An Overview of AdS/CFT

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The AdS/CFT Correspondence

Strongly interacting QFT in $d$-dimensions

$\leftrightarrow$

General relativity in (at least) $(d+1)$-dimensions

Gravity lives here: $(d+1)$-dimensional bulk

QFT lives here: $d$-dimensional boundary
The extra direction, $r$, should be thought of as *energy scale*. 

- Objects occurring on different scales live in different $r$-slices of bulk 
- AdS/CFT is the geometrization of Wilsonian RG flow.
What is AdS?

- AdS = Anti-de Sitter Space

\[ ds^2 = \frac{L^2}{r^2} (dr^2 + \eta_{\mu\nu} dx^\mu dx^\nu) \]

slice of Minkowski space

horizon at \( r \to \infty \)

UV boundary at \( r = 0 \)
Properties of AdS

\[ ds^2 = \frac{L^2}{r^2} \left( dr^2 + \eta_{\mu\nu} dx^\mu dx^\nu \right) \]

- Solution of Einstein’s equations with *negative* cosmological constant.
- Isometry group is translations + $SO(d,2)$
- Geodesics

lightlike geodesics hit the boundary
timelike geodesics don’t
The Basics of AdS/CFT
Generating Function

\[ Z_{\text{QFT}}[\phi_0] = \int \mathcal{D}A \exp \left( \frac{i}{\hbar} S_{\text{QFT}}[A] + \phi_0 \mathcal{O}(A) \right) \]
Idea: Make the Sources Come Alive

\[ \phi(\vec{x}, r) \rightarrow \phi_0(\vec{x}) \]
How to Calculate: GKPW Formula

(Gubser, Klebanov, Polyakov; Witten)

\[ Z_{QFT}[\phi_0] = Z_{\text{Quantum Grav}}[\phi \rightarrow \phi_0] \]

\[ \approx e^{iS_{\text{gravity}}(\phi)} \bigg|_{\phi \rightarrow \phi_0} \]

when classical gravity is valid

on-shell action for AdS bulk

boundary conditions
AdS/CFT Will Not Solve Your Favourite Theory

- Tricky Part: Find the map

\[ S_{\text{QFT}} \mapsto S_{\text{gravity}} \]

- We have several classes of well explored examples, but no proof.
  - e.g. Maximally supersymmetry Yang-Mills = IIB string theory on \( AdS_5 \times S^5 \)

- Simpler method: take your favourite gravity theory to define the QFT on the boundary

- Classical gravity means large N
  - Matrix large N, not vector large N
Building the Dictionary
<table>
<thead>
<tr>
<th><strong>Bulk</strong></th>
<th><strong>Boundary</strong></th>
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<tr>
<td>Fields, $\phi(\vec{x}, r)$</td>
<td>Operators, $\mathcal{O}(\vec{x})$</td>
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# The Dictionary

## Bulk
- Fields, \( \phi(x, r) \)
- Mass, \( m \)
- Spin, Charge

## Boundary
- Operators, \( \mathcal{O}(\vec{x}) \)
- Dimension, \( \Delta(\Delta - d) = m^2 L^2 \)
- Spin, Charge

### Some special fields

- Metric, \( g_{\mu\nu} \)
- Gauge Fields, \( A_\mu \)
- Stress Tensor, \( T_{\mu\nu} \)
- Conserved Current, \( J_\mu \)
A Scalar Field

- The Lagrangian for a scalar field in AdS is

\[ S_{\text{scalar}} = -\frac{1}{2} \int d^{d+1}x \sqrt{-g} \left( g^{AB} \partial_A \phi \partial_B \phi + m^2 \phi^2 \right) \]

- Special property of AdS: stability requires

\[ m^2 \geq -\frac{d^2}{4L^2} \]  

(Breitenlohner-Freedman bound)

- Relevant boundary operator: \( m^2 < 0 \)
- Marginal boundary operator: \( m^2 = 0 \)
- Irrelevant boundary operator: \( m^2 > 0 \)
Solution near the Boundary

- Solve the equation of motion near $r=0$

\[
\phi(\vec{x}, r) \rightarrow \left(\frac{r}{L}\right)^{\Delta_-} [\phi_0(\vec{x}) + \ldots] + \left(\frac{r}{L}\right)^{\Delta_+} [\phi_1(\vec{x}) + \ldots]
\]

\[
\Delta_\pm = \frac{d}{2} \pm \sqrt{(\frac{d}{2})^2 + m^2L^2}
\]

- $\phi_0(\vec{x})$ is source. (I lied before!)

- What is interpretation of $\phi_1(\vec{x})$
Can use GKPW formula to show that

$$\phi_1(\vec{x}) = \langle \mathcal{O}(\vec{x}) \rangle$$

- i.e. response in presence of source $\phi_0(\vec{x})$
Every Single “Holographic” Calculation Ever...

- Fix the source $\phi_0(\vec{x})$

- Fix another boundary condition in the interior of space
  - regularity
  - ingoing boundary condition at horizon

- Compute the response $\phi_1(\vec{x})$
What AdS/CFT is Good For
Finite Temperature

Boundary theory at Hawking temperature, $T$

Euclidean and Lorentzian signatures
Finite Density

Temperature, $T$
Chemical Potential, $\mu$

Reissner-Nordström Black Hole
Transport

Kovtun, Policastro, Son, Starinets

e.g. shear viscosity

\[ \frac{\eta}{s} = \frac{1}{4\pi} \]
Many Other Phenomena

- Superconductivity
- Non-Fermi Liquids
- Quantum Oscillations
- Quantum Hall Transitions
- Dynamical Lattice Formation
- Band Structure
- ...