Boomerang RG Flows with Intermediate Scaling

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Holography provides powerful tools to study strongly coupled CFTs deformed by an operator that explicitly breaks translations

$$L_{CFT} \to L_{CFT} + \int dx \phi_s(x) \mathcal{O}(x)$$

Why study?

- Allows momentum to dissipate and hence is a framework for having finite DC thermal and electric conductivities

- RG flows lead to novel ground states in IR
  - Metals with Drude peaks [Hartnoll,Hoffman][Horowitz, Santos,Tong][.
  - Novel metals without Drude peaks [Donos,JPG][Gouteraux][…..]
  - Insulators [Donos,Hartnoll][Donos,JPG][……]

- Novel RG flows with intermediate scaling

Q-Lattices [Donos,JPG]

- Exploit a global symmetry in the bulk to break translations leading to a system of ODEs rather than PDEs

- E.g. complex scalar field ϕ

$$\mathcal{L} = \sqrt{-g}(R - 2\Lambda + \partial\phi\partial\phi^* - V(|\phi|) + \dots)$$

Use global symmetry $\phi \to e^{i\theta}\phi$

to construct ansatz for bulk fields

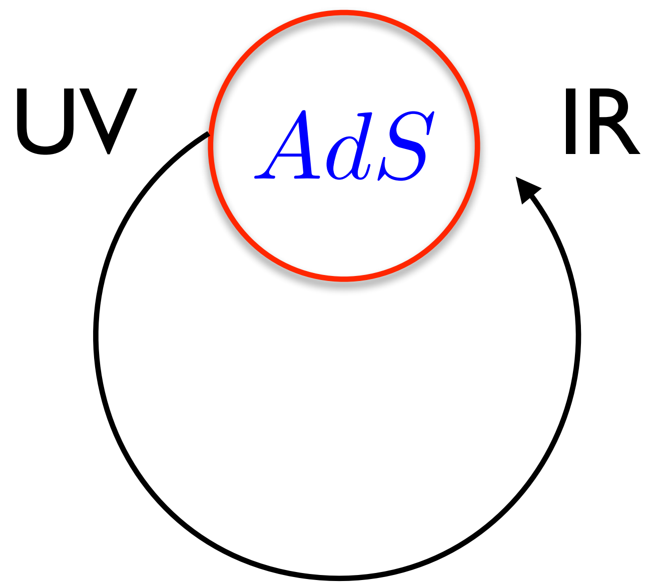
$$\phi = f(r)e^{ikz}$$

$$ds^2 = -U(r)dt^2 + U(r)^{-1}dr^2 + e^{2V_1(r)}(dx^2 + dy^2) + e^{2V_2(r)}dz^2$$

Plan

- Top-down Q-lattices: type IIB and D=11 SUGRA

- "Boomerang" RG flows



- Boomerang RG flows with intermediate scaling behaviour

Type IIB Constructions

- Recall

$$ds^2 = ds^2(AdS_5) + ds^2(X_5)$$

$$F_5 = Vol(AdS_5) + Vol(X_5)$$

with X_5 Einstein space

If $X_5 = S^5$ dual to N=4 SYM

If $X_5 = SE_5$ dual to N=1 quiver theory

- Consistent KK truncation of type IIB on X_5

$$\mathcal{L} = \sqrt{-g}(R + 12 - \frac{1}{2}\partial\phi^2 - \frac{1}{2}e^{2\phi}\partial\chi^2)$$

- The scalars ϕ, χ parametrise $SL(2,R)/U(1)$

- For each of the three conjugacy classes of $SL(2,R)$ there is a distinct Q-lattice construction

- Note $\tau = \chi + ie^{-\phi}$ in AdS vacua is a modulus dual to a marginal operator in the CFT

with $\tau \sim \frac{\theta}{2\pi} + i\frac{4\pi}{g_{YM}^2}$ in dual field theory

a) Linear dilaton

[Jain,Kundu,Sen,Sinha,Trivedi]

Take

$$\chi = 0$$

$$\phi = kz$$

$$ds^2 = -U dt^2 + U^{-1} dr^2 + e^{2V_1}(dx^2 + dy^2) + e^{2V_2} dz^2$$

- RG flow

UV $\qquad\qquad\qquad\qquad$ IR

$$AdS_5 \longrightarrow AdS_4 \times \mathbb{R}$$

- Problem: dilaton gets large for large z

b) Linear axion

[Azeyanagi, Li, Takayanagi]

Take $\chi = az \qquad \phi = \phi(r)$

$$ds^2 = -U dt^2 + U^{-1} dr^2 + e^{2V_1}(dx^2 + dy^2) + e^{2V_2} dz^2$$

- RG flow

UV AdS_5 with $\phi(r) \sim \mathcal{O}(r^{-2})$ as $r \to \infty$

$\downarrow$

IR $LLif_5$

$$ds^2 = \frac{11}{12}\left(\frac{dr^2}{r^2} + r^2(-dt^2 + dx^2 + dy^2) + r^{4/3} dz^2\right)$$

$$e^{\phi} = r^{2/3}$$

- Finite T black holes constructed $s \sim T^{8/3}$

[Mateos, Truncanelli]

c) IIB Boomerang flow [Donos,JPG,Sosa-Rodriguez]

Field redefinition

$$\chi = \frac{2\,\tanh\frac{\varphi}{2}\,\sin\alpha}{1+\tanh^2\frac{\varphi}{2}+2\,\tanh\frac{\varphi}{2}\,\cos\alpha} \qquad e^{-\phi} = \frac{1-\tanh^2\frac{\varphi}{2}}{1+\tanh^2\frac{\varphi}{2}+2\,\tanh\frac{\varphi}{2}\,\cos\alpha}$$

$$\mathcal{L} = \sqrt{-g}\left(R + 12 - \frac{1}{2}\partial\varphi^2 - \frac{1}{2}\sinh\varphi\,\partial\alpha^2\right)$$

Q-lattice ansatz

$$\varphi = \varphi(r) \qquad \alpha = kz$$

$$ds^2 = -U dt^2 + U^{-1}dr^2 + e^{2V_1}(dx^2+dy^2) + e^{2V_2}dz^2$$

Periodic in the z direction

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Periodic in the z direction

- RG flow

UV AdS_5 with $\varphi \to \lambda$ as $r \to \infty$

$$\alpha = kz$$

↓

RG flow specified by λ, k

Note: λ is dimensionless

IR

- RG flow

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RG flow specified by λ, k

Note: λ is dimensionless

IR AdS_5

- Perturbative RG flow $\lambda \ll 1$

Solve exactly linearised scalar equation of motion

as $r \to \infty$
$$\delta\varphi(r) = \lambda - \frac{\lambda\, k^2}{4\, r^2} + \cdots .$$

as $r \to 0$
$$\delta\varphi(r) = \lambda \sqrt{\frac{\pi}{8}} \left(\frac{k}{r}\right)^{3/2} e^{-k/r} + \cdots$$

Back reacts on metric at order λ^2

As $r \to 0$ find

$$ds^2 = r^{-2} dr^2 + r^2(-dt^2 + dx^2 + dy^2 + L_2^2 dz^2)$$

Get AdS_5 vacua in IR with renormalised length scale

- Perturbative RG flow $\lambda \ll 1$

  Solve exactly linearised scalar equation of motion

  as $r \to \infty$ $$\delta\varphi(r) = \lambda - \frac{\lambda k^2}{4 r^2} + \cdots .$$

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- RG flow for large λ

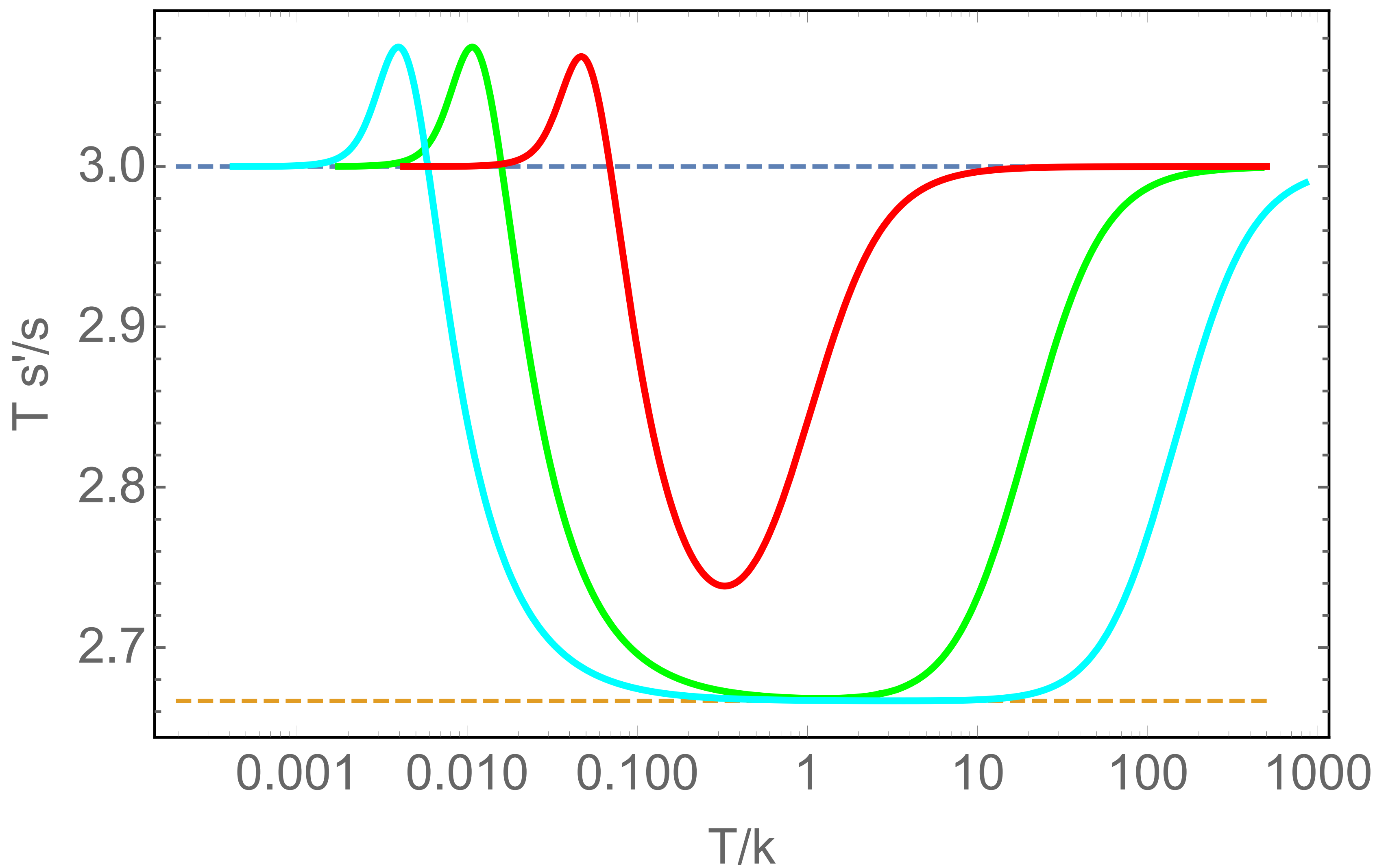
Still have boomerang RG flow? Need to investigate numerically

Construct finite T black hole solutions for various λ

For $T \gg k$ will look like AdS_5 Schwarzschild black hole

e.g. $s \propto T^3$

For $T \ll k_{IR}$??



Red $\lambda = 2$ Green $\lambda = 5$ Cyan $\lambda = 7$

Boomerang flow plus intermediate scaling fixed by

$LLif_5$ fixed point solution of [Azeyanagi,Li,Takayanagi]

D=11 Constructions [Donos,JPG,Rosen,Sosa-Rodriguez]

- Start with $AdS_4 \times S^7$

  Dual to ABJM theory if take a $\mathbb{Z}_k$ quotient

- Consistent KK truncation of D=11 on S^7

  N=8 $SO(8)$ gauged SUGRA in D=4

- Consider further truncation keeping sector invariant under $U(1)^4 \subset SO(8)$

  N=2 gauged SUGRA + 3 vector multiplets ('STU model')

  Includes 3 scalars X_i and 3 pseudoscalars Y_i

Package as: $X_i + iY_i = \lambda_i e^{i\sigma_i}$ and set gauge fields to zero

$$\mathcal{L} = \sqrt{-g}(R - \sum_i [\frac{1}{2}\partial\lambda_i^2 + \frac{1}{2}\sinh^2\lambda_i \partial\sigma_i^2] - V \ldots)$$

$$V = -8(\cosh\lambda_1 + \cosh\lambda_2 + \cosh\lambda_3)$$

X_i dual to $\Delta = 1$ scalar bilinear operators

Y_i dual to $\Delta = 2$ fermion bilinear operators

both are relevant operators

- RG flow ansatz

$$ds^2 = -U dt^2 + U^{-1} dr^2 + e^{2V}(dx^2 + dy^2)$$

$$X_2 = \gamma(r)\cos(kx), \qquad Y_2 = \gamma(r)\sin(kx)\,,$$
$$X_3 = \gamma(r)\cos(ky), \qquad Y_3 = \gamma(r)\sin(ky)$$
$$\Delta = 1 \qquad\qquad \Delta = 2$$

Periodic deformation in x and y directions

UV AdS_4 with $\gamma = \frac{\Gamma}{r} + \frac{\hat{\Gamma}}{r^2} + \ldots$ as $r \to \infty$

RG flow specified by Γ, k

Note: Γ is dimensionful

Perturbative flows: $\Gamma/k \ll 1$

Can show that they are boomerang flows heading back to AdS_4 in IR

For larger values of Γ/k continue to find boomerang flows as well as intermediate scaling

First recall hyperscaling violation (HSV) solutions: which depend on (z,θ)

[Huijse,Sachdev,Swingle]

$$ds^2 = \rho^{-(2-\theta)}\left(-\rho^{-2(z-1)}dt^2 + d\rho^2 + dx^2 + dy^2\right)$$

Scaling behaviour: $t \to \mu^z t, \quad (x,y) \to \mu(x,y), \quad \rho \to \mu\rho$

$$ds^2 \to \mu^\theta ds^2$$

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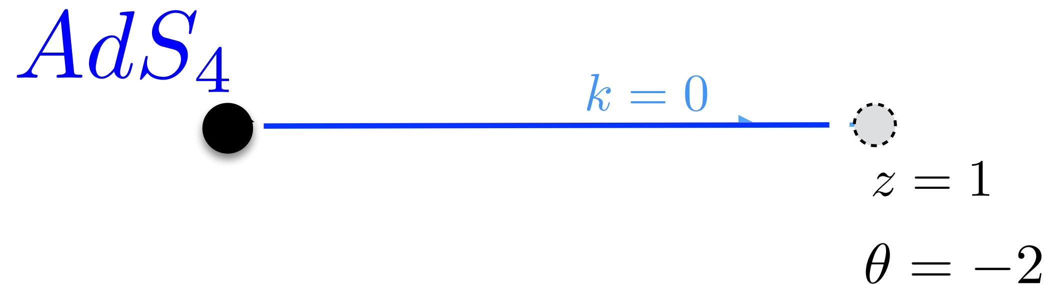
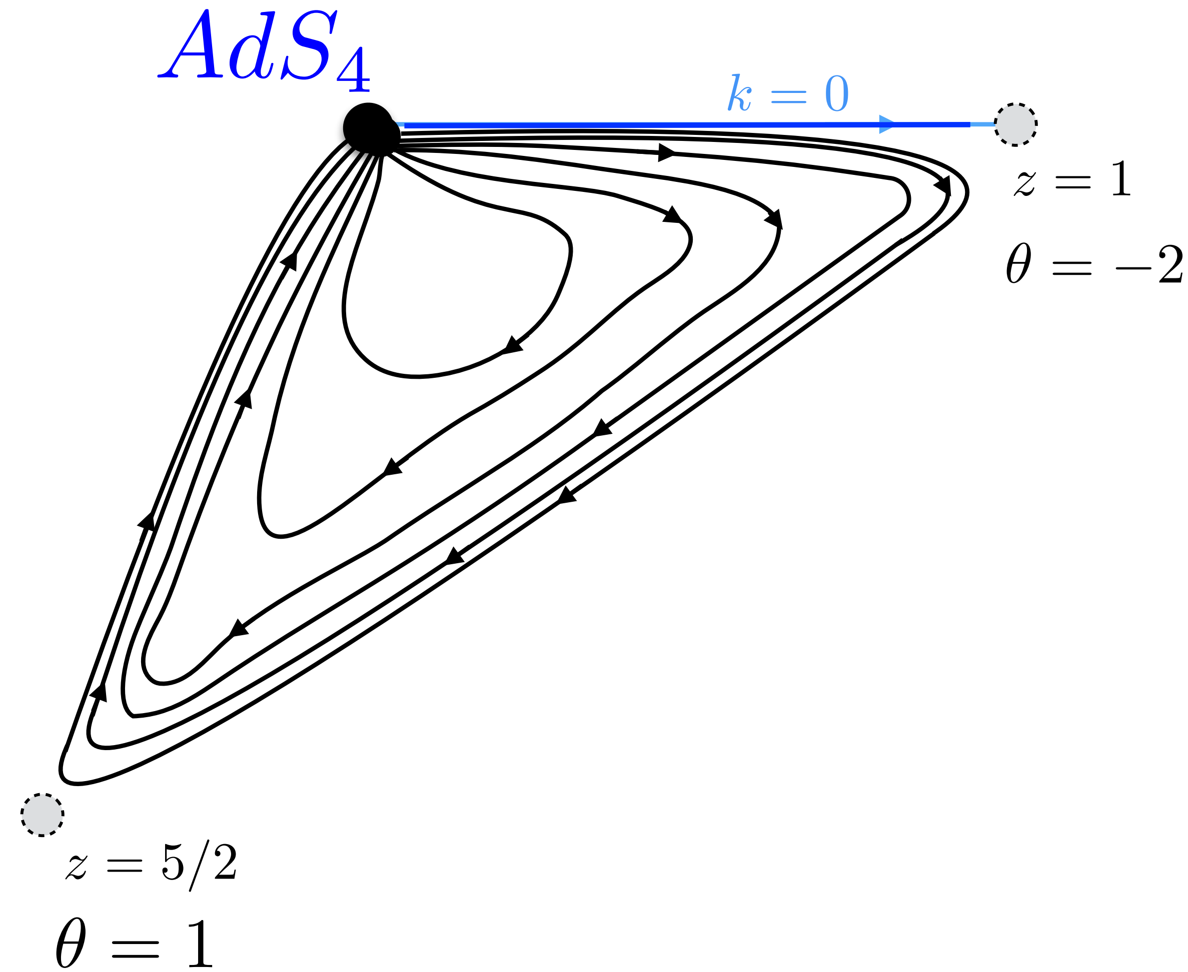


- $k = 0$ $\Gamma \neq 0$ flow is Poincare invariant, approaching HSV in IR

RG flows

labelled by Γ/k

are all boomerang flows



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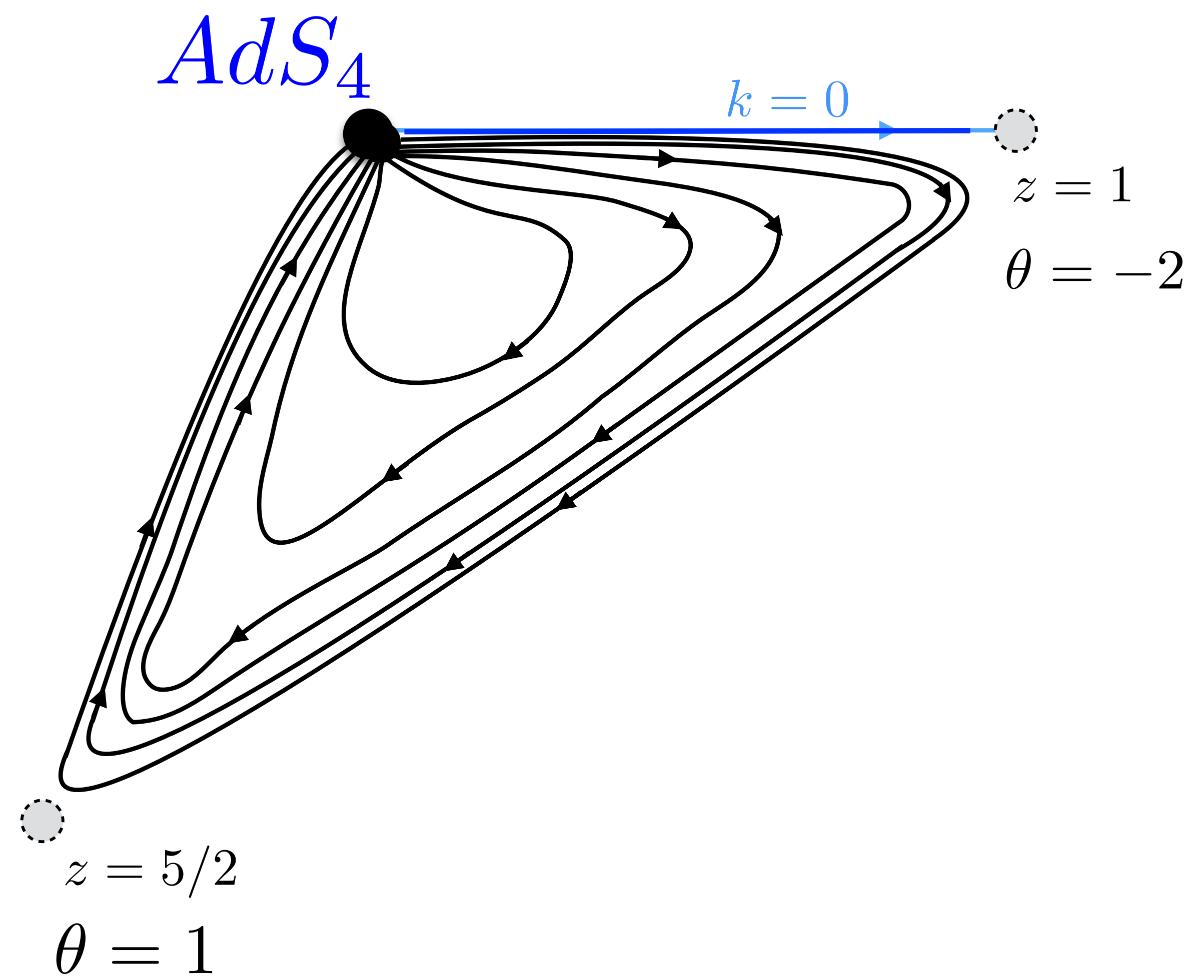

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Comments

- First intermediate scaling due to deformations by relevant op's

- Second intermediate scaling region appears: this HSV behaviour is not a solution to equations of motion and there are no flows to it: it is an exact solution of an auxiliary theory!

More Comments

- Have also constructed finite T black hole solutions

  Can clearly see intermediate scaling behaviour in $s(T)$

- Intermediate scaling manifest in some other observables

- DC conductivity

- Finite charge density

- Study in ABJM

Happy Birthday Chris!