Constraints on Gauss-Bonnet Cosmologies

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Outstanding Questions for the Standard Cosmological Model, March 26-29 2007

…cordance Cosmology

- Early Inflation
- de Sitter space
- During 10^{-30} sec

$$H_{infl} \leq 10^{-5} M_p$$

A rapid acceleration of the new-born Universe

- Current Acceleration
- Almost de Sitter space
- During a few billion years

$$H_{accel} \sim 10^{-60} M_P$$

A new and slow stage of inflation

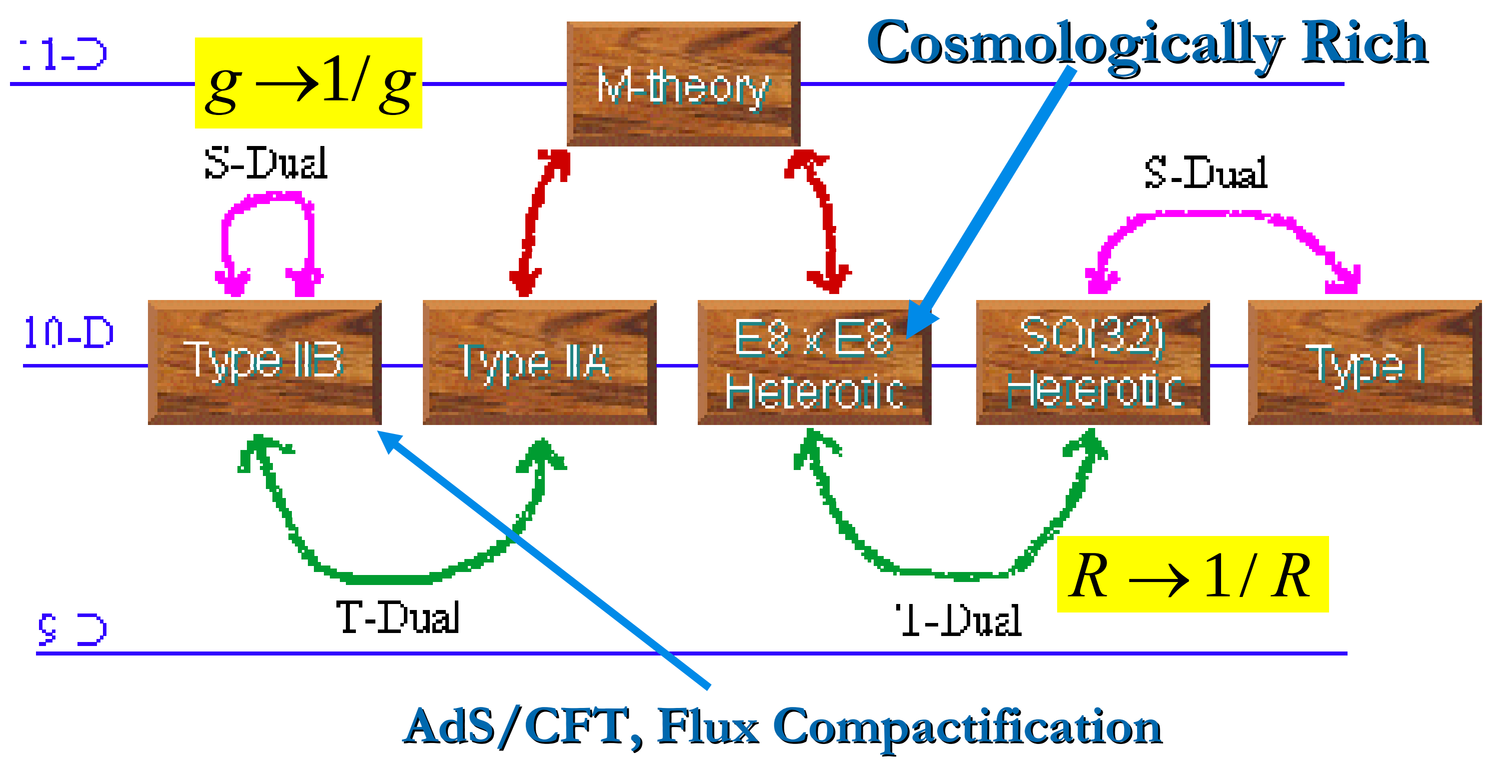
How did the Universe start, and how it is going to end?

- **There may exist a fundamental and simple, concise relation**

*** between past and future (inflation/dark energy)**

*** between close-by and far-away (QM/GR)**

…er Universe from string theory?



A Remarkable Progress in Physics (1995/1996)

How to get an almost de Sitter Universe?

A natural link between "cosmic acceleration" and "string theory" is still missing

Motivations

Gauss-Bonnet gravity is motivated by

- ✓ the form of a most general scalar-tensor theory,
- ✓ uniqueness of a gravitational Lagrangian in higher dimensions,
- ✓ the leading order curvature corrections in (heterotic) string theory,

$$L_{eff} = \frac{1}{2\kappa^2} R + \frac{\nabla S \nabla \bar{S}}{(S+\bar{S})^2} + 3\frac{\nabla T \nabla \bar{T}}{(T+\bar{T})^2} + \frac{1}{8}(\mathrm{Re}S)^2 R^2_{GB} + \frac{1}{8}(\mathrm{Im}S)^2 \varepsilon^{\mu\nu\rho\lambda} R_{\mu\nu}{}^{\sigma\tau} R_{\rho\lambda\sigma\tau}$$

$$\mathrm{Re}\, S \equiv \frac{2}{g_s^2} e^{\phi}, \ \phi \equiv \textit{string dilaton}$$

$$\mathrm{Im}\, S \equiv \tau \equiv \textit{pseudoscalar axion}$$

$$\mathrm{Re}\, T \equiv e^{2\sigma} \equiv \frac{1}{(\textit{compactification radius})^2}$$

Compactification

General 4+n dimensional Lagrangian of pure gravity

$$L_{4+n} \propto R - 2\Lambda + \alpha \Re^2$$

which is of second order in the curvature operator

$$\Re^2 \equiv R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD}$$

which are divergence free and have well defined and stable perturbations around the Minkowski vacuum. With the ansatz

$$ds^2_{4+n} = e^{-n\varphi(x)} g_{\mu\nu}(x) dx^\mu dx^\nu + dY_a dY^a e^{2\varphi(x)}$$

upon dimensional reduction, we get

$$L = \frac{1}{16\pi G}\left(R - (\nabla\varphi)^2 - 2V(\varphi) + f_1(\varphi)\begin{bmatrix} \alpha\Re^2 \\ + \beta g_{\mu\nu}\nabla^\mu\varphi\nabla^\nu\varphi \\ + \chi(\nabla\varphi)^2\nabla^2\varphi \\ + \delta(\nabla\varphi)^4 \end{bmatrix}\right.$$

celeration and string theory

Consider the one-loop corrected superstring action

$$S_g = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - V(\sigma,\phi) - \frac{\gamma}{2}(\nabla\sigma)^2 - \frac{\varsigma}{2}(\nabla\phi)^2 + [\lambda(\phi) - \delta\xi(\sigma)]R_{GB}^2\right]$$

I.P. Neupane
hep-th/0602097

modulus

a Brans-Dicke-like runway dilaton

present at string tree level

$$\frac{(\dot{a})^2\ddot{a}}{a^3} = 24H^2(\dot{H}+H^2) = R^2 - 4R^{ab}R_{ab} + R^{abcd}R_{abcd} = R_{GB}^2$$

Gauss-Bonnet curvature invariant

In a known example of string compactification

$$\lambda(\phi) = \lambda_0 e^{\phi/\phi_0} + \ldots..$$

$$\xi(\sigma) \approx \ln(2) - \frac{2\pi}{3}\cosh(\sigma/\sigma_0) + \ldots...$$

We do not have a precise knowledge about the potential; it may take into account non-perturbative effects: branes, fluxes or singularities in the internal spaces.

us, Absence of scalar-GB coupling

- This simplifies the model a lot

One defines $$S_\phi = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - V(\phi) - \frac{\gamma}{2}(\nabla\phi)^2\right]$$

$$X \equiv \kappa^2 \frac{\gamma}{2}(\dot{\phi}/H)^2,\ Y \equiv \kappa^2(V/H^2),\ \varepsilon \equiv \dot{H}/H^2$$

EOMs $$Y = 3 + \varepsilon,\ X = -\varepsilon$$

$$w \equiv -\frac{2\varepsilon}{3} - 1$$

Sufficiently Simple!

Equation of state

Different choice of ε implies different Y and hence different potentials!

X=0 and Y=3 is a de Sitter fixed point with w = - 1

so many possibilities?

Quadratic/Chaotic

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 + \ldots$$

Exponential potential

$$V(\phi) = V_0\, e^{-\lambda(\phi/\phi_0)}$$

Axion potential

$$V(\phi) = \Lambda^4\left(C \pm \cos\left(\frac{\phi}{\phi_0}\right)\right)$$

Inverse power-law

$$V(\phi) = \Lambda^4\left(\frac{\phi_0}{\phi}\right)^n$$

The issue cannot be merely to achieve a dark energy EOS

$$w_{DE} \approx -1$$

For the model to work a scalar field must relax its potential energy after inflation down to a sufficiently low value: very close to the observed value of CC

...g the common modulus field T

- ...mally coupled with a Gauss-Bonnet term

$$S_g = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - V(\phi) - \frac{\gamma}{2}(\nabla\phi)^2 - \frac{1}{8}f(\phi)\,\mathrm{R}^2_{\mathrm{GB}}\right]$$

$$\frac{(\dot{a})^2\ddot{a}}{a^3} = 24H^2(\dot{H}+H^2) = R^2_{GB} = R^2 - 4R^{ab}R_{ab} + R^{abcd}R_{abcd}$$

GB term is topological in 4-D, and, if coupled, no Ghost for Minkowski background. But cosmology requires FRW, inflation → Non-constant scalar-GB coupling!

$$N \equiv \ln[a(t)] \equiv \phi/\phi_0 + \mathrm{const}$$

$$f(\phi) = f_0 + f_1\, \mathrm{e}^{\beta(\phi/\phi_0)},$$

$$\mathrm{V}(\phi) = \frac{2(1-\delta)}{3\kappa^4 f'} = V_0\, e^{-\beta(\phi/\phi_0)}$$

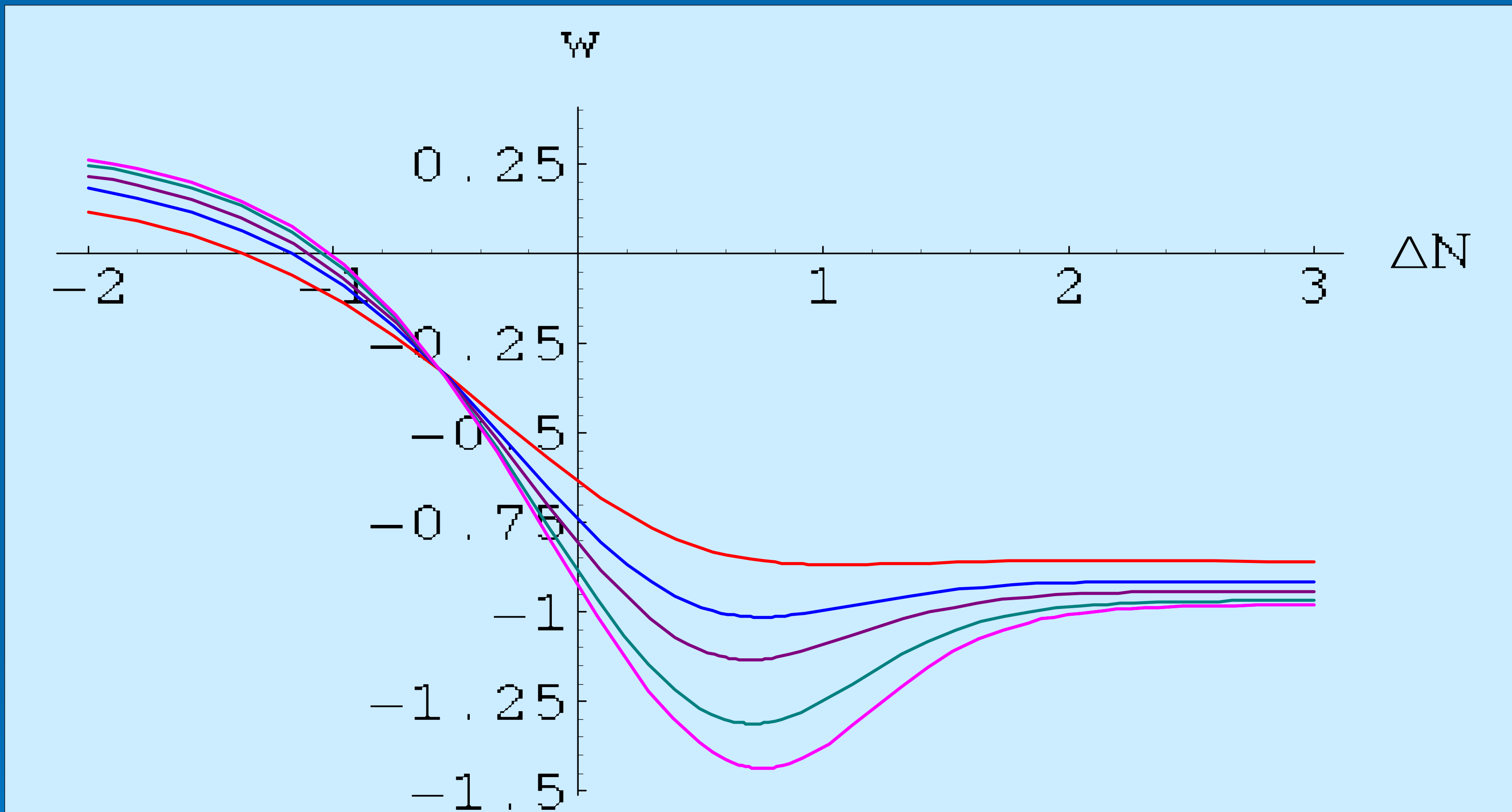
$$\beta \equiv 1 + 3\delta\,,\quad \delta \equiv \gamma\kappa^2\phi_0^2/2$$

- number of e-folds primarily depends on the field value

IPN hep-th/0602097 (CQG)
B.Leith and IPN hep-th/0702002

Consistent with string theory prediction, to the leading order

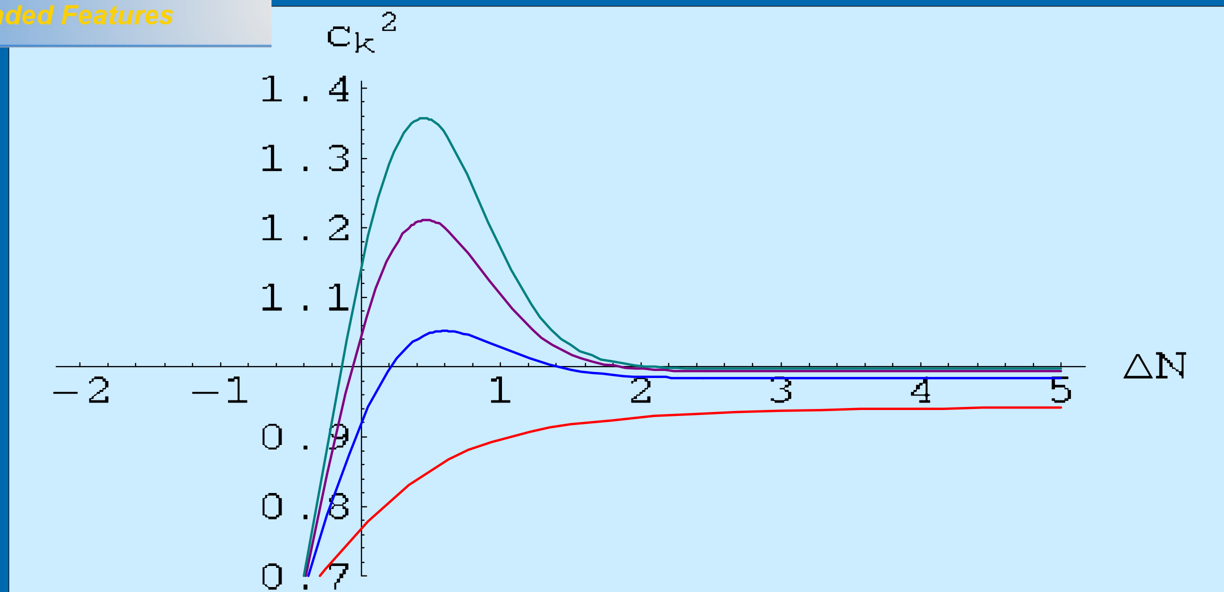
e barrier of cosmological constant

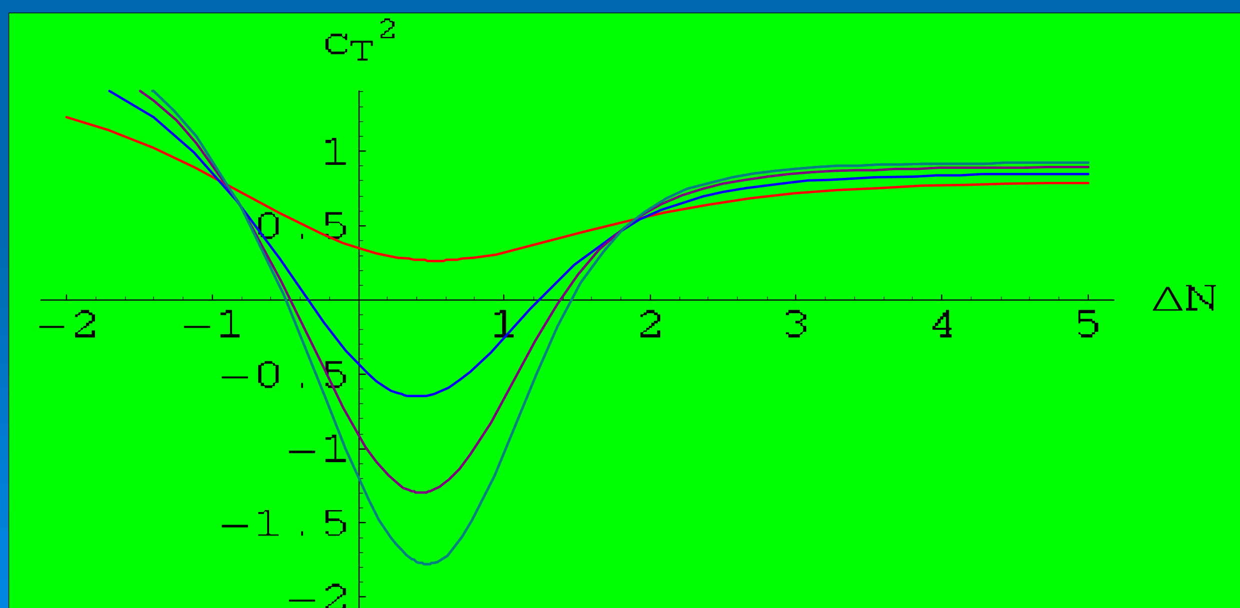


Equation of state parameter for the potential $V(\phi) = V_0\, e^{-2\phi/\phi_0}$

From top to bottom $\phi_0 = 4, 5, 6, 8, 10$

ics may be well behaved, but





ary solution: hep-th/0512262

Let $\Lambda(\phi) \equiv V(\phi) + \frac{1}{8} f(\phi) R_{GB}^2 + .. \equiv (3+\varepsilon) H^2(\phi)$

The Universe starts with $\varepsilon \geq -3$ and hence $w \leq 1$

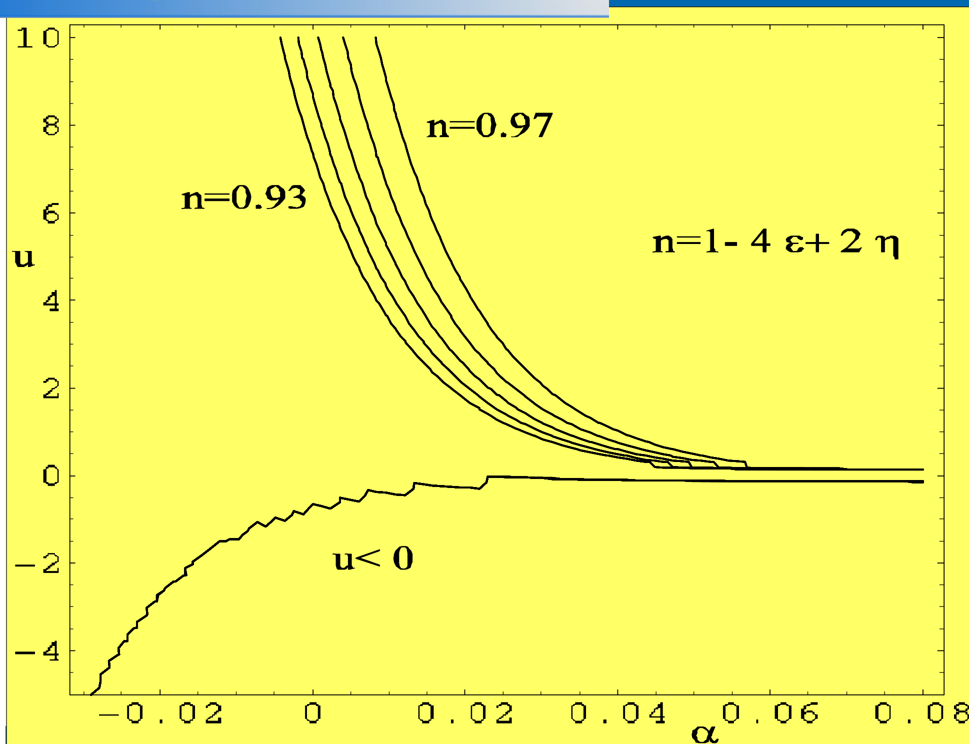
$$f(\phi) H^2 \equiv u(\phi) \equiv u_0 e^{\alpha N}$$

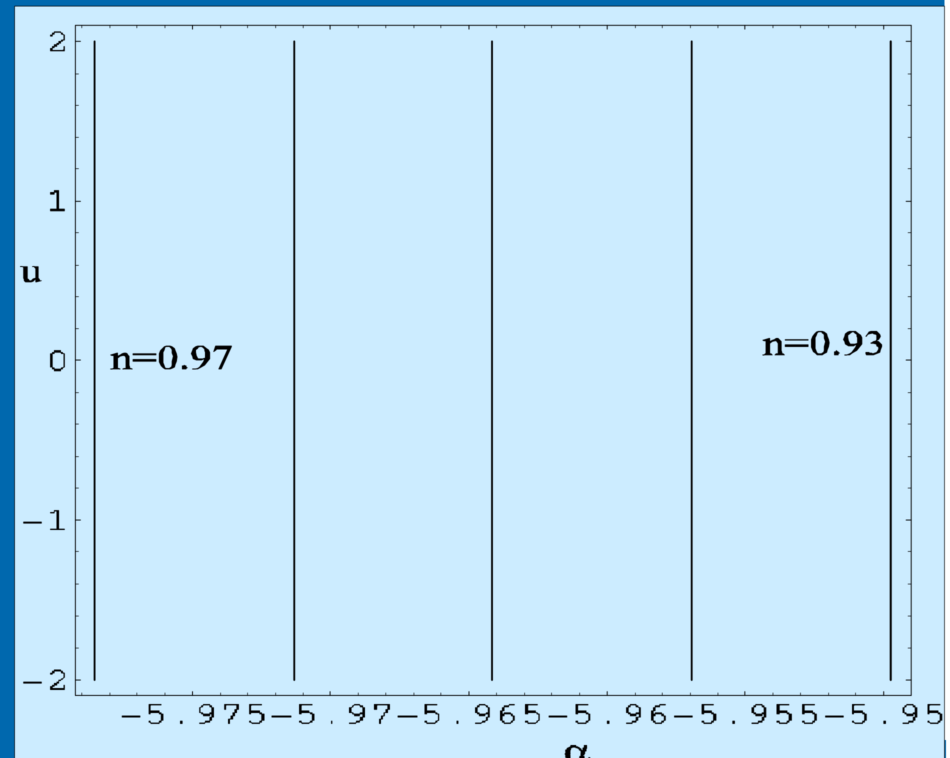
gives an explicit solution

$$H = e^{\int \varepsilon\, dN} = H_0 e^{-\beta_0 N} \cosh\beta(N + N_0)$$

$$\beta_0 \equiv 4 + \frac{\alpha}{4}, \ \beta \equiv \frac{1}{4}\sqrt{9\alpha^2 + 72\alpha + 208}$$

…tionary Observables



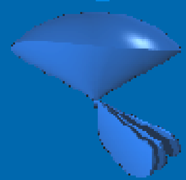


The spectral index $n_k - 1 \approx -4\varepsilon_H + 2\eta_H$ in the range [-0.07,-0.03]

Nature of the dark energy

CMB

+

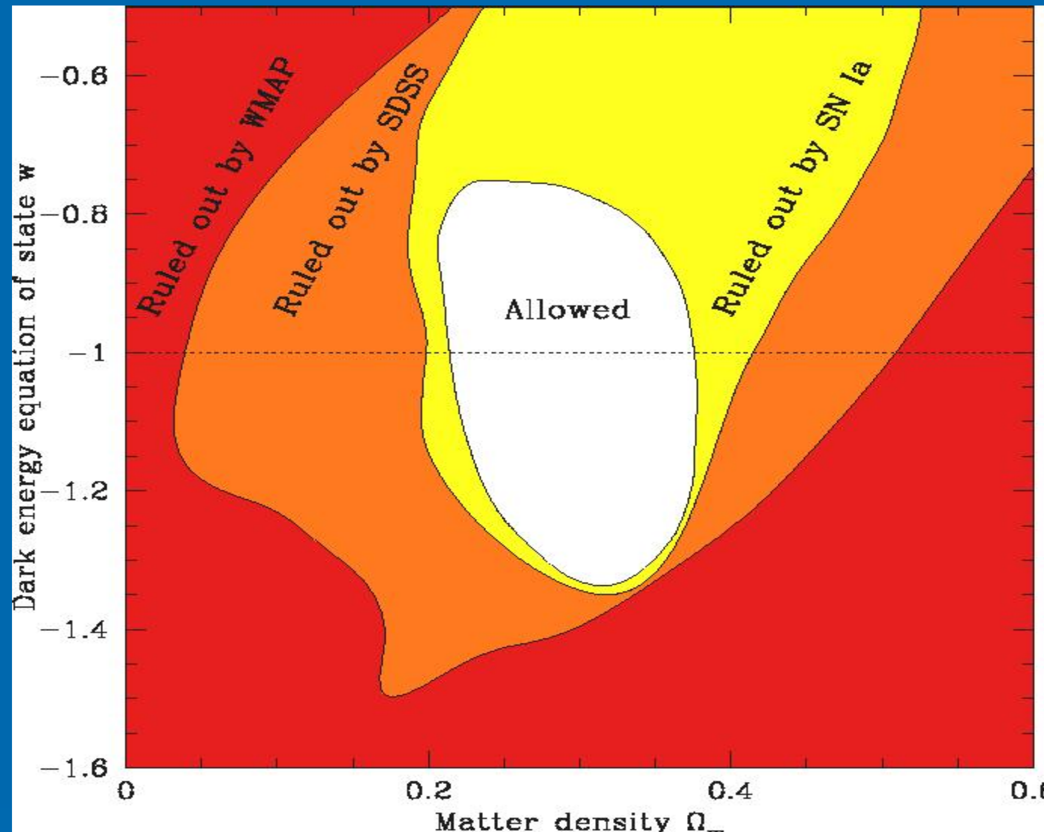


LSS

Recent claim that w < -1 preferred with evolution from w=0.

$$w_{eff} < -1?$$

Null dominant energy condition : energy doesn't propagate outside the light cone



A model with $w < -1$ negative kinetic energy

Tegmark et al. 2004

$$L = -\frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$|p| \leq |\rho| \Rightarrow -\rho \leq p \leq \rho$$

Instabilities cured by higher curvature terms? With stringy corrections, there is no need to introduce a wrong sign to the kinetic term to get $w < -1$!

…nical Quintessence or non-minimally coupled scalar field?

$$S = S_{grav} + S_{matter}$$

$$S_m = S\left(\sigma, A^2(\phi), \psi_m\right) = \int d^4x\sqrt{-g}\left(A^4(\phi)\right)(\rho_m + \rho_{rad} + \rho_s)$$

$$Q \equiv \frac{d \ln A(\phi)}{d(\kappa\phi)}$$

Local GR constraints on Q and its derivative loosely require imply that

$$Q^2 \leq 4 \cdot 10^{-5}, \quad \beta \equiv \frac{dQ}{d\phi} > -4.5$$

The second constraint can arise from various tests of the force of gravity within solar system and laboratories distances: $(dG/dt)/G$ **is less than** 10^{-12} **years**

B term help to lower w_{DE} close to -1 ?

In the absence of the GB term, i.e. with a canonical scalar field:

$$w_{eff} = w_m \Omega_m + w_\phi \Omega_\phi$$

$$\rho_{DE} = \tfrac{1}{2}\dot\phi^2 + V(\phi) + 3H^2 \dot f$$

$$p_{DE} = \tfrac{1}{2}\dot\phi^2 - V(\phi) - \frac{d}{dt}(H^2 \dot f) - 2H^2 \dot f$$

The energy condition

$$\rho_{DE} + p_{DE} = \dot\phi^2 - \frac{d}{dt}(H^2 \dot f) + H^2 \dot f \geq 0$$

may be violated when there is an appreciable change in Gauss-Bonnet energy density

And we don't need $\dot H > 0$ as for a phantom field

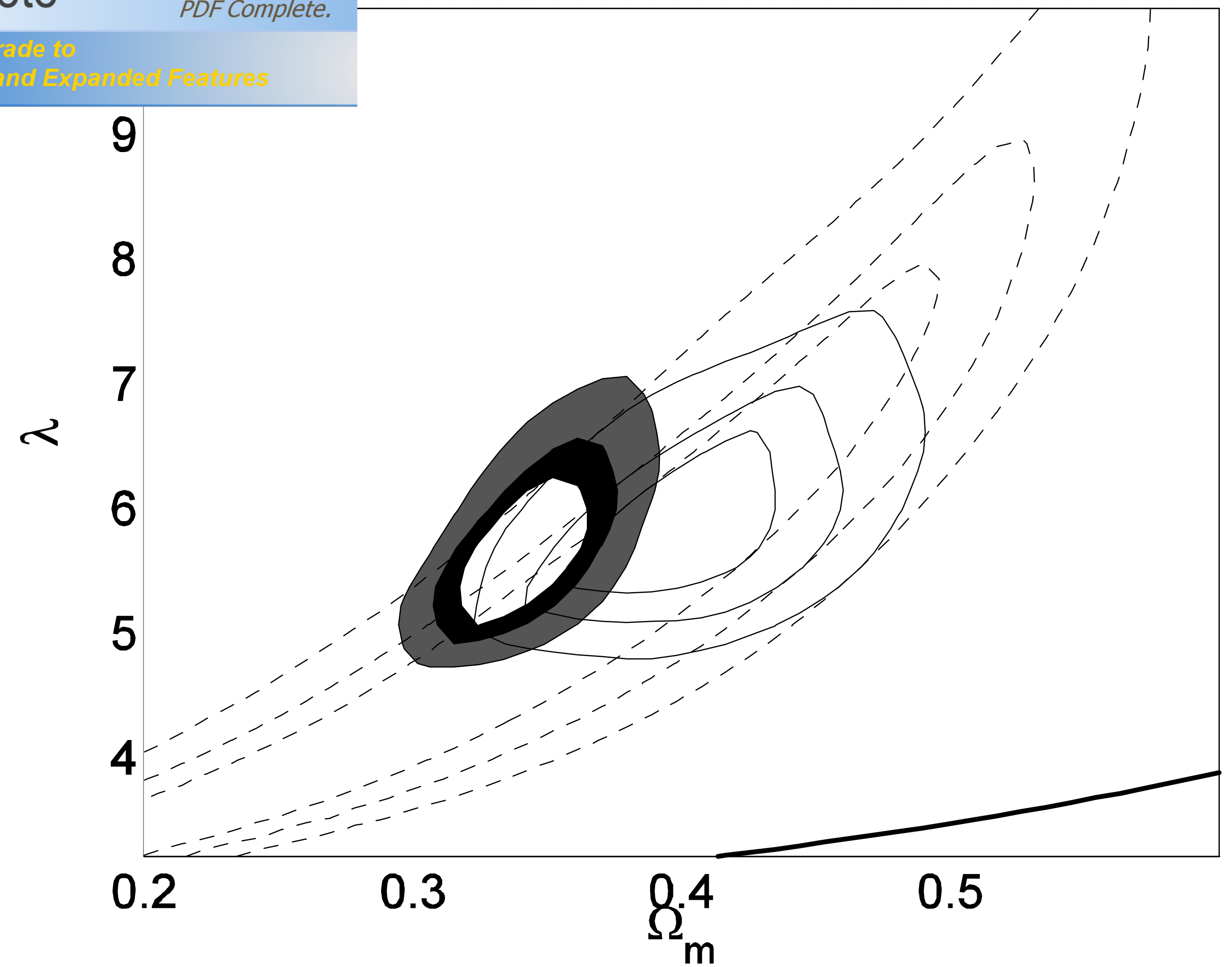
llustrative Simple Example

Simple exponential terms for both the scalar potential and the scalar-Gauss-Bonnet coupling:

$$V(\phi) = V_0\, e^{-\lambda\phi/m_P}, \qquad f_{,\phi} = f_0\, e^{\alpha\phi/m_P}$$

Perhaps too naïve choices (as the slopes of the potentials considered in a post inflation scenario are too large to allow the required number of e-folds of expansion in the Universe.

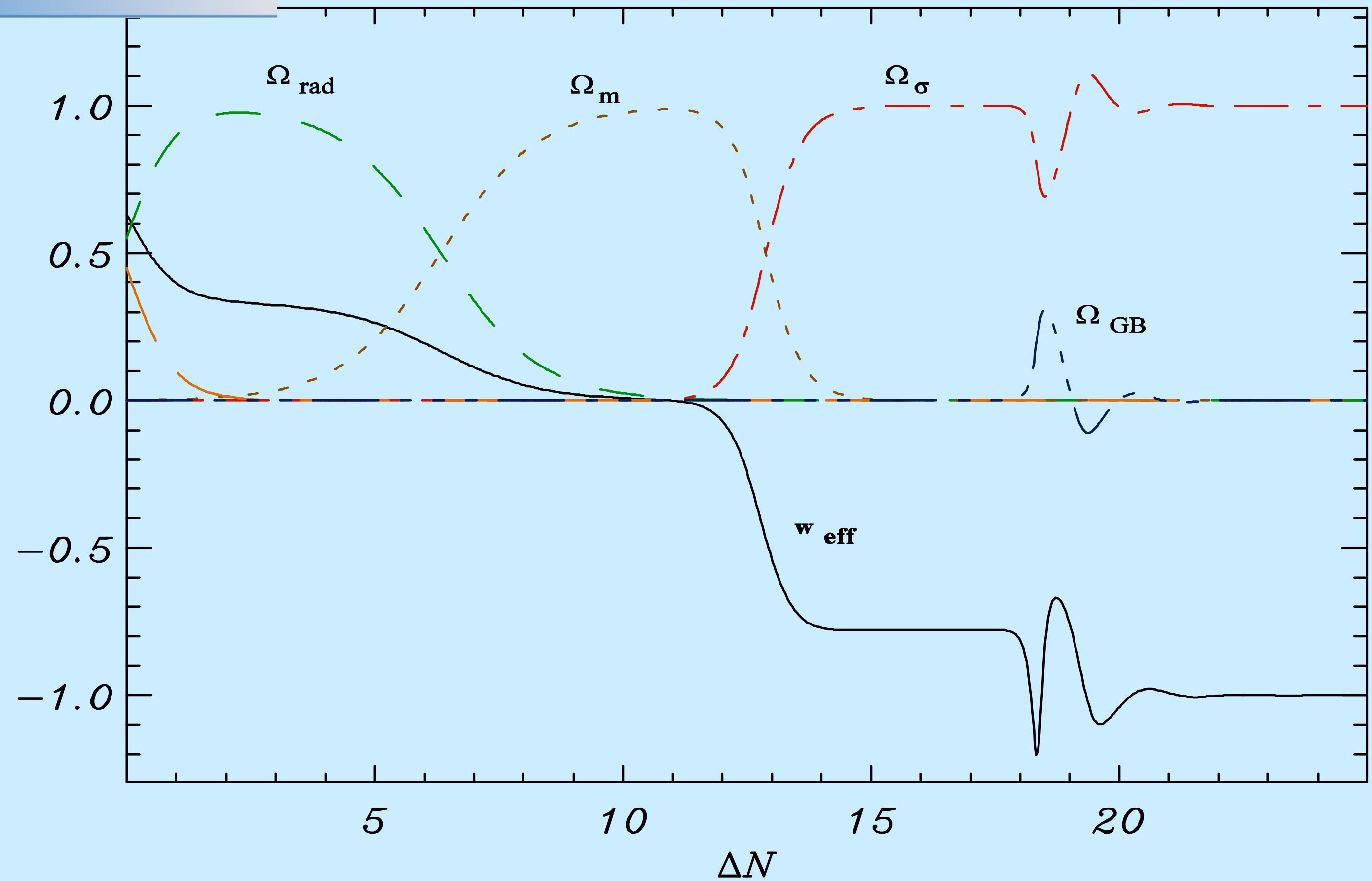
hold some validity as a post-inflation approximation



Dashed lines (SNe IA plus CMBR shift parameter)
Shaded regions (including baryon oscillation scale)

Koivisto & Mota hep-th/0609155

ing w = -1 due to stringy effects?



In the absence of (or trivial) GB-scalar coupling, a crossing between non-phantom $w \geq -1$ and phantom cosmology $w < -1$ is unlikely. But this is possible with scalar-GB coupling.

lowed a non-minimal coupling between
matter fields

$$Q \equiv \frac{d \ln A(\phi)}{d\phi}$$

We find

$$Q\phi' \equiv \frac{\Omega_r}{\Omega_m}$$

prime denotes a derivative with respect to e-folding time

For the present values of density fractions

$$p \equiv \ln[a] + const$$

$$\Omega_m \approx 0.27, \ \Omega_r \approx 10^{-4}$$

the effect of any non-minimal coupling is negligibly small unless that the (dark energy) field is rolling fast. For the validity of weak equivalence principle,

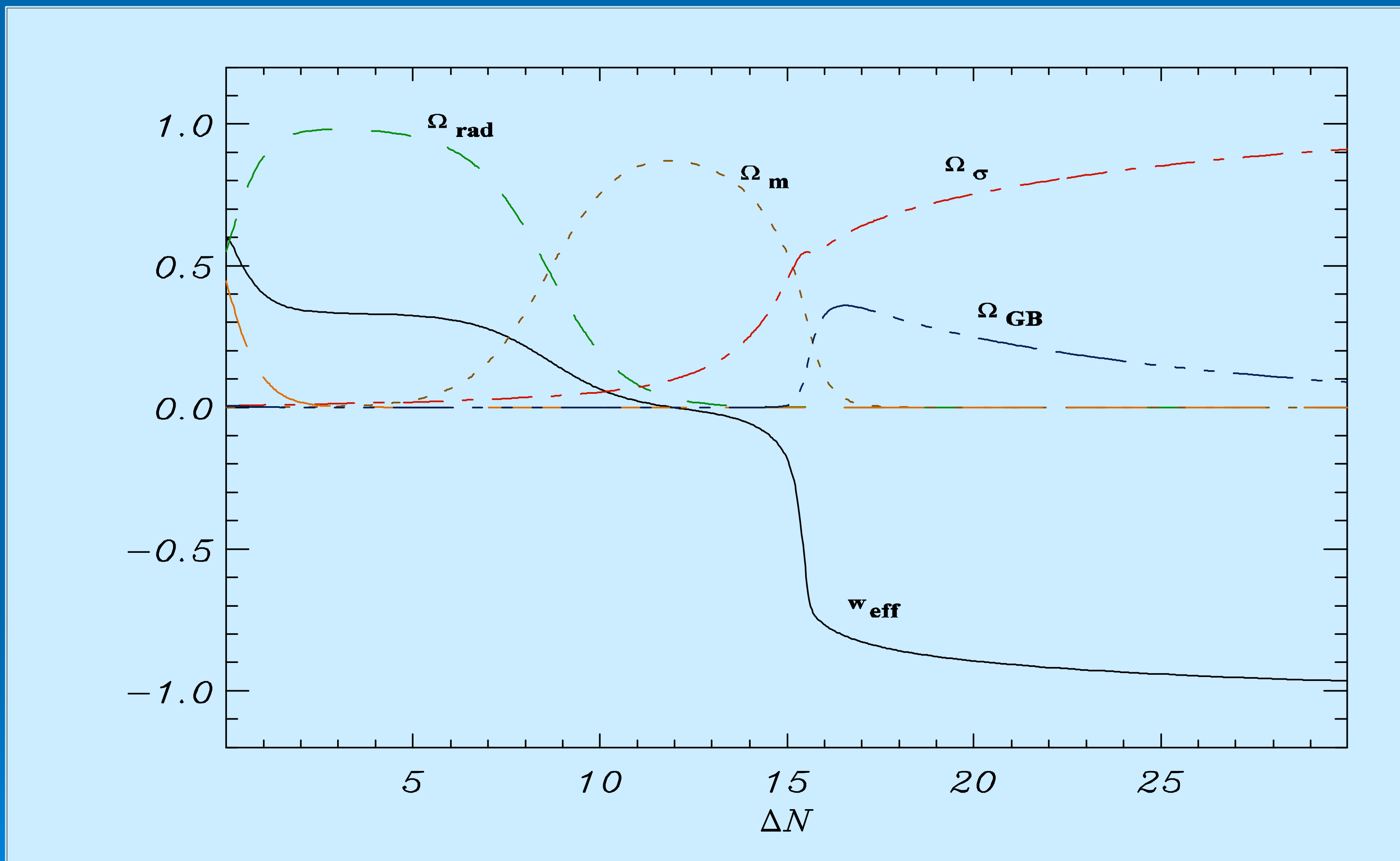
$$\phi' \leq 0.84, \quad Q \sim \frac{\delta\rho}{\rho} \leq 5.10^{-5}$$

Damour et al gr-qc/0204094 (PRL)

Especially on large cosmological scales

…rting a smooth progression to $w_{eff} = -1$

$$V(\psi) = H \quad (\psi)\left(\Lambda_0 + \Lambda_1 e^{\beta\phi}\right), \quad f(\sigma) \propto e^{\alpha\phi}$$



Evolution of the fractional densities and effective equation of state with

$$\alpha = 9, \ \beta = \sqrt{2/3}, \ \Lambda_0 = 10^{-8}$$

Issues of Ghost and Superluminal modes

- **A metric spacetime under quantum effect: perturbed metric about a FRW background**

$$ds^2 = -(1+2A)\,dt^2 + 2a\partial_i B\,dx^i dt + a^2\left[(1+2\psi)\delta_{ij} + 2\partial_{ij}E + 2h_{ij}\right]dx^i dx^j$$

One then defines a gauge invariant quantity, so-called a comoving perturbation

$$\Psi \equiv \psi - \frac{H}{\dot{\phi}}\delta\phi$$

$$S_{linear} \propto \int dt\; a^3 \left[-C(t)\,\Psi\ddot{\Psi} + \frac{D(t)}{a^2}\Psi\nabla^2\Psi \right]$$

Speed of propagation

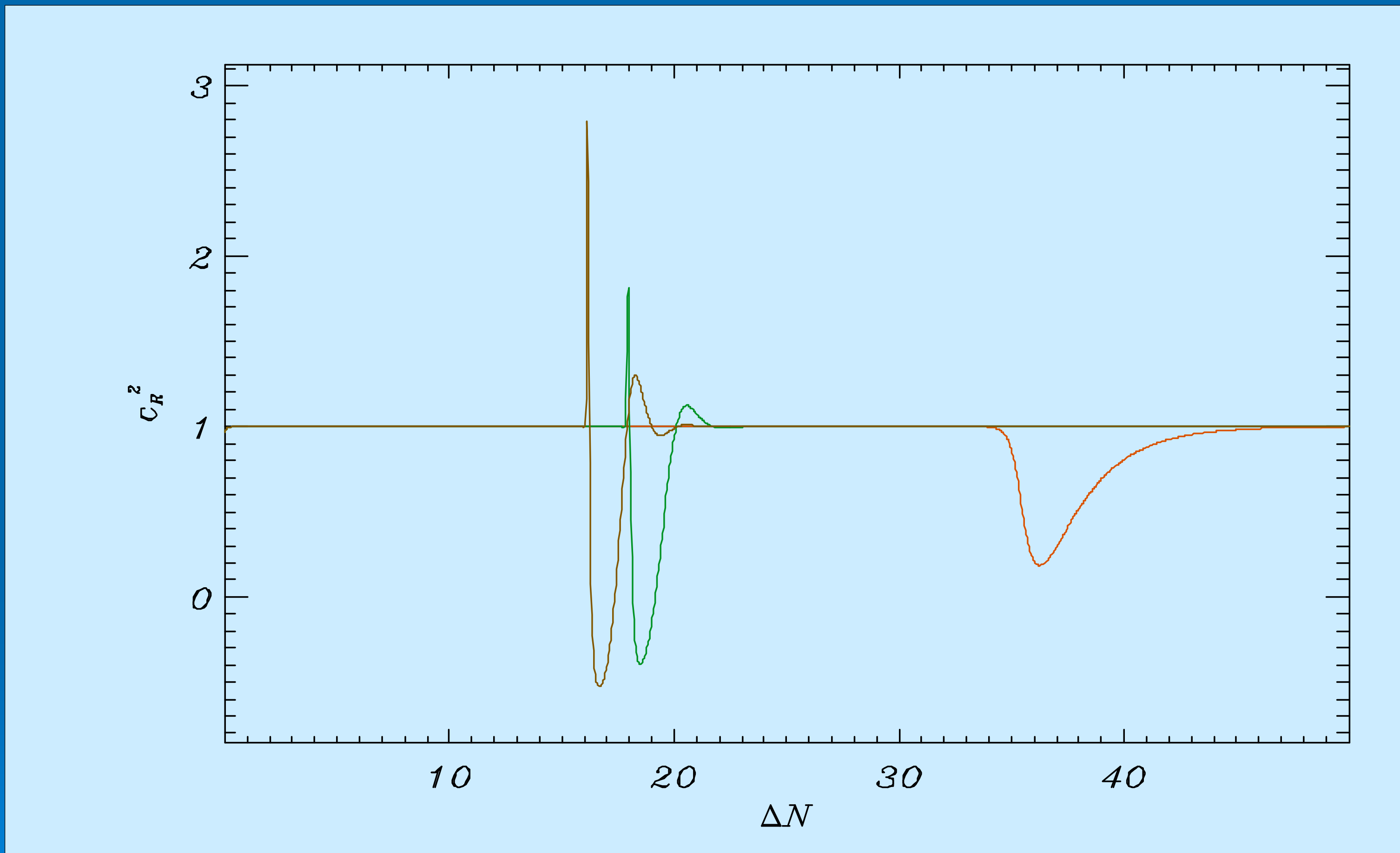
$$C_k^2 \equiv \frac{D(t)}{C(t)}$$

No-ghost and stability conditions: $C(t), D(t) > 0$ **and that** $0 < C_k^2 \le 1$

These conditions apply to scalar and tensor modes, while vector modes do not propagate

...on speed of a scalar mode

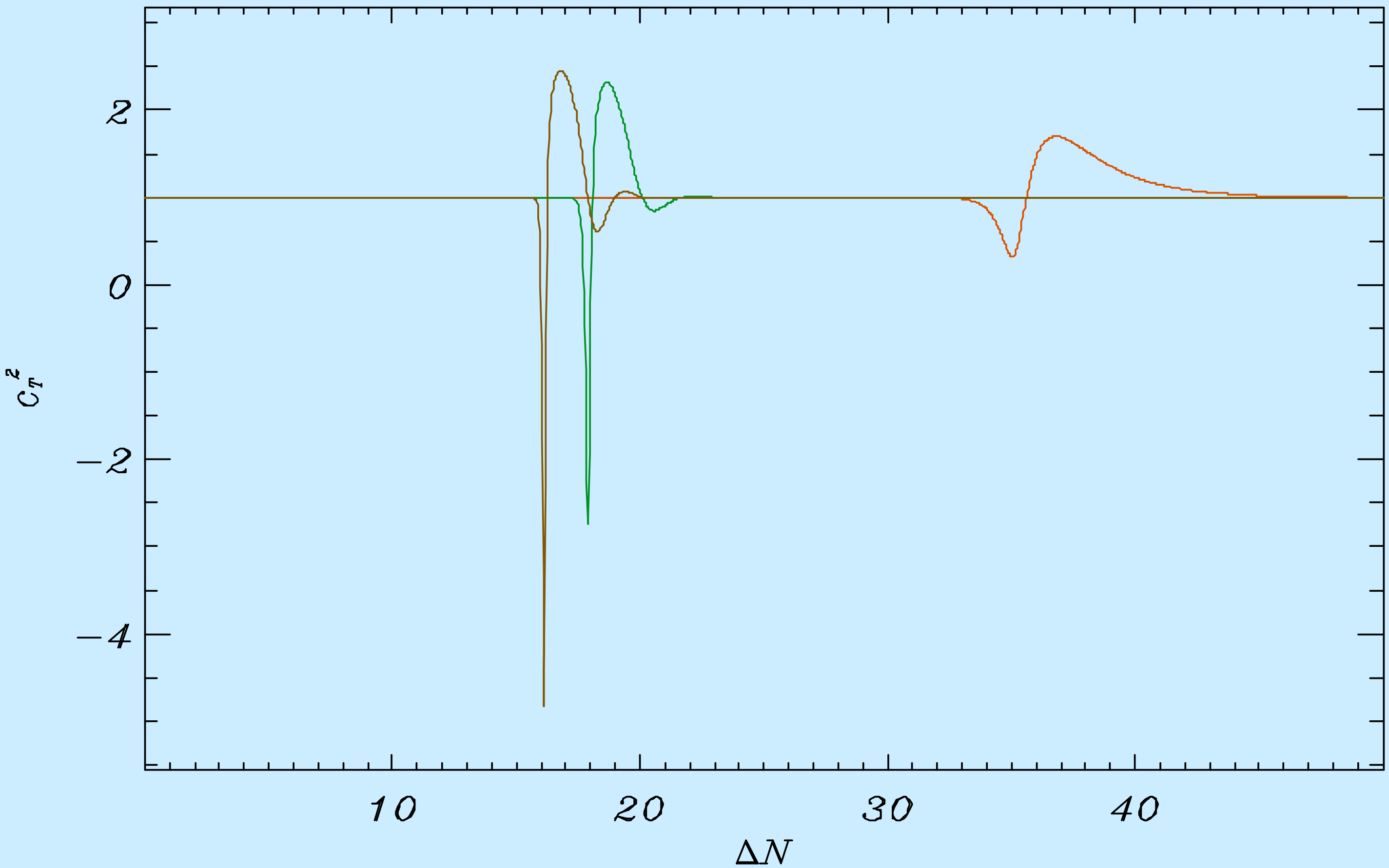
$$c_R^2 = 1 + \frac{\cdots [4\varepsilon(1-\mu) + \ddot{f} - \mu]}{[2\gamma(1-\mu)x^2 + 3\mu^2](1-\mu)} > 0, \quad c_T^2 = \frac{1-\ddot{f}}{1-\dot{f}H} > 0$$



$$f_{,\phi} = f_0\, e^{\alpha\phi} + \ldots, \quad V(\phi) = V_0\, e^{-\beta\phi} + \ldots$$

$\beta = \sqrt{2/3}$ and $\alpha = 12, 8, 3, \sqrt{2/3}$ (left to right)

gation speeds for a tensor modes



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effects of a Gauss-Bonnet term

The growth of matter fluctuations

$$\ddot{\delta} + 2\dot{\delta}H = 4\pi\tilde{G}\rho_m\delta$$

$$\Omega_f \equiv \mu = \dot{\phi}\dot{f}H$$

$\delta \equiv \dfrac{\delta\rho}{\rho}$ **is the matter density contrast**

$$\tilde{G} = G\left[1 + 3\Omega_f - \frac{\dot{\phi}}{H}\left(\frac{\ddot{\phi}}{\dot{\phi}^2} + \frac{f_{\phi\phi}}{f_\phi}\right)\Omega_f\right]$$

$$\frac{d\tilde{G}/dt}{\tilde{G}} < 0.01$$

$$|G_{now} - G_{nucleo}|/G_{now}(t_{now} - t_{nucleo}) < 10^{-12}\,yr^{-1}$$

It may work for some choice like

$$\phi \equiv \frac{\dot{\phi}}{H} \sim O(0.1) \qquad f(\phi) \sim e^{\alpha\phi/m_P} \quad \text{even if } \alpha \sim O(1)$$

Growth of matter perturbations

$$\left(\frac{\dot{\delta}}{\delta}\right)_{GB} \approx \left(\frac{\dot{\delta}}{\delta}\right)\left[1-\left(1+\frac{\dot{H}}{H^{2}}\right)\left(1+\frac{3\Omega_{m}}{4}\right)\Omega_{f}\right]$$

The limit on growth factor

$$\frac{\dot{\delta}}{\delta} \approx 0.51 \pm 0.1$$

implies that

$$|\Omega_{GB}| < 0.2$$

Summary

- Curvature corrections (coupled to a scalar field) easily account for an accelerated universe with quintessence, cosmological constant or phantom equation-of-state without a wrong sign kinetic field. Such terms may trigger the onset of late dark energy domination after a scaling matter era.
- Constraining cosmologies other than Lambda-CDM, using the available data may be difficult but seems promising.
- Gauss-Bonnet cosmologies to be compatible with recent astronomical data, the fraction of energy density associated with the coupled GB term should not exceed 0.2