

D-branes, Black Holes and $SU(\infty)$ Gauge Theory

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Abstract

We discuss an application of the known in QCD large N expansion to strings and supermembranes in the strong coupling. In particular we use the recently obtained master field describing $SU(\infty)$ gauge theory to argue that quantum extreme black holes obey quantum Boltzmann (infinite) statistics. This supports a topological argument by Strominger that black holes obey infinite statistics. We also speculate on a formulation of X -theory of strings and p -branes as theory of Grothendieck's motives. The partition function is expressed in terms of L -function of a motive. The Beilinson conjectures on the values of L -functions are interpreted as dealing with the cosmological constant problem.

Duality connects weak and strong coupling regimes in string theory [1]. We know how to deal with the weak coupling regime but one needs a nonperturbative method to control the strong coupling regime. Actually one knows the only one systematic nonperturbative method in quantum field theory. This is the large N expansion. In this talk we shall discuss some applications of the large N expansion to theory of superstrings and p -branes.

Recently a remarkable progress towards understanding of the black hole entropy in superstring theory has taken place. Strominger and Vafa [2] used the Dirichlet (D)-brane approach [3] to identify and to count the degenerate quantum states which have the same quantum numbers of certain extreme black holes. They have shown that the growth of the elliptic genus of an appropriate Kahler manifold agrees with the Bekenstein-Hawking entropy formula in the limit of large charges.

Quantum states describing black hole in the D-brane approach consist of a large number N of D-branes. Witten [4] has shown that the effective action of N parallel coincident Dirichlet p -branes is the dimensional reduction of the ten-dimensional $U(N)$ supersymmetric Yang-Mills theory to $p + 1$ -dimensions. Therefore it seems natural to use the known in QCD large N expansion [5, 6, 7] to study quantum black holes. The large N expansion works even in the strong coupling regime therefore in principle one can apply it to large black holes when

there are problems how to deal with strong interactions [8]. For 0-brane case one gets a model of the $SU(N)$ supersymmetric gauge quantum mechanics. In the limit $N \rightarrow \infty$ this model yields the supermembrane action and the gauge group $SU(\infty)$ approaches the group of area-preserving transformations corresponding to a membrane with topology of sphere [9, 10].

Here we shall argue that the Hilbert space for extreme black holes and membranes is the Boltzmannian Fock space defined by the following relations for creation and annihilation operators:

$$a_k a_l^+ = \delta_{kl} \quad (1)$$

(this is q -deformed commutational relation $a_k a_l^+ - q a_l^+ a_k = \delta_{kl}$ for $q = 0$). This follows from the general result obtained in [7] that the colourless sector of $SU(\infty)$ invariant theory in d -dimensional ($d \geq 1$) spacetime is described by states in the Fock space defined by relations (1). For $SU(N)$ gauge field $A_\mu(x)$ in d -dimensional Minkowski space-time one has the following basic relation

$$\lim_{N \rightarrow \infty} \frac{1}{N^{1+\frac{d}{2}}} < 0 | \text{tr} (A_{\mu_1}(x_1) \dots A_{\mu_n}(x_n)) | 0 > = (\Omega_0 | B_{\mu_1}(x_1) \dots B_{\mu_n}(x_n) | \Omega_0)$$

Here the limiting field $B_\mu(x)$, so called master field, is defined as a solution of the Yang-Feldman equation

$$B_\mu(x) = B_\mu^{(in)}(x) + \int D_{\mu\nu}^{ret}(x-y) J_\nu(y) dy, \quad (2)$$

where the free field $B_\mu^{(in)}(x)$ is quantized according to the rule (1). Here $|0 >$ is vacuum in the ordinary Bosonic Fock space and $|\Omega_0 >$ is vacuum in the Boltzmannian Fock space. The master field $B_\mu(x)$ does not have matrix indexes. It satisfies to the ordinary Yang-Mills equations but it is quantized according to the new rule. The gauge group for the gauge master field is an infinite dimensional group of unitary operators in the Boltzmannian Fock space. After the fixing the gauge one gets the BRST symmetry. The gauge group is approached by the group of area-preserving diffeomorphisms of the membrane.

BPS-saturated states in spacetime correspond to BPS-states of the D-brane worldvolume theory. One interprets the BPS-states as a black hole in space-time only for large charges, i.e. in the large N limit. But this limit is described by quantum theory in the Boltzmannian Fock space, as it has been discussed earlier. Thus quantum states of extreme black hole belong to the Boltzmannian Fock space. This supports a topological argument made by Strominger [11] that extreme black holes obey infinite statistics. He argued that black hole exchange is the exchange of two wormhole ends. This is not a diffeomorphism and therefore extremal black hole scattering resembles that of distinguishable particles.

Duality and D-branes led to a new insight into the structure of superstring theory but also pointed up that our current understanding of string theory is

only valid up to some scale. Probably string theory can not be fundamentally defined as a theory of extended objects of any kind - including strings [12]. It was suggested [13, 14] that theory of Grothendieck's motives [16, 17, 18] can be useful in attempts to find a fundamental "X-theory" of strings and p-branes. Motives are defined by algebraic correspondences modulo homological equivalence. Motivic cohomology is a kind of universal cohomology theory for algebraic varieties. Realizations of a motive M over the field of rational numbers Q are linear spaces over Q and over the field of l -adic numbers Q_l . If X is a smooth projective algebraic variety over Q then the realization of its motive is given by the Betti $H_B(X)$, De Rham $H_{DR}(X)$ and l -adic cohomology $H_l(X)$ of X . Category of motives is isomorphic to category of finite dimensional representations of proalgebraic motivic Galois-Serre group G . In the "X-theory" the group G plays the role of conformal group in the ordinary string theory. Motivic X-theory deals with algebraic varieties over the field of rational numbers Q , so the theory is background independent and it is not based on a spacetime continuum. The partition function in X-theory is given by L -function of a motive. L -function of a motive M is defined by the Euler product: $L(M, s) = \prod_p \det(1 - p^{-s} \text{Frob}_p | H_l(M)^{I_p})^{-1}$, where Frob_p is a Frobenius element in G_{Q_p} and I_p is the inertia group in G_{Q_p} . One assumes the existence of the meromorphic continuation and functional equation for $L(M, s)$.

String partition function can be expressed as inverse to the Mellin transform of L -function of a motive. Motive M of bosonic strings has been constructed by Deligne and it was used in the proof of the Ramanujan conjecture: $|\tau(p)| \leq 2p^{11/2}$ [17, 19]. The Ramanujan function $\tau(n)$ is defined by the relation $\Delta(q) = q \prod_n (1 - q^n)^{24} = \sum_n \tau(n) q^n$. One has: $\Delta(q)^{-1} = \text{tr} q^H$ where $H = L_0 - 1/24$ and L_0 is the Virasoro operator. L -function of the motive M is the Dirichlet series:

$$L(M, s) = \sum_n \tau(n) n^{-s} = \prod_p (1 - \tau(p) p^{-s} - p^{11-2s})^{-1}, \quad (3)$$

It is amusing that the motive M is eleven-dimensional. Theory of motives shows geometrical aspects of modular functions widely used in string theory [20, 21, 22]

To understand the vanishing of the cosmological constant we have to consider values of partition function. In X-theory this means the investigation of values of L -functions of motives. Therefore the mystery of the cosmological constant is ultimately reduced to the Beilinson conjectures [18] on values of L -functions.

For a further discussion of arithmetical physics see [23, 24, 25, 26, 27, 28]

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