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Dp-brane thermodynamics from super Yang-Mills in the 't Hooft limit

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Abstract:
Thermal maximally supersymmetric $SU(N_c)$ Yang-Mills (SYM) in $(1+p)$-dimensions for $0 \leq p \leq 3$, taken in the large N_c 't Hooft limit, is conjectured by Maldacena duality to be dual to Dp-branes at finite temperature in the decoupling limit. These may be described semiclassically by certain supergravity black brane solutions, whose properties then make predictions for the N_c and temperature, T, dependence of certain SYM quantities. Specifically the free energy is predicted to go as $\sim N_c^2 T^{\frac{2(7-p)}{(5-p)}}$ and the thermal expectation value of the scalars to go as $\sim T^{\frac{2}{5-p}}$. Following the arguments of Horowitz and Martinec in the context of Matrix theory black holes, and more recently Smilga, in the D0-brane context, we consider the effective canonical partition function for the low energy moduli of the theory. Simple estimates then predict precisely the correct N_c and T dependence of the free energy and scalar expectation values.

Contents

1. Introduction

Maldacena duality has enormous potential to allow us to understand the most basic questions about quantum gravity. Once one can understand the recovery of the graviton in a well defined theory of quantum gravity, an important

The black brane solution in supergravity is,

$$\begin{aligned} ds^2 &= \alpha' \left(\frac{U^{\frac{7}{2}}}{2\pi\sqrt{b\lambda}}(-f dt^2) + 2\pi\sqrt{b\lambda} \left(U^{-\frac{7}{2}} \frac{dU^2}{f} + U^{-\frac{3}{2}} d\Omega^2 \right) \right) \\ f(U) &= 1 - \frac{U_0}{U} \end{aligned} \tag{1.1}$$

where the radial coordinate U is taken to correspond to the expectation value of the scalars in the SYM. The horizon is at $U = U_0$, implying the thermal scale for the scalars is U_0 in the SYM. Supergravity then predicts the low temperature thermodynamic behaviour,

$$\frac{E}{\lambda^{3-p}} = c\, N^2 \left(\frac{T}{\lambda^{3-p}} \right)^{\frac{2(7-p)}{5-p}} \qquad c = \left(\frac{2^{21} 3^{12} 5^2}{7^{19}} \pi^{14} \right)^{1/5} \simeq 7.41. \tag{1.2}$$

for energy density E and temperature T, valid for $T/\lambda^{3-p} \ll 1$ but finite in the large N limit, and the relation,

$$U_0^{7-p} = c\, \frac{E\lambda^4}{N^2} \qquad c = \left(\frac{2^{21} 3^{12} 5^2}{7^{19}} \pi^{14} \right)^{1/5} \simeq 7.41. \tag{1.3}$$

Work of Smilga [**?**] provides an argument for the origin of this power law dependence on t.

The Horowitz-Polchinski correspondence point [?] is expected at temperatures of order $t \sim 1$ and a supergravity description breaks down due to curvature corrections near the horizon. In the ultra low temperature limit where we scale T with N, then for $t \ll 1/N^{-10/21}$ the dilaton is large at the horizon and string quantum corrections become important so that supergravity again breaks down, and one must move to an M-theory description. We emphasise here that we are interested in the usual 't Hooft scaling so that $t \sim O(1)$ limit as $N \to \infty$ where supergravity is valid (for $t \ll 1$), rather than the M-theory limit.

2. Low energy effective thermal action for moduli

We begin with maximally supersymmetry $U(N_c)$ Yang-Mills theory. Since we will be interested in finite temperature T, we use the Euclidean time formalism, with τ the Euclidean time with period $\beta = 1/T$. Then bosonic part of the action is given as,

$$\mathcal{S}_B = \frac{1}{g_{YM}^2} \int_0^\beta d\tau \int_0^\infty dx^p \, \mathrm{Tr} \left[F_{\mu\nu}^2 + D_\mu \Phi^i D_\mu \Phi^i + \left[\Phi^i, \Phi^j\right]^2 \right] \tag{2.1}$$

and the fermionic action is,

$$\mathcal{S}_F = \frac{1}{g_{YM}^2} \int_0^\beta d\tau \int_0^\infty dx^p \, \mathrm{Tr} \, \Psi \left(\gamma^\mu D_\mu\right) \Psi \tag{2.2}$$

where Φ^i for $i = 1, \ldots, (9-p)$ are scalars and ψ is a Majorana-Weyl that both transform in the adjoint. As usual, $F_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$, and $D_\mu = \partial_\mu + i[A_\mu, \cdot]$, and A_μ is the vector potential transforming in the adjoint. These adjoint fields are represented as $N_c \times N_c$ Hermitean matrices.

A set of classical vacua of this theory are gauge equivalent to the configurations,

$$A_{ab}^\tau = a\delta_{ab} \;, \quad \Phi_{ab}^i = \phi_a^i \delta_{ab} \tag{2.3}$$

for constants ϕ_a^i and P, with all other fields being zero, and $a, b = 1, \ldots, N_c$ are the colour indices of the adjoint Hermitean matrices. Such a configuration is a classical vacuum that breaks the $U(N_c)$ symmetry. In the dual Dp-brane interpretation diagonal components of the matrices Φ^i, the ϕ_a^i, are thought to represent the transverse displacement of the N_c branes. The constant a describes the holonomy of the gauge field about the Euclidean time circle,

$$U = P e^{i \int_0^\beta d\tau A^\tau} = e^{i\beta a} \tag{2.4}$$

and hence $a \sim a + 2\pi/\beta$. Note that since the p spatial directions are non-compact, there are no gauge moduli associated to them. We may promote these constants to slowly varying moduli fields, $a(\tau, x)$ and $\phi^i(\tau, x)$, and then consider the thermal field theory of these moduli degrees of freedom.

The same classical vacua exist in the non-supersymmetric (or quenched) theory without fermions, or in the supersymmetric case with fermions. In the non-supersymmetric case thermal quantum corrections generate a potential on this moduli space that dominates all other dynamics. In the maximally supersymmetric case of interest here, this thermal potential (and in fact also the correction to the classical kinetic term) while generated, interestingly does not dominate the dynamics, and other IR effects are of greater importance, leading to the non-trivial scaling of free energy and scalar vevs that are predicted from supergravity. Then one finds that there are regimes in moduli space - were the brane separations are all large - where this moduli theory is weakly coupled and can be a good description, and furthermore one can compute the leading corrections to it in a controlled loop expansion which in the quantum mechanics case $p = 0$ is the Born-Oppenheimer approximation.

In the appendix of this paper we use such a loop expansion to determine the effective action of these moduli as,

$$\mathcal{S}_{moduli} = \mathcal{S}_{classical} + \mathcal{S}_{1-loop} + \mathcal{S}_{2-loop} + \dots \tag{2.5}$$

where the classical action is,

$$\mathcal{S}_{classical} = \frac{N}{\lambda} \int_0^\beta d\tau \int_0^\infty dx^p \sum_a \partial_\mu \phi^i_a \partial_\mu \phi^i_a \tag{2.6}$$

and we note that there is no classical kinetic term for the holomony field $a(\tau, x)$. The 1-loop correction computed in the appendix, depends only on the differences $\Delta^i_{ab}(\tau, x) \equiv \phi^i_a - \phi^i_b$ between moduli fields, due to the translation invariance of the brane, and can be organised into a derivative expansion as,

$$\mathcal{S}_{1-loop} = \int_0^\beta d\tau \int_0^\infty dx^p \sum_{a<b} \left(L'_4 + \frac{1}{\beta} e^{-\beta|\Delta_{ab}|} \cos \frac{2\pi a}{R} \left(L_0 + L_2 + L_4 \right) + \dots \right) \tag{2.7}$$

where the zeroth order and second order derivative terms are,

$$\begin{aligned} L_0 &= \left(\frac{\beta}{|\Delta_{ab}|} \right)^{-\frac{p}{2}} \left(c_1 + O\left(\frac{1}{|\Delta_{ab}|^2} \right) \right) \\ L_2 &= \partial_\mu \Delta^i_{ab} \partial_\mu \Delta^i_{ab} \left(\frac{\beta}{|\Delta_{ab}|} \right)^{2-\frac{p}{2}} \left(c_2 + O\left(\frac{1}{|\Delta_{ab}|^2} \right) \right) \\ &\quad + \left(\Delta^i_{ab} \Delta^j_{ab} \partial_\mu \Delta^i_{ab} \partial_\mu \Delta^j_{ab} \right) \left(\frac{\beta}{|\Delta_{ab}|} \right)^{3-\frac{p}{2}} \left(c_3 + O\left(\frac{1}{|\Delta_{ab}|^2} \right) \right) \end{aligned} \tag{2.8}$$

and the fourth derivative terms are,

$$L'_4 = \frac{1}{(4\pi)^{p/2}} \Gamma(\frac{7-p}{2}) \frac{\left(\partial_\mu \Delta^i_{ab} \partial_\mu \Delta^i_{ab} \right)^2}{|\Delta^i_{ab}|^{7-p}} \left(1 + O\left(\frac{1}{|\Delta_{ab}|^2} \right) \right) \tag{2.9}$$

and

$$L_4 = (\partial_\mu \Delta^i_{ab} \partial_\mu \Delta^i_{ab})^2 \left(\frac{\beta}{|\Delta_{ab}|}\right)^{4-\frac{p}{2}} \left(c_4 + O\left(\frac{1}{|\Delta_{ab}|^2}\right)\right)$$
$$+ \left(\partial_\nu \Delta^k_{ab} \partial_\nu \Delta^k_{ab}\right) \left(\Delta^i_{ab} \Delta^j_{ab} \partial_\mu \Delta^i_{ab} \partial_\mu \Delta^j_{ab}\right) \left(\frac{\beta}{|\Delta_{ab}|}\right)^{5-\frac{p}{2}} \left(c_5 + O\left(\frac{1}{|\Delta_{ab}|^2}\right)\right)$$
$$+ \left(\Delta^i_{ab} \Delta^j_{ab} \partial_\mu \Delta^i_{ab} \partial_\mu \Delta^j_{ab}\right)^2 \left(\frac{\beta}{|\Delta_{ab}|}\right)^{6-\frac{p}{2}} \left(c_5 + O\left(\frac{1}{|\Delta_{ab}|^2}\right)\right) \tag{2.10}$$

and the dots ... represent terms of higher derivative order and $|\Delta_{ab}| = \sqrt{\Delta^i_{ab}\Delta^i_{ab}}$. This calculation is reliable when the massive degrees of freedom that are integrated out are weakly coupled, and this occurs when the moduli are well separated, so,

$$1 \ll \beta |\Delta^i_{ab}| \tag{2.11}$$

and hence there terms of order $O(1/|\Delta_{ab}|^2)$ are subdominant in the above expressions.

We emphasise that the term L'_4 is qualitatively different from the terms L_0, L_2, L_4 in that the latter all arise from thermal corrections and are subsequently suppressed by a factor $e^{-\beta|\Delta_{ab}|}$, whereas the term L'_4 arises already in the zero temperature theory. Indeed we shall see that these thermal corrections are actually dominated by this zero temperature term in the naive scaling limit we now discuss.

3. Estimates for free energy and scalar vevs

We now use similar estimates to those of Horowitz and Martinec in order to deduce the N_c and T scaling of the quantities of interest. Having computed the effective action in a regime where is applies, we now argue that the theory actually receives its dominate contribution from the region where this loop expansion becomes strongly coupled, and hence certain 1-loop terms are of the same scale as the leading classical kinetic terms.

Firstly we estimate all the ϕ^i_a to be of the same order which we write ϕ. Furthermore we estimate that the differences between these moduli are also of the same scale, so that,

$$\Delta^i_{ab} \sim \phi \tag{3.1}$$

Then given that the only explicit dimensional scale is temperature, it is natural to assume,

$$\partial_\mu \phi^i_a \sim \partial_\mu \Delta^i_{ab} \sim \frac{1}{\beta}\phi \tag{3.2}$$

We now equate the leading classical term with the 1-loop term L'_4, and later argue this dominates the other thermal 1-loop terms with the same or less derivatives. The classical term is estimated as,

$$\frac{N}{\lambda}\int_0^\beta d\tau \int_0^\infty dx^p \sum_a \partial_\mu \phi^i_a \partial_\mu \phi^i_a \sim \frac{N}{\lambda}\int_0^\infty dx^p \, \frac{N}{\beta}\phi^2 \tag{3.3}$$

where we use $\int d\tau \sim \beta$, $\sum_a \sim N$ and $\partial_\mu \phi^i_a \sim \phi/\beta$. We estimate the 1-loop L'_4 term as,

$$\int_0^\beta d\tau \int_0^\infty dx^p \sum_{a<b} \frac{1}{(4\pi)^{p/2}} \Gamma(\frac{7-p}{2}) \frac{\left(\partial_\mu \Delta^i_{ab} \partial_\mu \Delta^i_{ab}\right)^2}{|\Delta^i_{ab}|^{7-p}} \sim \int_0^\infty dx^p \, \frac{N^2}{\beta^3} \frac{1}{\phi^{3-p}} \tag{3.4}$$

where we have taken, $\int d\tau \sim \beta$, $\sum_{a<b} \sim N^2$ and $\partial_\mu \phi^i_a \sim \phi/\beta$.

Now equating the classical and 1-loop term yields the estimate,

$$\frac{N^2}{\beta} \phi^2 \sim \frac{N^2}{\beta^3} \frac{1}{\phi^{3-p}} \tag{3.5}$$

and so,

$$\phi \sim \beta^{-\frac{2}{5-p}} \tag{3.6}$$

This scaling implies that,

$$\beta |\Delta^i_{ab}| \sim \beta \phi \sim \beta^{\frac{3-p}{5-p}} \tag{3.7}$$

Then the classical and 1-loop action for $\mathcal{L}'_4$ are estimated as,

$$S_{classical}, S_{\mathcal{L}'_4} \sim \frac{N}{\lambda} \int dx^p \, \frac{N}{\beta} \phi^2 \sim \int dx^p N^2 \beta^{-\frac{9-p}{5-p}} \tag{3.8}$$

The remaining 1-loop terms we estimate as,

$$\begin{aligned} \int_0^\beta d\tau \int_0^\infty dx^p \sum_{a<b} \Big(\frac{1}{\beta} e^{-\beta|\Delta_{ab}|} \cos \frac{2\pi a}{R} \left(L_0 + L_2 + L_4\right) + \ldots \Big) &\sim \int dx^p e^{-\beta\phi} \left(\frac{\phi}{\beta}\right)^{\frac{p}{2}} \left(1 + O(\beta\phi)\right) \\ &\sim \int dx^p e^{-\beta^{\frac{3-p}{5-p}}} \beta^{-\frac{7-p}{5-p}} \end{aligned} \tag{3.9}$$

Hence we may consistently ignore these over the classical and $\mathcal{L}'_4$ term provided that $1 \ll \beta$, ie. we are in the low temperature limit. Now the free energy, F, is given as $\beta F =< S >$, and hence we may estimate the free energy density f as,

$$\beta f \sim N^2 \beta^{-\frac{9-p}{5-p}} \tag{3.10}$$

and thus,

$$f \sim N^2 T^{-\frac{2(7-p)}{5-p}} \tag{3.11}$$

This is precisely the parametric dependence predicted by the supergravity. Note that the free energy density and energy density ϵ should have the same parametric scaling so that,

$$\epsilon \sim N^2 T^{-\frac{2(7-p)}{5-p}} \tag{3.12}$$

The thermal vev of the scalars, U, is measured as,

$$U = \sqrt{< \frac{1}{N}\mathrm{Tr}\Phi^i\Phi^i >} \sim \sqrt{< \frac{1}{N}\sum_a \phi_a^i\phi_a^i >} \sim \phi \tag{3.13}$$

Then, we find,

$$U \sim \phi \sim \beta^{-\frac{2}{5-p}} \sim \left(\frac{\epsilon}{N^2}\right)^{7-p} \tag{3.14}$$

which is precisely the relation predicted by the supergravity.

4. Discussion

Acknowledgements

A. Canonical partition function 1-loop effective action