length determined by 'island-hopping'. This latter is a property of the network as a whole, as slow to find by ancient MBA sailors as by JAVA programmers today. However it is a physically reasonable approach which requires our network-centric viewpoint.

With this in mind, we shall largely restrict ourselves to the four models that correspond to these four options of gravitational/non-gravitational exchange terms with direct distance/network distance fall-off.

Long-term trends ('landscape buckling')

When we think of H as describing a 'landscape' or 'contour map' there is a problem. It is of finite size, since the coordinates are either the fractions of the resources exploited or the normalised exchange links. As such they cannot be negative, and are effectively bounded by unity. We should therefore think of H as defining a map in which only one 'page' corresponds to the 'real world'. The minima of H in this 'real world' page may be usual minima (valley bottoms), but may be at the edge of the 'page' as the landscape slopes towards it.

Taking the map as a whole, for a smooth landscape the lowest point is on the horizontal (otherwise you could move further downhill). Our first step is therefore to identify the horizontal features of the landscape. The positions in which the 'ground' is horizontal are termed the extremal positions of the system and are the solutions to

$$\delta H = 0.$$

δH is the change in H on varying the parameters a little (i.e moving slightly in the landscape). On the horizontal the variation in H does not change on slight movement in position, to first approximation. Only then do we check to see whether they are on our 'real world' page.

There are a large number of possible parameters and looking for characteristic behaviour in this multi-dimensional space is difficult, particularly when we have to resort to numerical modelling. For n sites we have an n -dimensional coordinate map. In H , E alone contains $n(n-1)$ directional link parameters, in addition to λ .

Empirically, there are two ways to proceed:

1. To look for analytic solutions to (necessarily) simplified models, to see typical behaviours (of a statistical nature). Since our results here are algebraic, we can look for algebraic patterns without having to worry as to whether the numerical values that we have given to site and link parameters can be justified.
2. While algebraic solutions can show how patterns of behaviour arise they are, by definition, too generic to be directly applicable to a system as specific as the EBA Aegean. To this end we need to develop numerical simulations of increasingly larger systems, with none of the simplifications of the algebraic models, until we can tackle 'realistic' island sites, with specific parameter choices.

ANALYTIC (MEAN-FIELD) SOLUTIONS

Before attempting any numerical modelling with the real island parameters, it is useful to see some of the behaviour that might arise, using simple analytic approximations.

For this section we restrict ourselves to algebraic solutions, in which we ignore volatility. As an extreme example that is surprisingly illuminating, we take the case in which all sites are of equal size and which interact in a similar way. For example, we could imagine a group of equal islands each interacting at equal strength with each other or a ring of equal coastal sites that interact with their nearest neighbours only. For such a simple system all site exploitation variables are identical and our landscape, as originally posed, becomes essentially one-dimensional. It is therefore more sensible to use the dual description of the system landscape in which the (unique) site utilisation becomes the altitude and in which λ and κ become that latitude and longitude, respectively. We understand λ as characterising the average strength of the trading links and κ as a measure of insularity in the individual sites.

We find two different types of characteristic behaviour according to the nature of the trading links, as encoded in the term E in equation (1). One choice, which we term our Gravity Model⁷ is to make the term in E for exchange between the i th and j th site to be such that trade from sites with large resources enhances E , as does trade to sites with large resources (larger consumer market). However, this is not the only way to postulate an exchange term. More simply, it could be that an exchange/trade term at a site only depends on the existence of links to other sites, and is insensitive to the resources/population available on the

⁷ This was Model B and when with direct distances was the numerical Model 1.

site itself. We term this our Market Led Model⁸. We take these possibilities in order of complexity.

Market Led Model

Our Market Led Model is exactly solvable, without the need for taking sites to have equal behaviour. As λ increases from zero for fixed κ there is a monotonic growth in average site exploitation from under-exploitation to full exploitation. This growth is faster for smaller κ , but there is no sharp transition, or jump in the growth. Alternatively, if λ is held fixed and we increase κ (i.e. individual site activity becomes more important than intersite trade), then all sites undergo medium exploitation.

Gravity Model

In this case we can only solve analytically with the assumption of equal site behaviour. We have not discussed the constraints encoded in the term C in (1), which show how H incurs a cost whenever exploitation increases or new trading links are added. They are now important in determining behaviour. Roughly, provided λ is large enough then, as λ increases from zero for fixed κ , there is a monotonic growth in average site exploitation from under-exploitation to full exploitation, as in our Market Led Model. Again, as in our Market Led Model, provided λ is large enough then, if λ is held fixed and we increase κ , all sites undergo medium exploitation as trading links become unimportant. The major difference occurs when λ (trading strength) decreases for small fixed κ (low self-sufficiency). Then, for only a small reduction in trading strength, exploitation of resources can collapse from full exploitation to no exploitation which, naively, we might infer as site abandonment.

What is particularly interesting is that such simple models can lead to discontinuous and dramatic behaviour naturally. If we drop the assumption of equal site behaviour we expect to see this collapse happening to some sites rather than others, rather than a total collapse, for reasons that depend on the details of the inter-site interactions. In this regard we note the following observation by Broodbank et al. (in press):

“For the southern Aegean islands in the late Second and Third Palace periods, an age of intensifying trans-Mediterranean linkage and expanding political units, there may often have been precariously little middle ground to hold between the two poles of (i) high profile connectivity, wealth and population, or (ii) an obscurity and relative poverty in terms of population and access to wealth that did not carry with it even the compensation of safety from external groups”.

We note that these rapid collapses are not induced by volatility but correspond to a smooth buckling of the landscape in which a valley becomes an unstable col, for example. This is reminiscent of the (often

⁸ This was Model A and, with network distances, was the numerical Model 3.

misapplied) *catastrophe theory* of the 70's. The introduction of volatility would make the situation even more complicated, and our models are not yet well enough developed to be able to include it analytically.

NUMERICAL MODELLING

We are beginning to apply the models discussed above to realistic data. However, *a priori* it is difficult to make sensible estimates for the model parameters so we have to search for robust ranges where features are visible, much as we have to might choose the right scale and coverage when choosing a map for a problem in real life. However we will take a collection of 19 sites significant in the MBA including representatives from Crete, Asia Minor, the Cyclades and Dodecanese. These are shown in figure 1.



Figure 1: Geographical location of the sites used in numerical examples.

1. Knossos	2. Malia	3. Phaistos	4. Kommos
5. Ayia Triadha	6. Palaikastro	7. Zakros	8. Gournia
9. Chania	10. Akrotiri	11. Phylakopi	12. Kastri
13. Naxos	14. Kea	15. Karpathos	16. Rhodes
17. Kos	18. Miletus	19. Iasos	

Figure 2: Table of site names against numerical label.

PPA in the MBA Aegean

Keeping the directional nature of the algorithm, we get results such as those shown below. It is straightforward to vary the number of outgoing edges per site. Here we show three per site. Note that counting incoming arrows shows some sites with more than three: Malia and Phaistos have the most, with five incoming connections. Knossos and Aya Triadha are examples with four, while Kastri is the only site with no incoming edges.

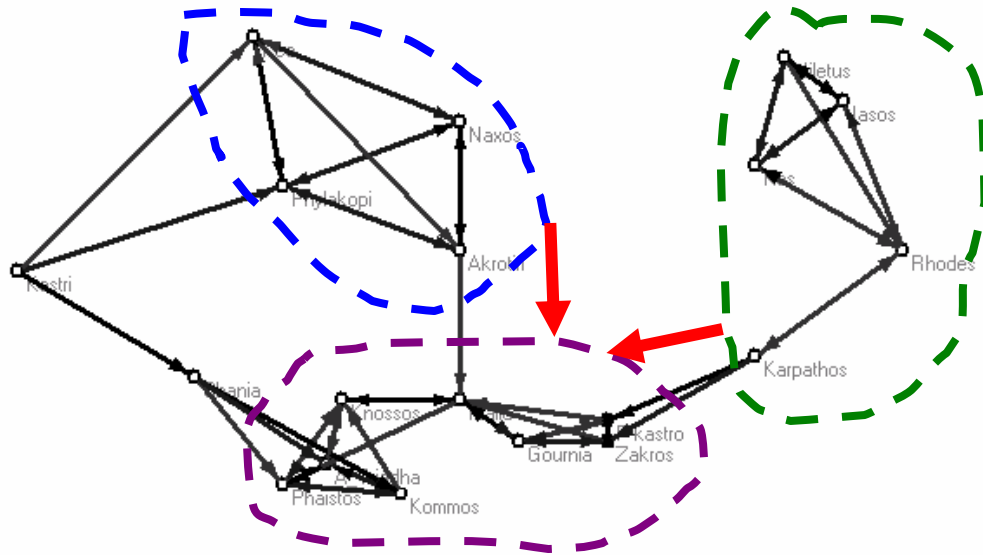


Figure 3: PPA of realistic data shown with approximate geographical locations for sites. The strongly connected cores are shown linked with large red arrows denoting that the Cycladean (top left blue group) and Dodecanese (top right green group) cores are only weakly linked to the Cretan core (bottom purple group).

In keeping the directionality of the edges added in PPA, we have gone beyond the analysis of Broodbank. PPA by its very nature tends to produce *strongly connected* cores, close knit networks where each site can reach every other following the links and respecting the directions on the links. In our figure 3 we see three: the Cycladean core (Naxos, Akrotiri, Phylakopi and Kea), the Dodecanese core (Karpathos, Rhodes, Iasos, Miletus and Kos) and the Cretan core from Phaistos, Knossos to P-Kastro and Zakros via Malia. These strongly connected cores are clustered where there is a high concentration of habitable land in the EBA examples of Broodbank. In our case they are clustered round sites where sites are found at a higher density than neighbouring regions.

Outlying regions are *weakly connected* to such strongly connected core areas. Typically a site in a weakly connected region can reach any of the central points of a neighbouring strongly connected core, but there is no route from the centre to the outliers unless one is allowed to go against the direction of the arrows. Thus Kastri and Chania are only weakly connected to other areas, there is no link from the Cyclades or main Cretan core to these places unless one goes against the arrows. We also note that in this sense there is a natural hierarchy of strong connectedness. The two strongly connected cores of the Cyclades and the Dodecanese are themselves only weakly connected to the Cretan core.

That is there are arrows from the Cyclades and the Dodecanese to Crete but not vice-versa.

A useful way to study these results by eye is to abandon the geographic layout used for the sites. In the next figure we show the same results rearranged on graph theoretic grounds.

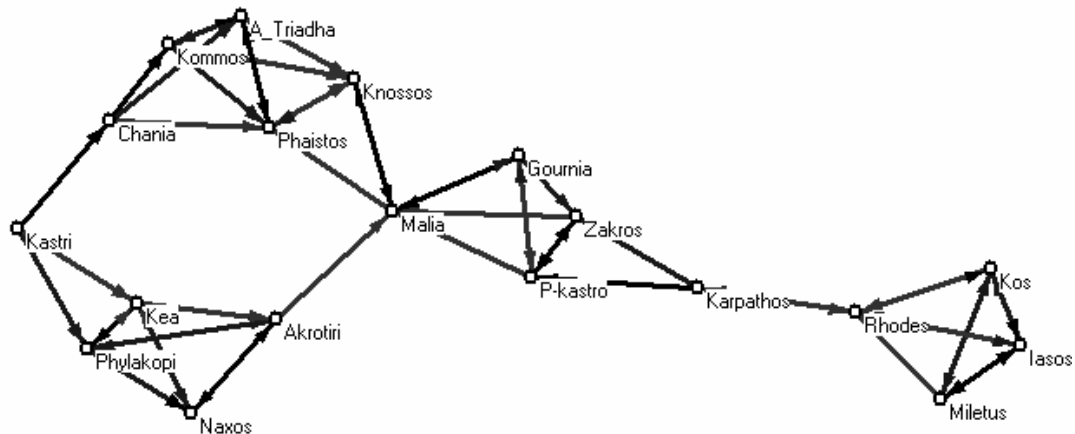


Figure 4: PPA with three directed edges per site, arranged using an energy (Kamada-Kawai free) scheme in the Pajek package (PAJEK, 2005)

That is, it is the relationship of the sites as defined by the edges, and not their physical position, which is used to find a suitable display. It often highlights aspects of the connections that are not so obvious in the geographical layout. There are now four very noticeable groups with Malia being a bridge between two halves of the Cretan strongly connected core noted above. Take Malia away and the Cretan core as well as network as a whole splits into two completely disconnected pieces. In a similar way Akrotiri, Kastri and Karpathos play a key role in keeping the network at least weakly connected. With such a small group, one can detect the relevant features of the network by eye and by hand, but there are specific group theoretic measures, which can confirm these identifications or perform them in more complicated examples.

However we can also make an interesting observation about PPA. That is that the regions where there are sites with high degree, the aspect highlighted in Broodbank, cluster around local maxima in site density. That is if we count the number of sites within a radius of say 100km as shown in figure 5, we find that the largest density is 8, centred on Malia, and this is the centre on a large global peak in the density extending across most of the Cretan sites. However there are two smaller peaks, one in the centre of the Cyclades with a peak density of just over 3, and another in the centre of the Dodecanese with peak value just over 3.5. When PPA assigns edges to site to its three nearest neighbours, it is most likely to find these in the nearest high density site region – the local maxima. Thus while it can be a useful tool, PPA is really emphasising zeros in the derivatives (slope) of local site density.

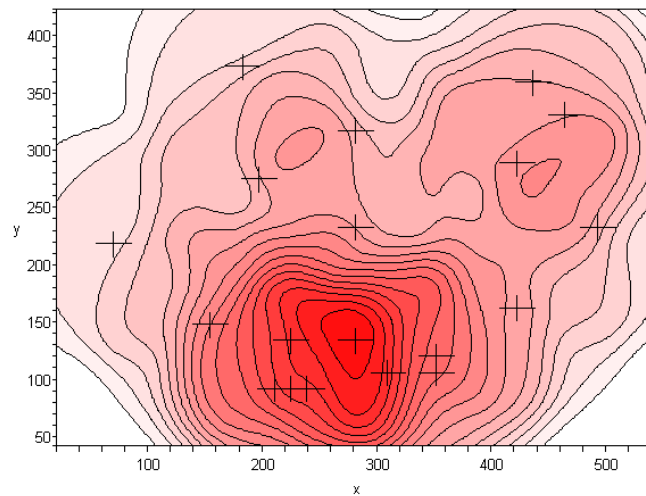


Figure 5: The density of sites within a radius of 100km, coordinates in km. The crosses mark the positions of sites. White indicates zero density, the darkest red is a density of just over 8 centred on Malia.

Market Led Model

Even for our Market Led Model, the simplest algebraically, there are a lot of parameters to study. It is quite difficult to find an appropriate range for our purposes. For instance, in this model using network distances we have for a certain range of values the result shown in the next figure.

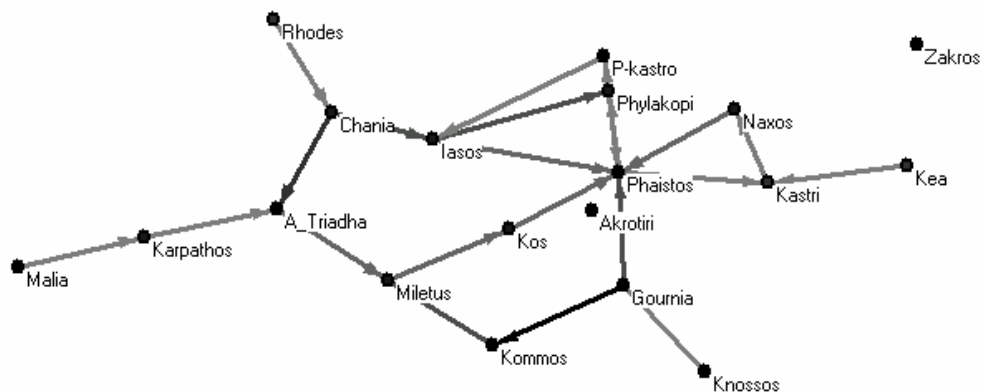


Figure 6: Non-geographical layout of sites in our Market Led Model with network distances used and for other exemplary parameters.

The model fails to always make connections to the most close neighbours for these values, and in any case most edge values are small (0.1 or below when 1 would be the norm). This suggests we have not found the appropriate parameter range. Work is ongoing.

Discussion

The results above are preliminary and are on a small number of sites. Increasing the number of sites used in our models is easy and will be done in the immediate future. The models discussed here are just being tested but the elements discussed here will be used in later refinements. Further or alternative sophistication appears unnecessary to date. Nevertheless they do illustrate our general approach and we feel the Hamiltonian (cost function) of equation 1 is sufficient for the MBA.

To date we have not investigated any sophisticated distinction between the potential sizes (available resources?) of sites though this is available in our existing numerical models and remains to be studied. This would allow us to include some aspects of more traditional site-centric viewpoints alongside the network and edge orientated features we have emphasised in our discussion. For instance we can distinguish between small rocky outcrops of strategic position but little resources and those sites with vast local potential.

In terms of analysis we have just been scratching the surface thus far. In our PPA example by retaining the directionality of the links, we have noted one simple way we can extend Broodbank's use of PPA, namely keeping the implicit directionality of the links. This enables us to use the network concepts of strongly and weakly connected cores. The non-geographical display of figures 4 and 6 is based implicitly on the *degree* of each vertex (the number of edges connected to a vertex). The same aspect was used by Broodbank but there are a number of other local measures of a vertex's importance that could be used, such as the *cluster coefficient*. A more interesting route would be to use the global network properties to rank the importance of sites, much as internet search engines have to rank different pages of the web (a network made of web links between web pages which form the vertices). We have already used the network connections in the Hamiltonian (energy function) of our Market Led Model and such global aspects for which the connections across the network play a central role remain to be exploited in this field. The ideas of *strongly* and *weakly connected* cores noted in our PPA discussion are one such example and this leads us towards the various ways of finding groups or communities from the connections in a network (e.g. Moody and White 2003). We can further exploit the EBA data and PPA analysis of Broodbank as a test of our work.

An important but time consuming aspect of the project is to develop graphical interactive representations of the data for the analysis, based on whatever network concepts seem appropriate. The aim is to exploit other network features to rearrange the way we display the results to downplay the geographical location of the sites and emphasise other edge or general network features of the problem. Again our non-geographical display of the PPA results in figure 4 is our prototype in contradistinction to the traditional geographical layout of the same data in figure 3. With such an approach, we hope to overcome our natural spatial preconceptions about the inter-relationships of different sites.

Finally, we hope to go beyond our present equilibrium approach to study more dynamical issues. The possible existence of very different low energy networks which have similar energies, representing distinct yet reasonable organisational solutions, can be exploited. Suppose in our models we find that sites on the north coast of Crete are invariably the most highly ranked by a variety of network measures, with Malia usually ranked ahead of Knossos. By 'heating up' our network (increasing the size of 'energy' fluctuations) or introducing temporary disruption in other ways (such as removal of some nodes) we can see if the network slides back into the same solution or new solutions. So while we might not be able to predict the collapse of Minoan civilisation, we might be able to highlight factors that favoured the subsequent rise of mainland influence (Evans et al, in preparation).

Final remarks

At the very least, the approaches we are pursuing encourage us to look much more closely and explicitly at site interactions and connectivity as sources of change. At present we have progressed a little beyond the heuristic, seeking both analytic solutions and numerical simulations. Rather than race into overambitious applications, our aim is to progress step by step; the importance of this has been underlined in other ISCOM interdisciplinary collaborations between scientists and social scientists. By starting with Broodbank's Proximal Point Analysis of the EBA Cyclades, which is both simple and effective, we oblige ourselves to demonstrate the utility of more sophisticated techniques. There is little point in applying sophisticated techniques if they do not provide more analytical power – the risk, if you like, of using a sledgehammer to crack nuts. We believe that more powerful and flexible techniques can be very usefully developed to tackle the more complex interaction dynamics we see in the Middle Bronze Age. And the additional flexibility that comes with greater modelling sophistication should also ensure a much broader applicability – not just to prehistory and history, but to the contemporary world too.

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