Constraints on SUSY Hybrid Inflation with Strings

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Introduction

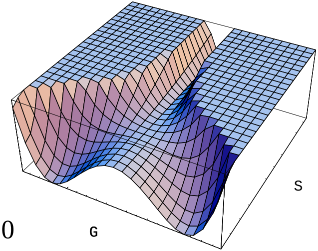
- Presently, CMB data is well fitted by a set of six standard parameters

$$\{\Omega_b h^2, \Omega_c h^2, \tau_R, \theta_A, \log P_{\mathcal{R}}, n_s\}$$

- $P_{\mathcal{R}}$ and n_s are predicted by an inflationary model, *i.e.* $V(\phi)$.
- Models which depend on two parameters are of special interest $\Rightarrow$ no large degeneracies between parameters.
- SUSY hybrid models satisfy this requirement. Furthermore, they are well motivated from particle physics.
- SUSY hybrid models: $n_s \gtrsim 0.98$. WMAP3: $n_s = 0.956 \pm 0.016$.
- Spontaneous symmetry breaking at the end of hybrid inflation $\Rightarrow$ cosmic strings. Need to be taken into account when making predictions for the CMB.

F-Term Inflation

Copeland, Liddle, Lyth, Stewart, Wands (1994); Dvali, Shafi and Schaefer (1994).



- Superpotential: $W = \kappa\widehat{S}(\widehat{\overline{G}}\widehat{G} - M^2)$.
- S: inflaton. G, $\overline{G}$: waterfall fields.
- Tree level potential:
  $$V_0 = \kappa^2\left[\left|\overline{G}G - M^2\right|^2 + \left|S\overline{G}\right|^2 + \left|SG\right|^2\right].$$
- During inflation, $\langle|S|\rangle > M$, $\langle|G|\rangle = \langle|\overline{G}|\rangle = 0$
  $V = \kappa^2 M^4$.
- Two parameters: κ and M.
- We are interested in strings. Assume that G is gauged under a $\mathrm{U}(1)$.

Additional Corrections to the F-Term Potential

- One-loop Coleman-Weinberg correction due to spontaneous SUSY breaking during inflation
  ($\dim G,\ \overline{G} = 1$):

$$V_{\mathrm{CW}} = \frac{\kappa^4}{32\pi^2}\left\{(S^2+M^2)^2\ln\left(1+\frac{M^2}{S^2}\right)+(S^2-M^2)^2\ln\left(1-\frac{M^2}{S^2}\right)+2M^4\ln\frac{\kappa^2 S^2}{Q^2}\right\}.$$

- "Minimal" SUGRA: $V_{\mathrm{SUGRA}} = 32\pi^2\kappa^2 M^4 \frac{S^4}{m_{\mathrm{pl}}^4}$.
  From canonical Kähler potential.

Full Potential

$$V = V_0 + V_{\mathrm{CW}} + V_{\mathrm{SUGRA}}$$

- Everything given in terms of κ and M.

D-Term Inflation Binetruy, Dvali (1996); Halyo(1996).

- Superpotential: $W = \kappa \widehat{S}\widehat{\overline{G}}\widehat{G}$.
- D-term: $D = \frac{g}{2}\left(|G|^2 - |\overline{G}|^2 + m_{\rm FI}^2\right)$, where $m_{\rm FI}$ denotes the Fayet-Iliopoulos mass, g the $\mathrm{U}(1)$ gauge coupling.
- Tree-level potential: $V_0 = \kappa^2\left[|\overline{G}G|^2 + |S\overline{G}|^2 + |SG|^2\right] + \frac{1}{2}D^2$.
- Inflation: $\langle G\rangle = \langle\overline{G}\rangle = 0$, $\langle|S|\rangle > \frac{g^2}{4\kappa^2}m_{\rm FI}^2$, $V = \frac{g^2}{8}m_{\rm FI}^2$.
- Coleman-Weinberg Potential:

$$V_{\rm CW} = \frac{1}{32\pi^2}\left\{(\kappa^2 s^2 + \frac{g^2}{4}m_{\rm FI}^2)^2 \ln\left(1 + \frac{g^2}{4\kappa^2}\frac{m_{\rm FI}^2}{s^2}\right) + (\kappa^2 s^2 - \frac{g^2}{4}m_{\rm FI}^2)^2 \ln\left(1 - \frac{g^2}{4\kappa^2}\frac{m_{\rm FI}^2}{s^2}\right) + \frac{g^4}{8}m_{\rm FI}^4 \ln\frac{\kappa^2 s^2}{Q^2}\right\},$$

  where $s = S\mathrm{e}^{8\pi\frac{S^2}{m_{\rm Pl}^2}}$ within minimal SUGRA.
- Results turn out not to depend on g for $g \lesssim 9\times 10^{-2}$.

Adiabatic Perturbations

- Calculate $P_{\mathcal{R}}$ and n_s at the scale $k = 0.05\mathrm{Mpc}^{-1}$.
- Need to know the value σ_e of the inflaton field at the time when the scale k exits the horizon ($\sigma = \sqrt{2}\mathrm{Re}[S]$).
- Number of e-foldings:

$$N_e \approx 50 + \frac{1}{3}\log\frac{T_R}{10^9\mathrm{GeV}} + \frac{2}{3}\log\frac{V^{1/4}}{10^{15}\mathrm{GeV}}$$

$$N_e = \int_{t_e}^{t_c} dt\, H = \frac{8\pi}{m_{pl}^2}\kappa^2 M^4 \int_{\sigma_c}^{\sigma_e} d\sigma \left(\frac{\partial V}{\partial \sigma}\right)^{-1} .$$

- Amplitude of power spectrum:

$$\sqrt{P_{\mathcal{R}}(k)} = \frac{2^{\frac{7}{2}}\sqrt{\pi}}{\sqrt{3}m_{pl}^3} \frac{V^{\frac{3}{2}}(\sigma)}{\partial V/\partial\sigma}\bigg|_{\sigma=\sigma_e} .$$

- Scalar spectral index: $n_s \approx 1 - 2\frac{m_{pl}^2}{8\pi}\frac{V''}{V}\Big|_{\sigma_e}$.
- Can calculate $\sqrt{P_{\mathcal{R}}(k)}$ and n_s from the model parameters κ and M (or m_{FI}, respectively).

Cosmic String Tension

D-term

$G\mu = 2\pi \left(\frac{m_{\rm FI}}{m_{\rm pl}}\right)^2$ (Bogomol'nyi strings).

F-Term

$G\mu = 2\pi \left(\frac{M}{m_{\rm pl}}\right)^2 \epsilon(\beta) \propto$ amplitude of string-induced spectrum.

$\beta = \kappa^2/(2g^2)$, $G = \frac{1}{m_{\rm Pl}^2}$

g: gauge coupling, 0.7 as suggested by Grand Unification

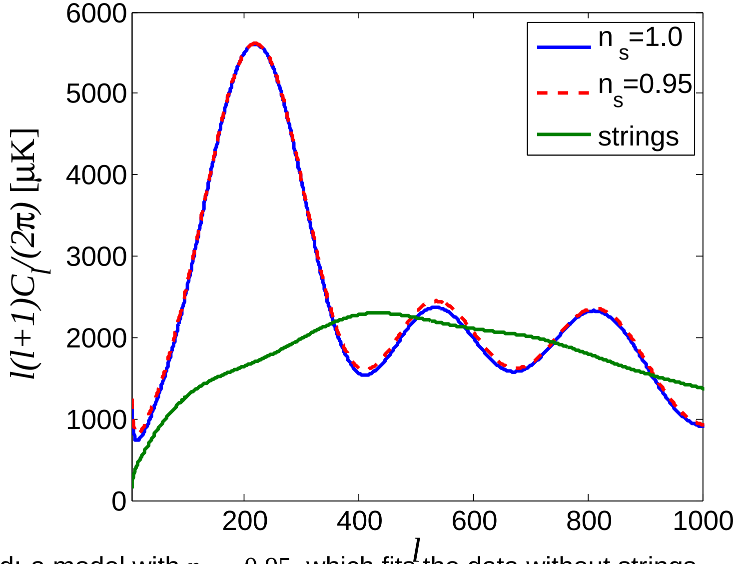
$$\epsilon(\beta) = \begin{cases} 1.04\beta^{0.195} & \text{for} \quad \beta > 10^{-2} \\ 2.4/\log(2/\beta) & \text{for} \quad \beta \leq 10^{-2} \end{cases}$$

Hill, Hodges Turner (1987)

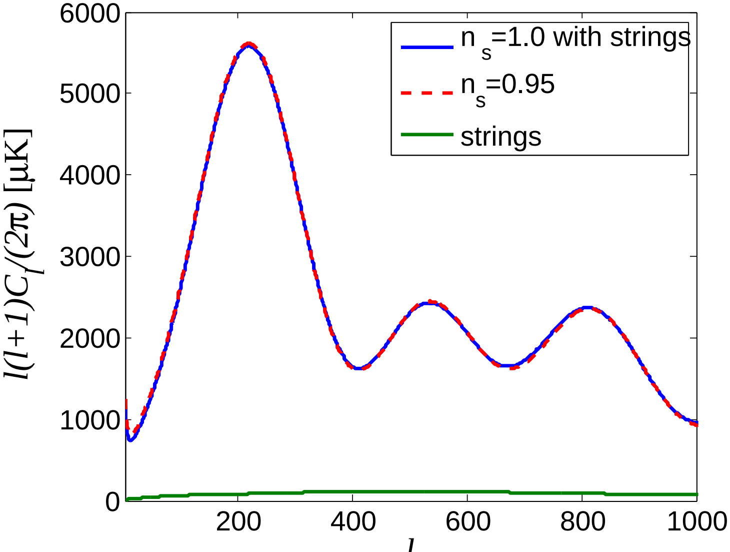
- ϵ smaller than one for the interesting range of parameters. $\Rightarrow$ String constraints more severe in D-Term models.

String Contribution to Perturbation Spectrum

- Strings may give a subdominant contribution ($\lesssim 10\%$) to the power spectrum. Battye, Weller (2000).
- The amplitude of the string spectrum is $\propto G\mu$.
- $G\mu \lesssim 3 \times 10^{-7}$. Wyman, Pogosian, Wasserman (2005, 2006); Fraisse (2005); Seljak, Slosar, McDonald (2006); Bevis, Hindmarsh, Kunz, Urrestilla (2006) .
- Calculate the string-induced spectrum from a moving segement model based on Albrecht, Battye, Robinson (1997); Pogosian, Vachaspati (1999).



- Red: a model with $n_s = 0.95$, which fits the data without strings.
- Blue: a model with $n_s = 1.00$, which fits the data when strings are added (NB: also the additional fit parameters differ).



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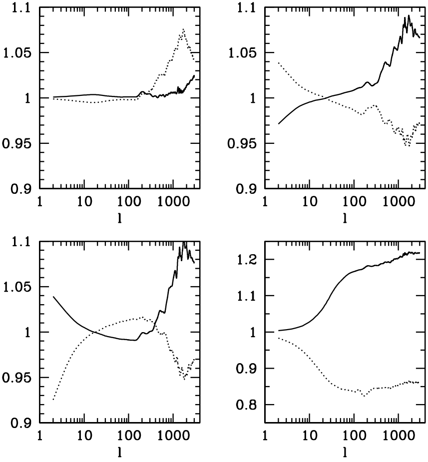
Cosmic Parameters and String Spectrum



Ratio of String spectrum at 3σ upper (solid) and lower (dashed) bound of cosmic parameters to string spectrum for best-fit value.

$\Omega_b h^2 = (0.0244, 0.0196)$ $\Omega_m h^2 = (0.0148, 0.100)$
$h = (0.82, 0.61)$ $\tau_R = (0.172, 0)$

Variation less than 20%

- When the string contribution is $\lesssim 10\%$, the error in the TT spectrum is $\lesssim 2\%$.
- *cf.* 1% accuracy rendered by the CAMB or CMBFAST codes.
- Calculate just one string spectrum (the amplitude of which is $\propto G\mu$).

MCMC Analysis

- Use COSMOMC code Lewis, S. Bridle (2002).
- Parameters $\{\Omega_{\rm b}h^2, \Omega_{\rm c}h^2, \tau_{\rm R}, \theta_{\rm A}, \log P_{\mathcal{R}}, n_{\rm s}, G\mu\}$
- For Determination of Model Parameters, derive $\{\log P_{\mathcal{R}}, n_{\rm s}, G\mu\}$ from $\{M, \log\kappa\}$ for F-term, $\{m_{\rm FI}, \log\kappa\}$ for D-term inflation.
- For comparison, analyze models with $G\mu \neq 0$ and $G\mu = 0$.

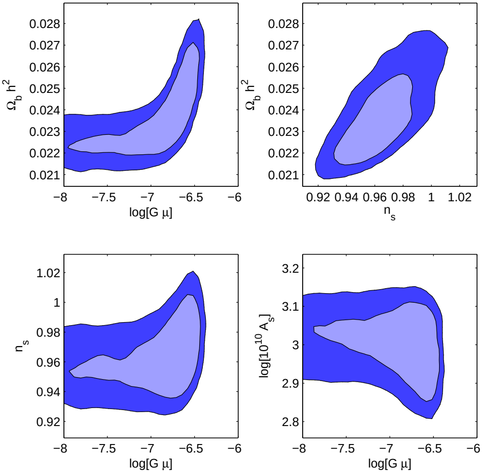
Flat Priors

Parameter	Prior
$\Omega_{\rm b}h^2$	(0.005, 0.1)
$\Omega_{\rm c}h^2$	(0.01, 0.99)
$\theta_{\rm A}$	(0.5, 10)
$\tau_{\rm R}$	(0.01, 0.9)
$\log(10^{10}P_{\mathcal{R}})$	(2.7, 5.0)

Parameter	Prior
n_s	(0.5, 1.5)
$\log\kappa$	(-5.0, -0.3)
$\log(T_{\rm R}/10^9{\rm GeV})$	(-6.0, 1.0)
c_H^2	(-0.25, 0.03)
$\log g$	(-2.0, 0.0)

Strings allow Blue Spectra $\{\Omega_b h^2, \Omega_c h^2, \tau_R, \theta_A, P_{\mathcal{R}}, n_s, G\mu\}$

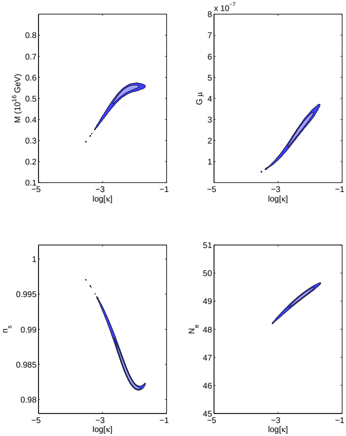
Standard Six Parameter Fit with Strings in Addition



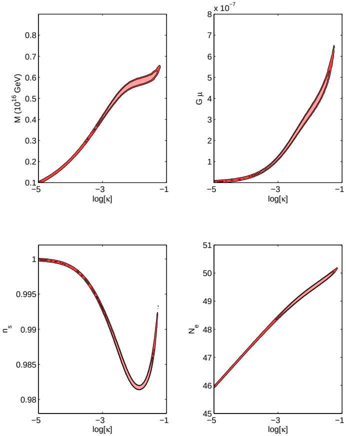
- For large $G\mu$, values as large as $n_s \approx 1.02$ are within 2σ contour.
- Imposing BBN constraints, $\Omega_b h^2 = 0.020 \pm 0.002$, we find $G\mu < 2.2 \times 10^{-7}$ and $n_s = 0.953 \pm 0.015$. (*cf.* Bevis, Hindmarsh, Kunz, Urrestilla (2007))
- Including strings fits the data slightly (but not significantly) better.

F-Term Inflation with Minimal SUGRA

$\{\Omega_\mathrm{b}h^2, \Omega_\mathrm{c}h^2, \tau_\mathrm{R}, \theta_\mathrm{A}, \log\kappa, M\}$, $g = 0.7$, $T_\mathrm{R} = 10^9\mathrm{GeV}$ $G\mu$ derived from κ, M and g.



$\log\kappa = -2.34 \pm 0.38$
$M = (0.518 \pm 0.059) \times 10^{16}\mathrm{GeV}$



Without strings.

Nonminimal SUGRA

A Remedy for the Spectral Index Crisis

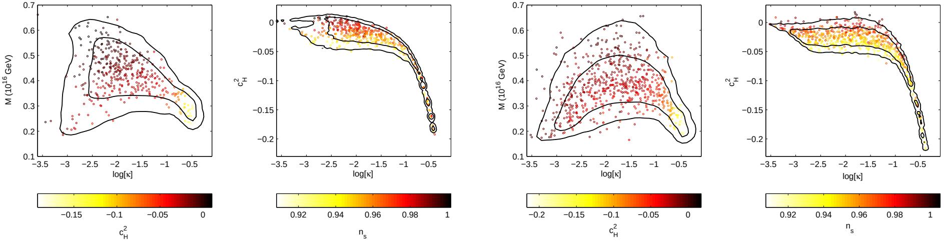
- For a non-canonical Kähler potential, mass-square terms of the order $V/(m_{\rm Pl}^2)$ arise.
- When these mass-squares are negative, they can "redden" the spectrum.

  Bastero-Gil, King, Shafi (2006)
- Parametrise these by c_H^2:

$$V_{\rm NMSUGRA} = c_H^2 H^2 S^2 \,.$$

F-Term Inflation with Nonminimal SUGRA

$\{\Omega_b h^2, \Omega_c h^2, \tau_R, \theta_A, \log\kappa, M, c_H^2\}$, $g = 0.7$, $T_R = 10^9\text{GeV}, a_S = 1\text{TeV}$



Best-Fit Values

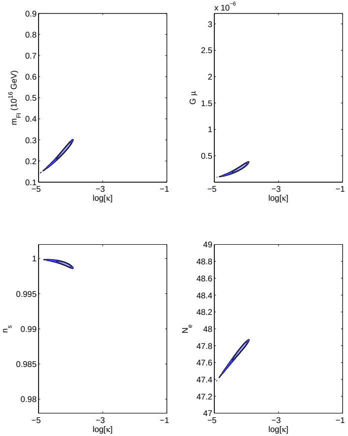
$\log\kappa = -1.87 \pm 0.66$
$M = (0.417 \pm 0.093) \times 10^{16}\text{GeV}$
$c_H^2 = -0.030 \pm 0.035$

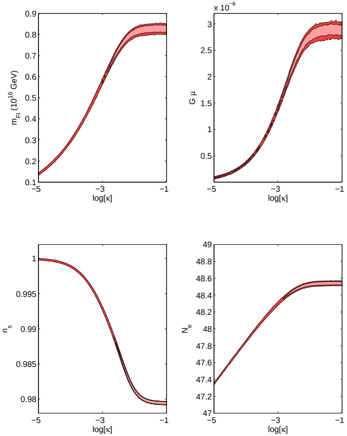
Introducing c_H^2 is yet not necessary in order to resolve the spectral index problem for hybrid inflation with strings.

For Comparison

Without strings.

D-Term Inflation $\{\Omega_b h^2, \Omega_c h^2, \tau_R, \theta_A, \log\kappa, m_{FI}\}\,,\ g = 0.001\,,\ T_R = 10^9\mathrm{GeV}$





$$\log\kappa = -4.24 \pm 0.19$$
$$m_{FI} = (0.24 \pm 0.03) \times 10^{16}\mathrm{GeV}$$

Without strings.

Conclusions

- We have computed up to date precision constraints on the parameters κ and M, m_{FI} of SUSY hybrid inflation models.
- Determination of important GUT parameters from cosmic observations (given the models are correct).
- Crucial is the improved WMAP3 data, allows in particular for strong bounds on n_{s}.
- Puts the simplest SUSY hybrid models without strings under pressure.
- A string contribution of order 10% to the CMBR power spectrum does however allow for larger values of n_{s}.

Conclusions

Model	Parameter Set	# Par.	$\log\kappa$	$(M, m_{\rm FI})/10^{16}\rm GeV$	$-\Delta 2\log\mathcal{L}$
6Par	$\{\Omega_b h^2, \Omega_c h^2, \tau_R, \theta_A, P_{\mathcal{R}}, n_s\}$	6	–	–	0
6Par & $G\mu$	$\{\Omega_b h^2, \Omega_c h^2, \tau_R, \theta_A, P_{\mathcal{R}}, n_s, G\mu\}$	7	–	–	-2.7
F	$\{\Omega_b h^2, \Omega_c h^2, \tau_R, \theta_A, \log\kappa, M\}$	6	-2.34 ± 0.38	0.518 ± 0.059	-2.2
FNoStr	$\{\Omega_b h^2, \Omega_c h^2, \tau_R, \theta_A, \log\kappa, M\}$	6	-2.40 ± 0.88	0.495 ± 0.139	2.9
FNMSUGRA	$\{\Omega_b h^2, \Omega_c h^2, \tau_R, \theta_A, \log\kappa, M, c_H^2\}$	7	-1.87 ± 0.66	0.417 ± 0.093	-2.9
D	$\{\Omega_b h^2, \Omega_c h^2, \tau_R, \theta_A, \log\kappa, m_{\rm FI}\}$	6	-4.24 ± 0.19	0.245 ± 0.031	-0.5
DNoStr	$\{\Omega_b h^2, \Omega_c h^2, \tau_R, \theta_A, \log\kappa, m_{\rm FI}\}$	6	-2.10 ± 0.89	0.730 ± 0.171	2.4